

# Data Structures and Algorithms for Engineers

## Module 1: Introduction

Lecture 1: Levels of abstraction. The software development life cycle.  
Formalisms for representing algorithms.

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Muḥammad ibn Mūsā al-Khwārizmī

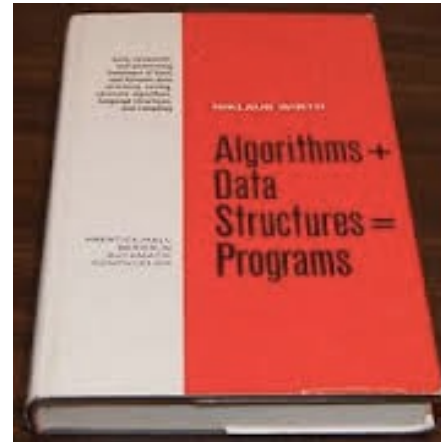
محمد بن موسى الخوارزمي

Born approximately 780, died between 835 and 850  
Persian mathematician and astronomer  
from the Khorasan province of present-day Uzbekistan

The word **algorithm** is derived from his name



# Algorithms + Data Structures = Programs



## Niklaus Wirth, 1976

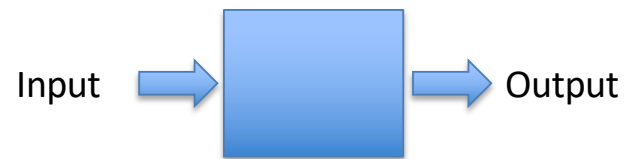
Inventor of Pascal and Modula programming languages  
Winner of Turing Award 1984



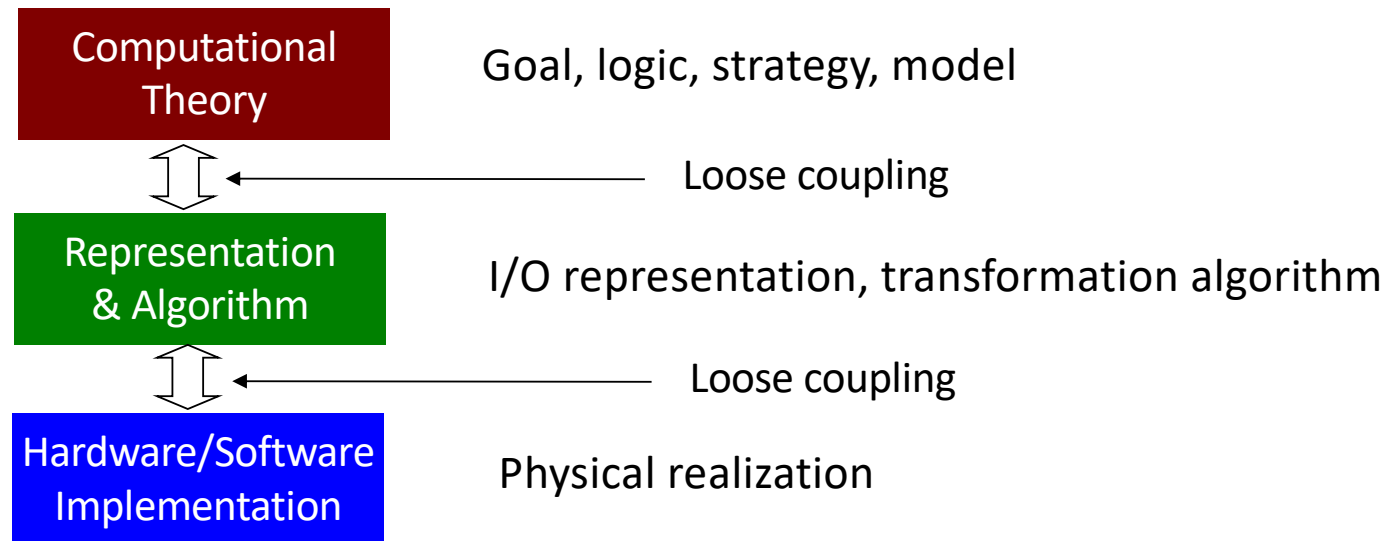
1969



**Information Processing:  
Representation & Transformation**



# Marr's Hierarchy of Abstraction / Levels of Understanding Framework



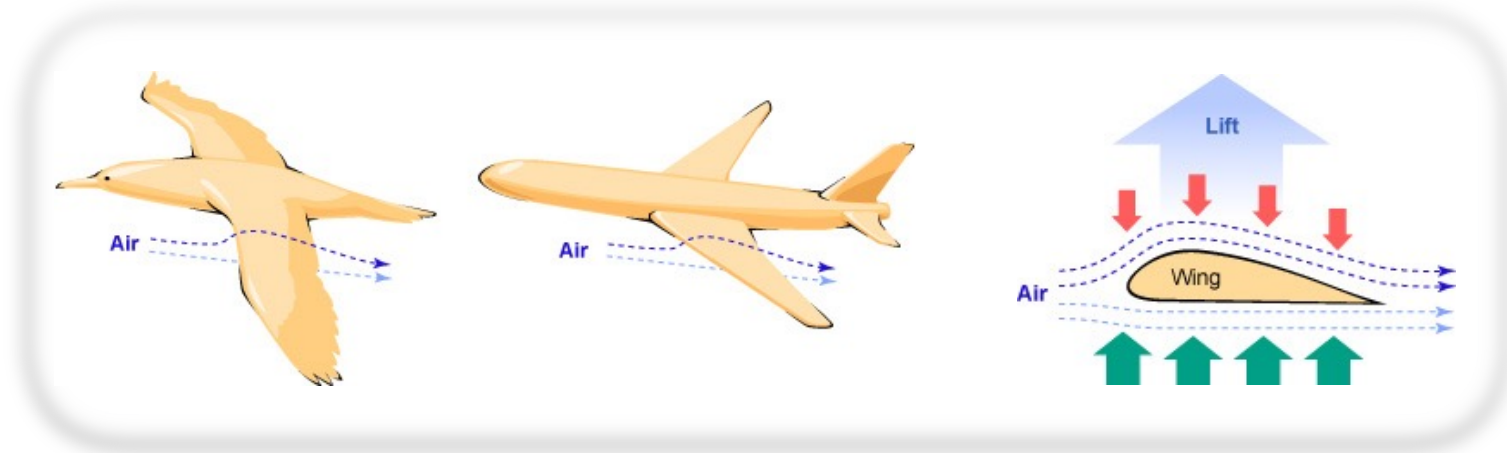
D. Marr and T. Poggio. "From understanding computation to understanding neural circuitry", in E. Poppel, R. Held, and J. E. Dowling, editors, *Neuronal Mechanisms in Visual Perception*, volume 15 of *Neurosciences Research Program Bulletin*, pages 470–488. 1977.  
D. Marr. *Vision*. Freeman, San Francisco, 1982.  
T. Poggio. The levels of understanding framework, revised. *Perception*, 41:1017–1023, 2012.

## Marr's Hierarchy of Abstraction / Levels of Understanding Framework

“Trying to understand perception by studying only neurons is like trying to understand bird flight by studying only feathers: it just cannot be done.

In order to understand bird flight, we have to understand aerodynamics; only then do the structure of feathers and the different shapes of birds' wings make sense”

Marr, D. *Vision*, Freeman, 1982.



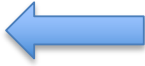
Computational  
Theory



Representation  
& Algorithm



Hardware/Software  
Implementation



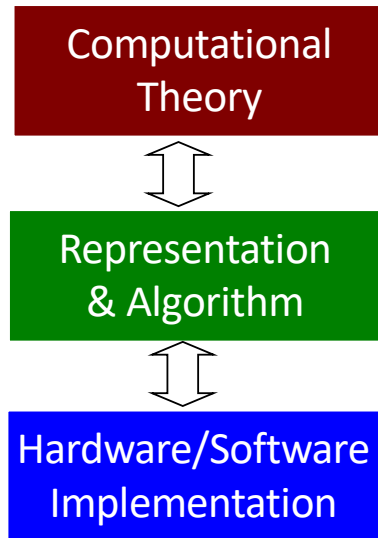
## Sorting a List

Given a sequence of  $n$  keys  $a_1, \dots, a_n$

Find the permutation (reordering)  
such that  $a_i \leq a_j$

$1 \leq i, j \leq n$

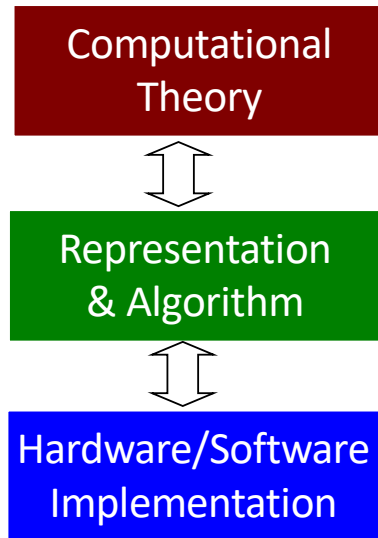




Sorting a List

- Bubble Sort
- Insertion Sort
- Quick Sort
- Merge Sort, ...

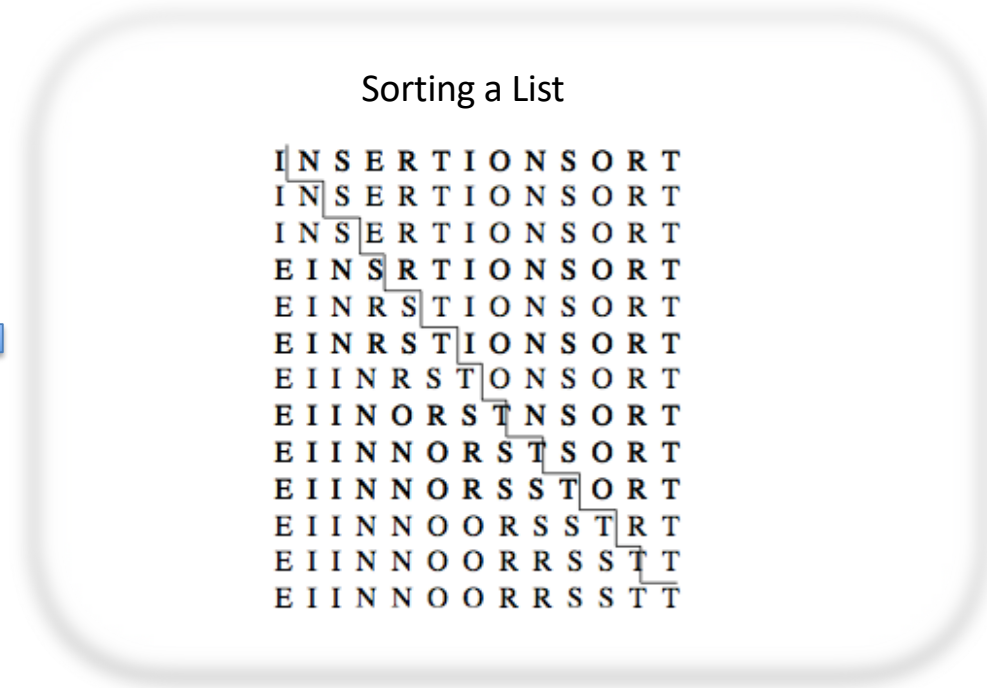
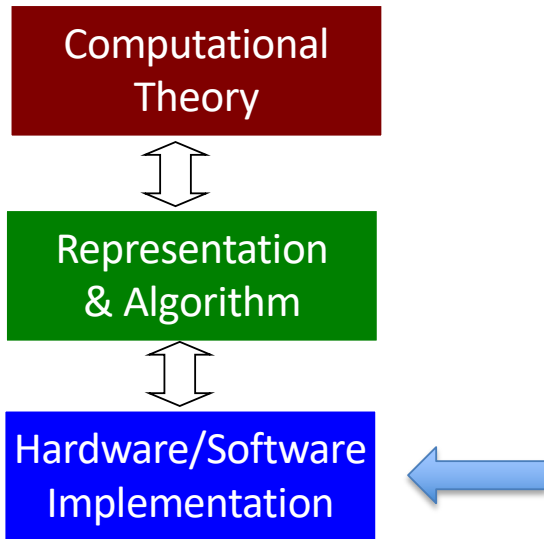
Key point: different computational complexity



## Sorting a List

```
insertion_sort(item s[], int n)
{
    int i,j;                /* counters */

    for (i=1; i<n; i++) {
        j=i;
        while ((j>0) && (s[j] < s[j-1])) {
            swap(&s[j],&s[j-1]);
            j = j-1;
        }
    }
}
```



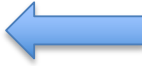
Computational Theory



Representation & Algorithm



Hardware/Software Implementation



### Fourier Transform

$$\begin{aligned}\mathcal{F}(f(x, y)) &= F(\omega_x, \omega_y) \\ &= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x, y) e^{-i(\omega_x x + \omega_y y)} dx dy\end{aligned}$$

$$\begin{aligned}\mathcal{F}(f(x, y)) &= F(\omega_x, \omega_y) \\ &= F(\omega_x \Delta\omega_x, \omega_y \Delta\omega_y) \\ &= \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x, y) e^{-i(\frac{\omega_x x}{M} + \frac{\omega_y y}{N})}\end{aligned}$$

Computational Theory



Representation & Algorithm



Hardware/Software Implementation



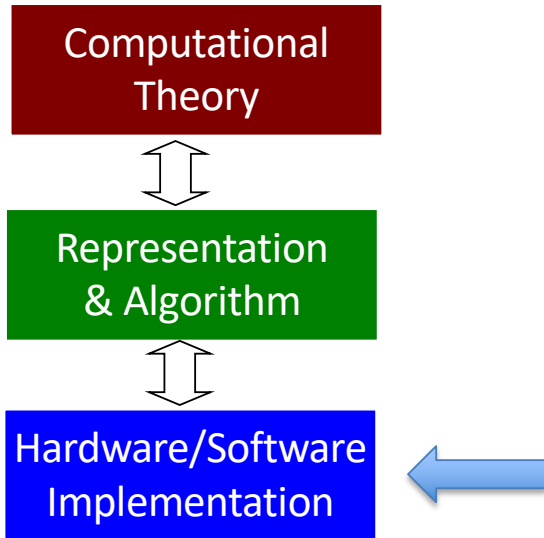
## Fourier Transform

DFT: Discrete Fourier Transform

FFT: Fast Fourier Transform

FFTW: Fastest Fourier Transform in the West

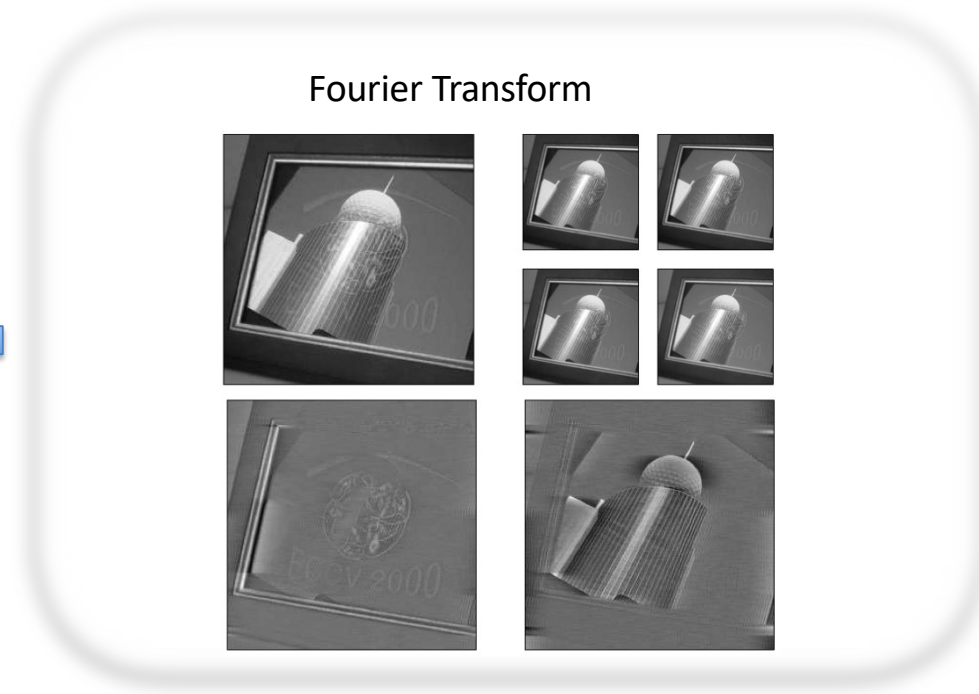
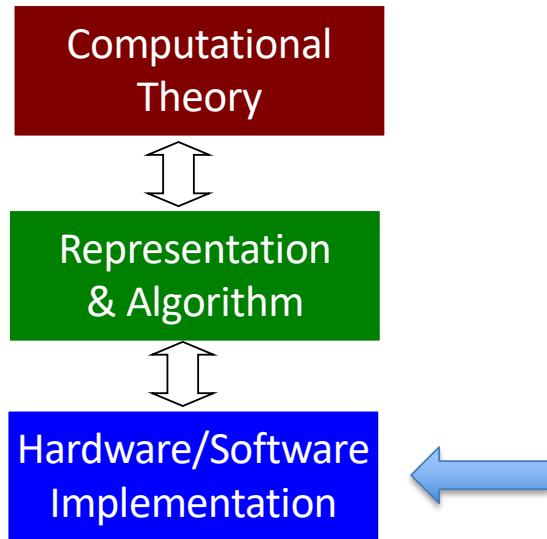
**Key point: different computational complexity**



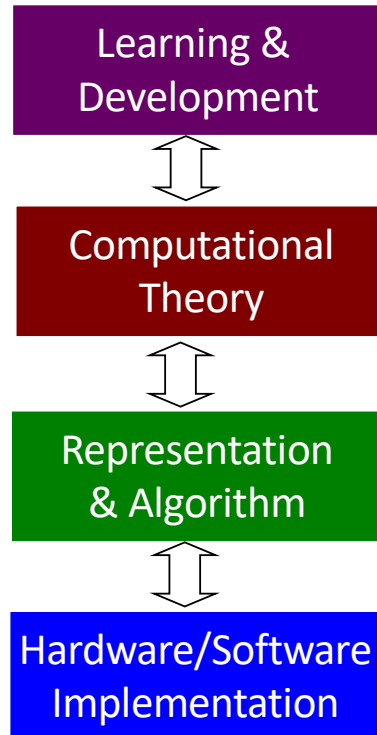
## Fourier Transform

```
main()
{
    unsigned long i;
    int isign;
    float *data1,*data2,*fft1,*fft2;

    data1=vector(1,N);
    data2=vector(1,N);
    fft1=vector(1,N2);
    fft2=vector(1,N2);
    for (i=1;i<=N;i++) {
        data1[i]=floor(0.5*cos(i*2.0*PI/PER));
        data2[i]=floor(0.5*sin(i*2.0*PI/PER));
    }
    twofft(data1,data2,fft1,fft2,N);
    printf("Fourier transform of first function:\n");
    prntft(fft1,N);
    printf("Fourier transform of second function:\n");
    prntft(fft2,N);
    /* Invert transform */
    isign = -1;
    four1(fft1,N,isign);
    printf("inverted transform = first function:\n");
    prntft(fft1,N);
    four1(fft2,N,isign);
    printf("inverted transform = second function:\n");
    prntft(fft2,N);
    free_vector(fft2,1,N2);
    free_vector(fft1,1,N2);
    free_vector(data2,1,N);
    free_vector(data1,1,N);
    return 0;
}
```



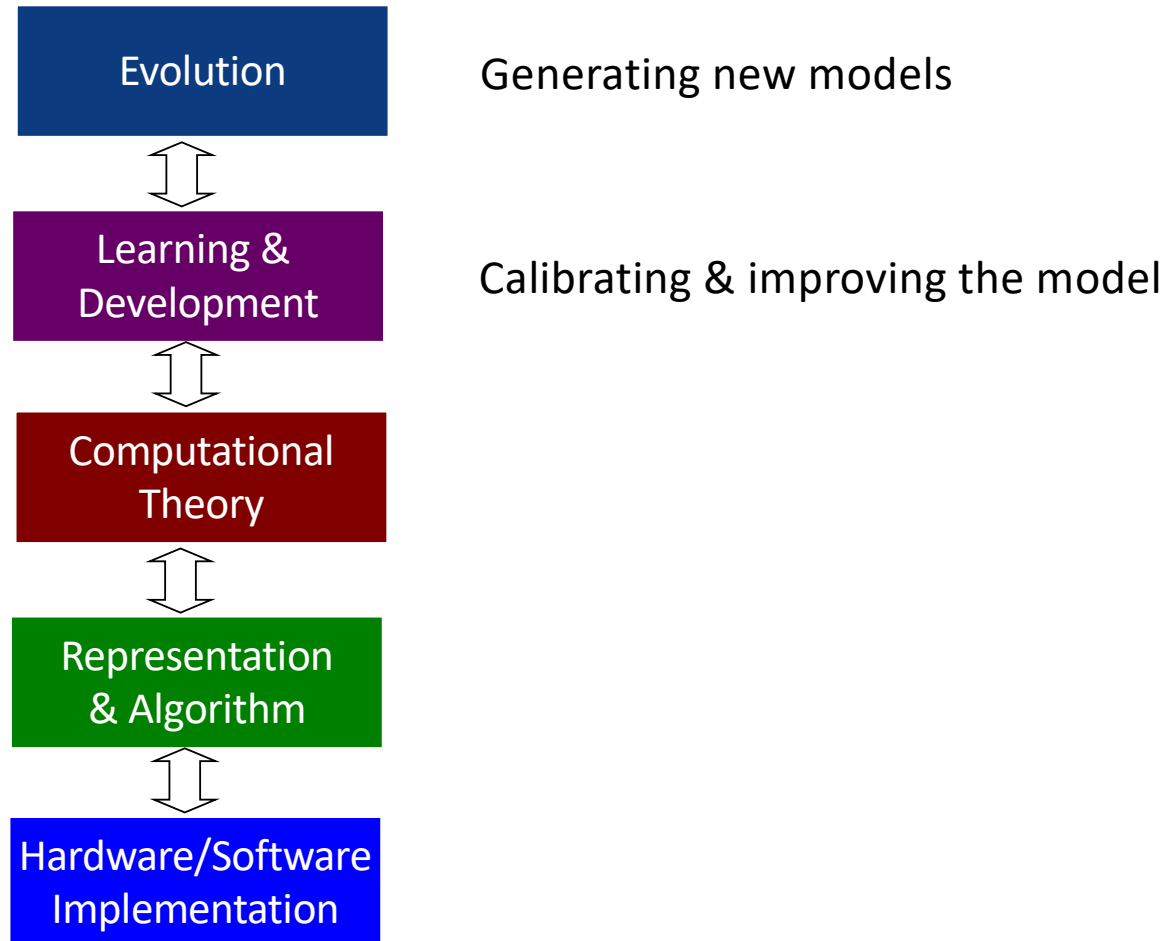
# Marr's Levels of Understanding Framework updated 2012 by T. Poggio



Calibrating & improving the model

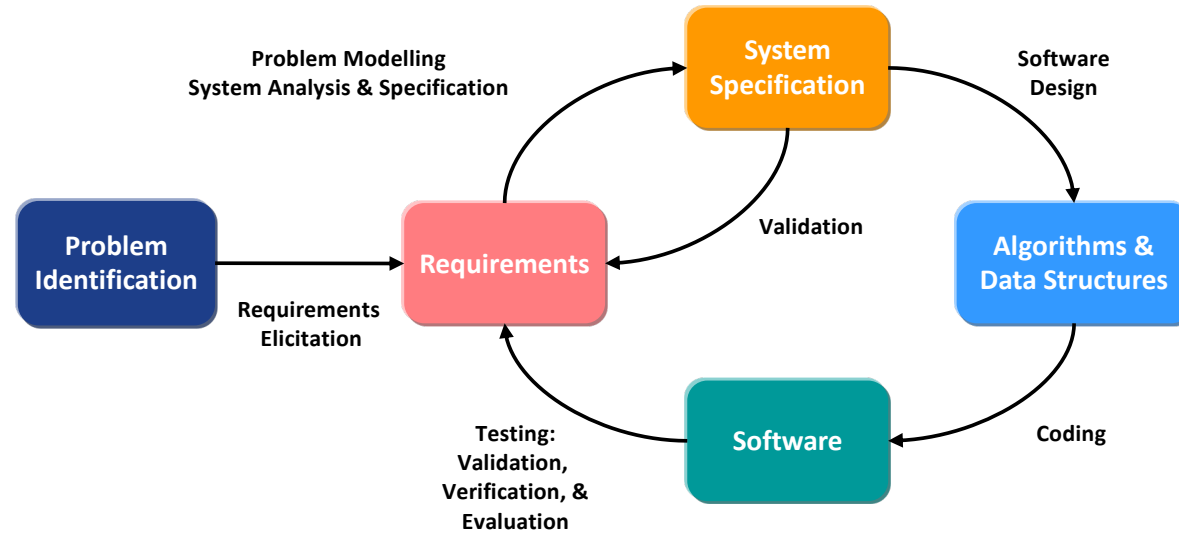


## Marr's Levels of Understanding Framework updated 2012 by T. Poggio

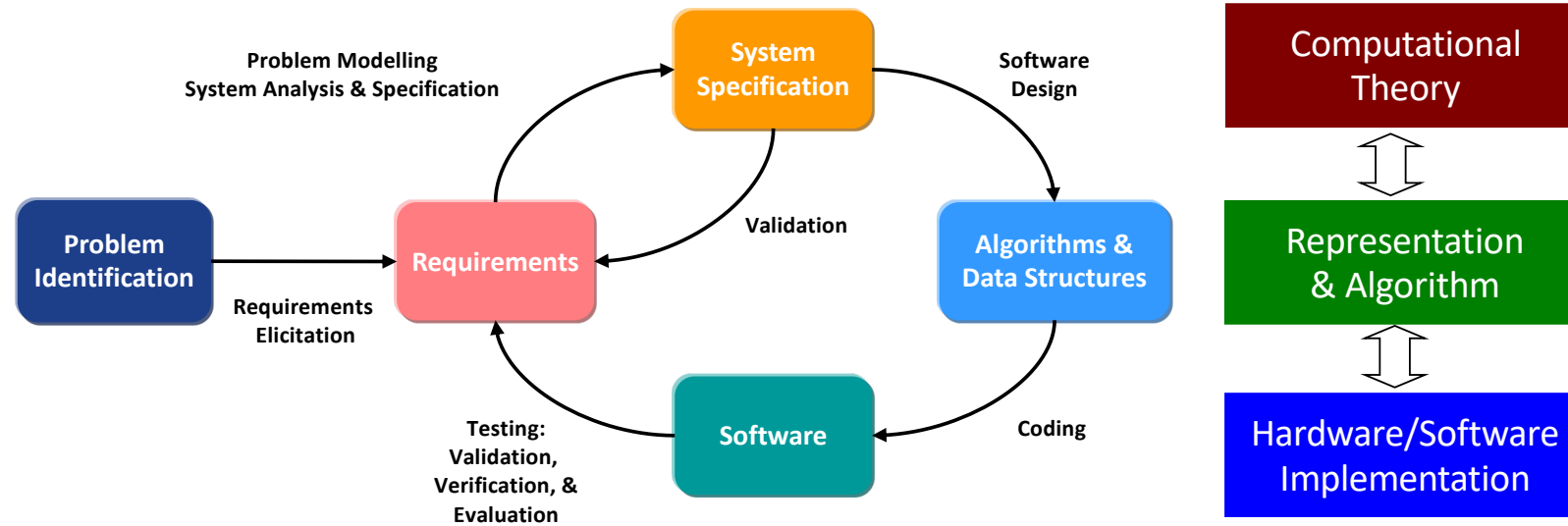


# Life Cycle Models

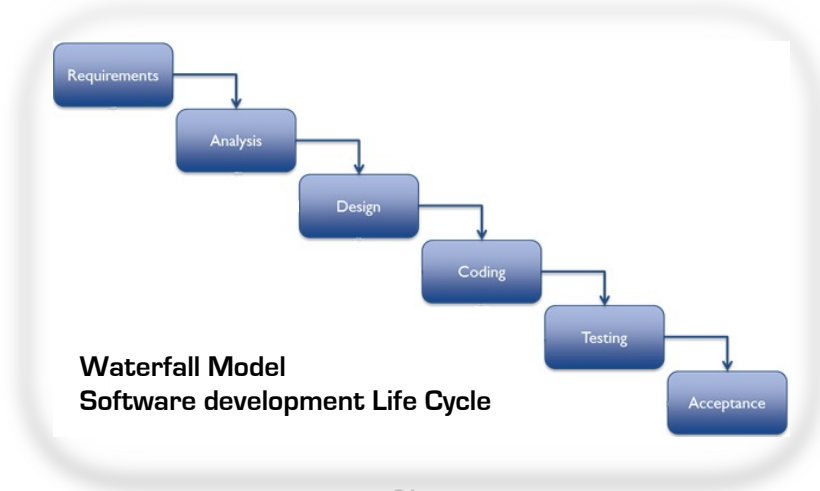
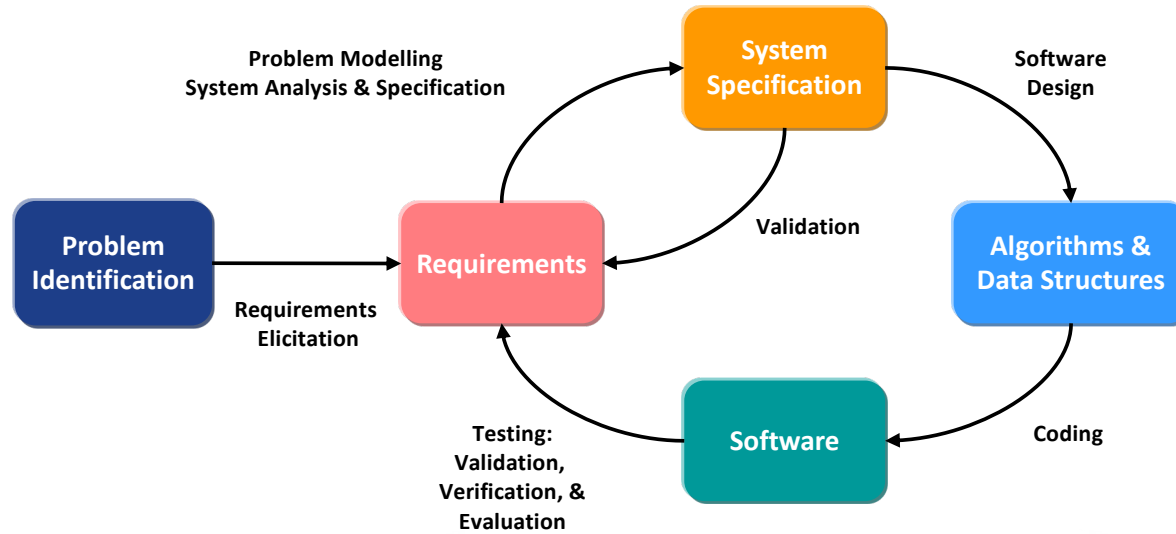
# The Software Development Life Cycle



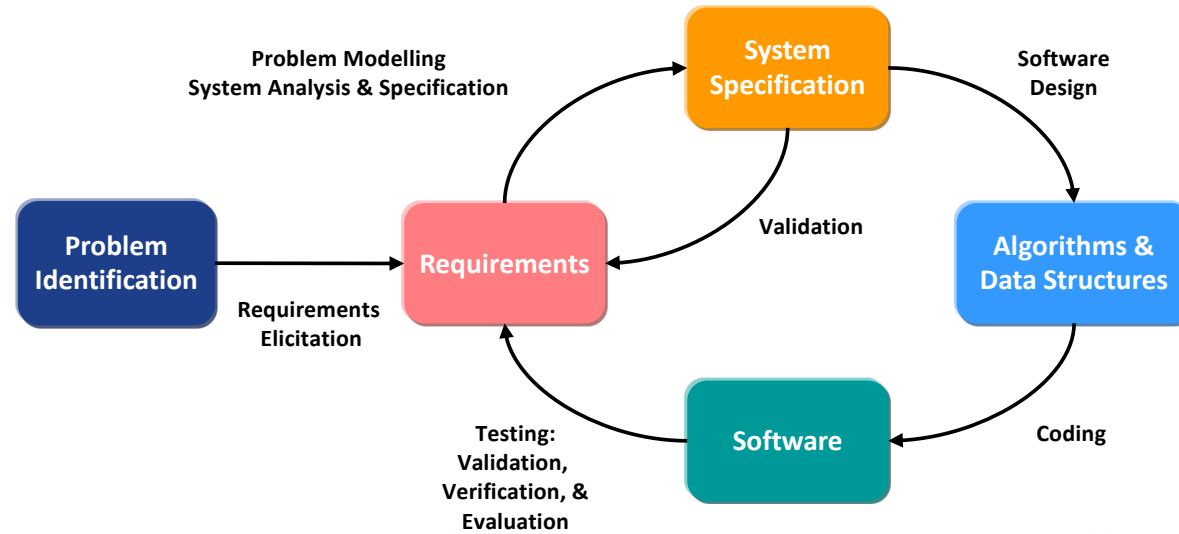
# The Software Development Life Cycle



# The Software Development Life Cycle



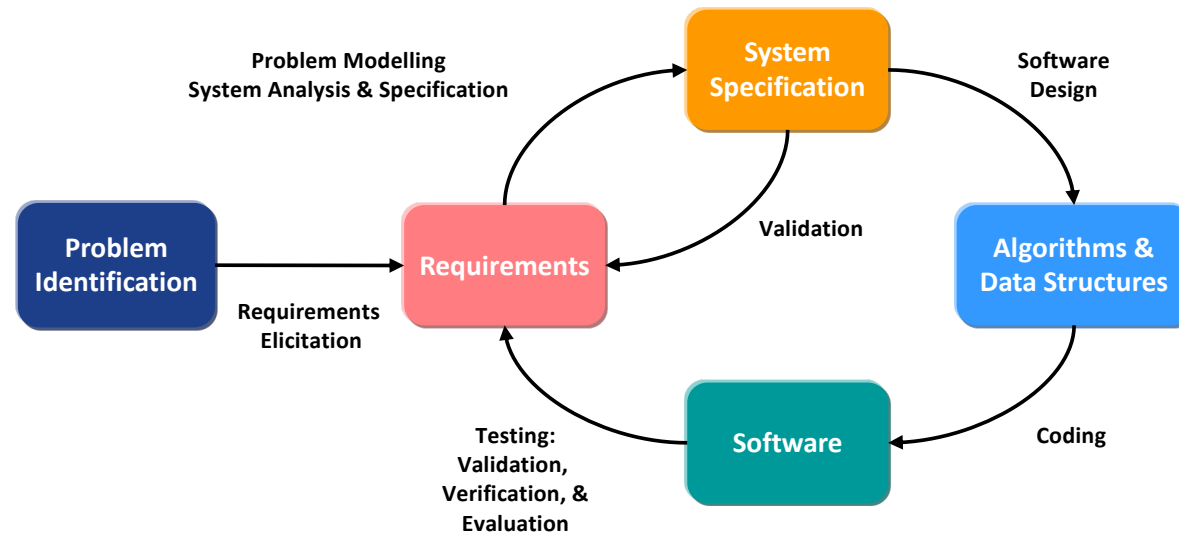
# The Software Development Life Cycle



## Life Cycle Models (Software Process Models):

- Waterfall (& variants, e.g. V)
- Evolutionary
- Re-use
- Hybrid
- Spiral
- ...

# The Software Development Life Cycle



## Software Development Methodologies:

### Structured

**Yourdon Structured Analysis (YSA)**

**Jackson Structured Analysis (JSA)**

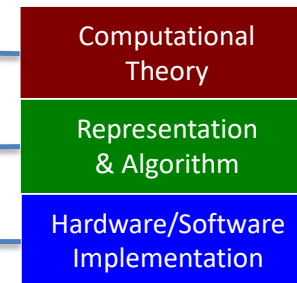
**Structured Analysis and Design Technique (SADT)**

**Object-oriented analysis, design, programming**

**Component-based software engineering (CBSE)**

# Software Development Life Cycle

1. Problem identification
2. Requirements elicitation
3. Problem modelling
4. System analysis & specification
5. System design
6. Module implementation and system integration
7. System test and evaluation
8. Documentation





# Software Development Life Cycle

## 1. Problem identification

- Normally requires experience
- Theoretical issues: appropriate models (problem domain)
- Technical issues: tools, OS, API, libraries (solution domain)

# Software Development Life Cycle

## 2. Requirements elicitation

- Talk to the client (by talk, I mean counsel and coach)
- Document agreed requirements

**What** it does, **what** it doesn't do, **how** the user is to use it or **how** it communicates with the user, **what** messages it displays, **how** it behaves when the user asks it to do something it expects, and especially **how** it behaves when the user asks it to do something it doesn't expect

- Validate requirements with client
- Repeat until mutual understanding converges
- But beware ...

# Software Development Life Cycle

## 2. Requirements elicitation

Customer to a software engineer:

“I know you believe you understood  
what you think I said,  
but I am not sure you realize  
that what you heard is not what I meant”.

R. Pressman

# Software Development Life Cycle

## 3. Problem modelling

- Identify **theory** needed to model and solve the problem
  - Ideally, identify several, compare them, and choose the best (i.e., most appropriate)
  - Use criteria derived from your functional and non-functional requirements
- Create a rigorous – ideally **mathematical** – description
  - Graph theory, Fourier theory, linear system theory, information theory, ...
- If you don't have a model, you aren't doing engineering
  - Connecting components (or lines of code) together is not engineering
  - Without a model, you can't analyze the system and make firm statement about
    - Robustness
    - Operating parameters
    - Limitations

# Software Development Life Cycle

## 4. System analysis & specification

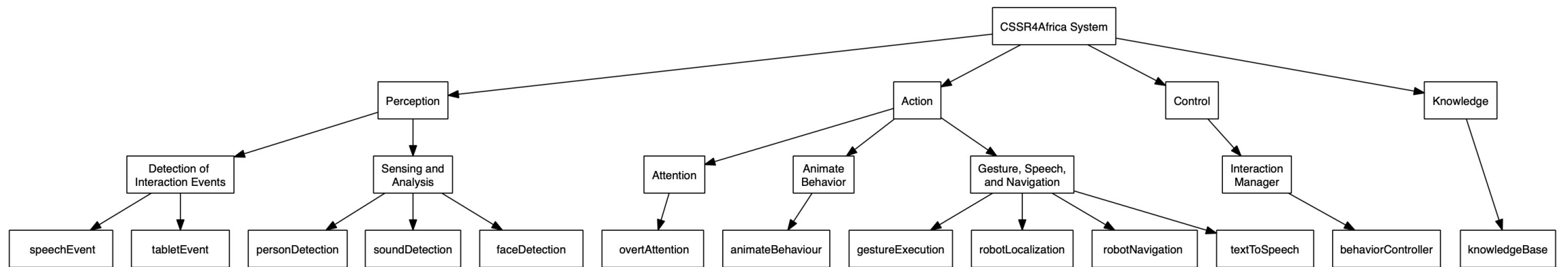
- Identify
  - The system functionality
  - The operational parameters (conditions under which your system will operate, including required software and hardware systems)
  - Limitations & restrictions
  - User interface or system interface
- Including
  - Functional model
  - Data model
  - Process-flow model
  - Behavioural model

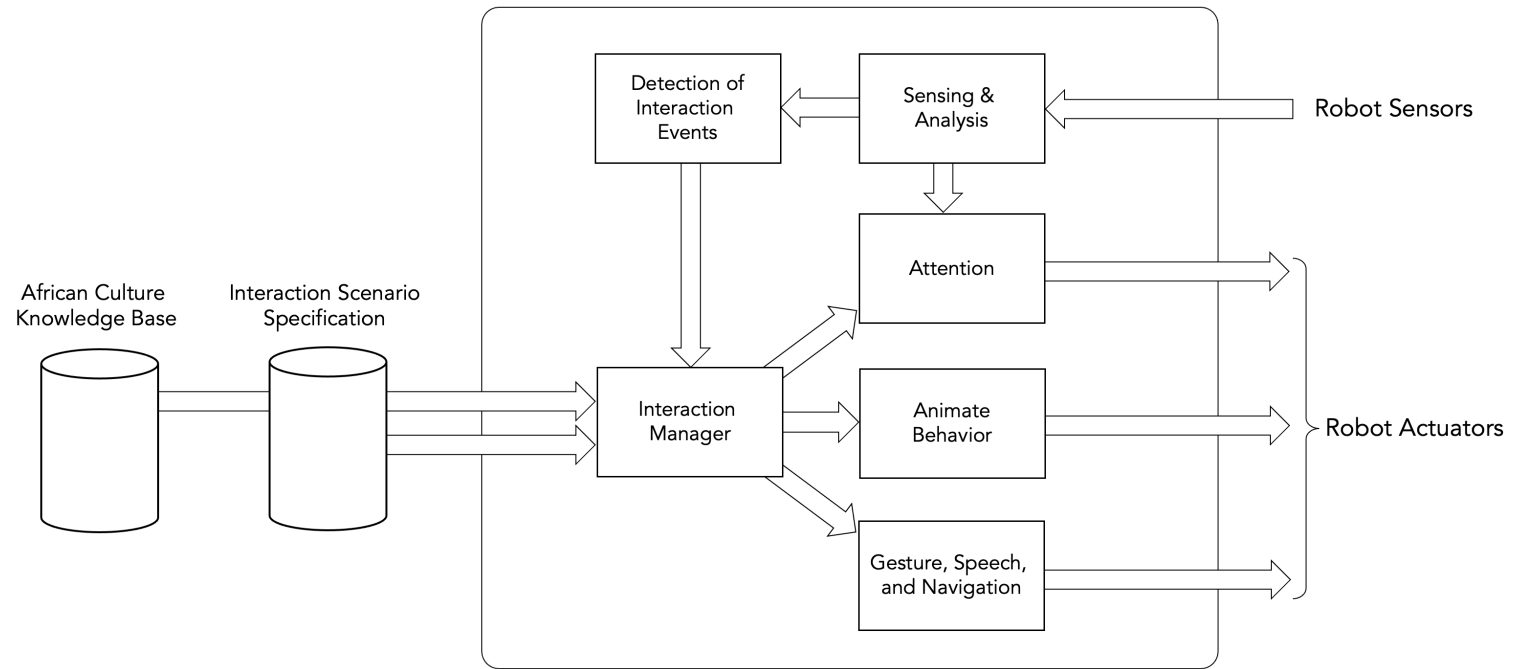
# Software Development Life Cycle

## 4. System analysis & specification

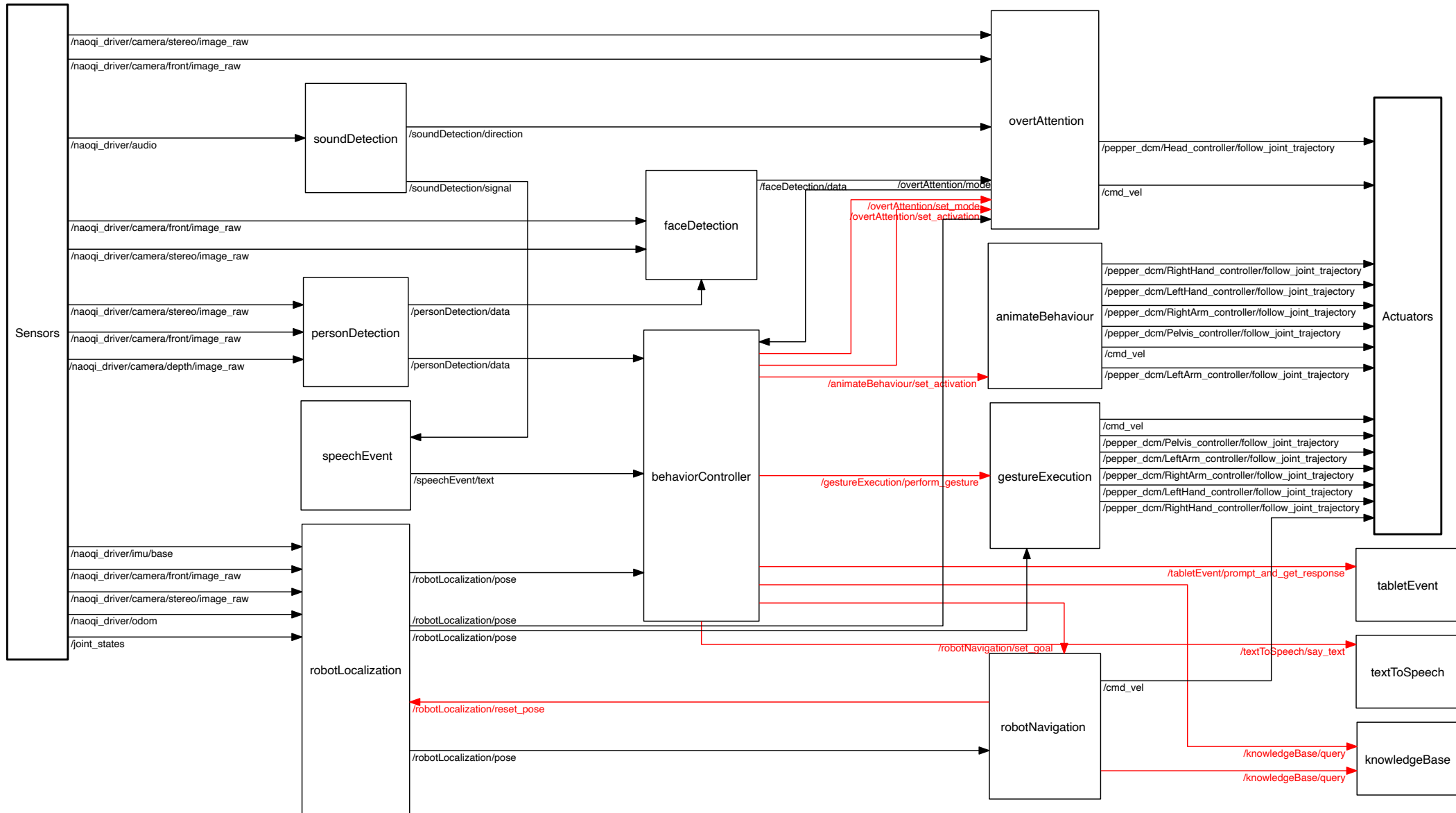
### Functional model

- Hierarchical functional decomposition **tree**
- Modular decomposition (typically)
- Each leaf node in the tree:
  - Short description of functionality, i.e. the input/output transformation
  - Information (data) input
  - Information (data) output
- **System architecture** diagram
  - Network of components at first or second level of decomposition









# Software Development Life Cycle

## 4. System analysis & specification

Modular decomposition ... Dave Parnas



“In this context "module" is considered to be a responsibility assignment rather than a subprogram. The modularizations include the design decisions which must be made before the work on independent modules can begin.”

D.L. Parnas, On the Criteria To Be Used in Decomposing Systems into Modules, Communications of the ACM, Vol. 15, No. 12, Dec 1972

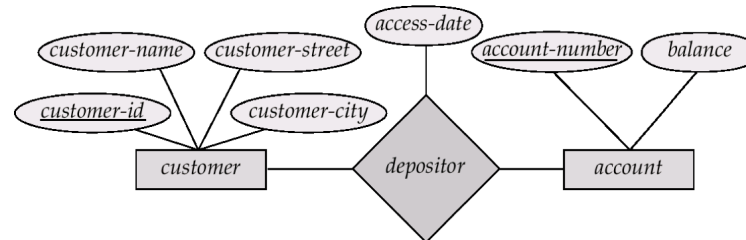
Also responsible for the concepts of **data hiding** and **encapsulation**, cf. ADTs in Lecture DSA02-04

# Software Development Life Cycle

## 4. System analysis & specification

### Data model

- Data entities (not data structures) to represent
  - Input, temporary, output data
- Data dictionary
  - What the data entities mean
  - How they are composed
  - How they are structured
  - Valid value ranges
  - Dimensions (e.g., velocity m/s)
  - Relationships between data entities
- Entity-relationship model



# Software Development Life Cycle

## 4. System analysis & specification

### Process-flow model

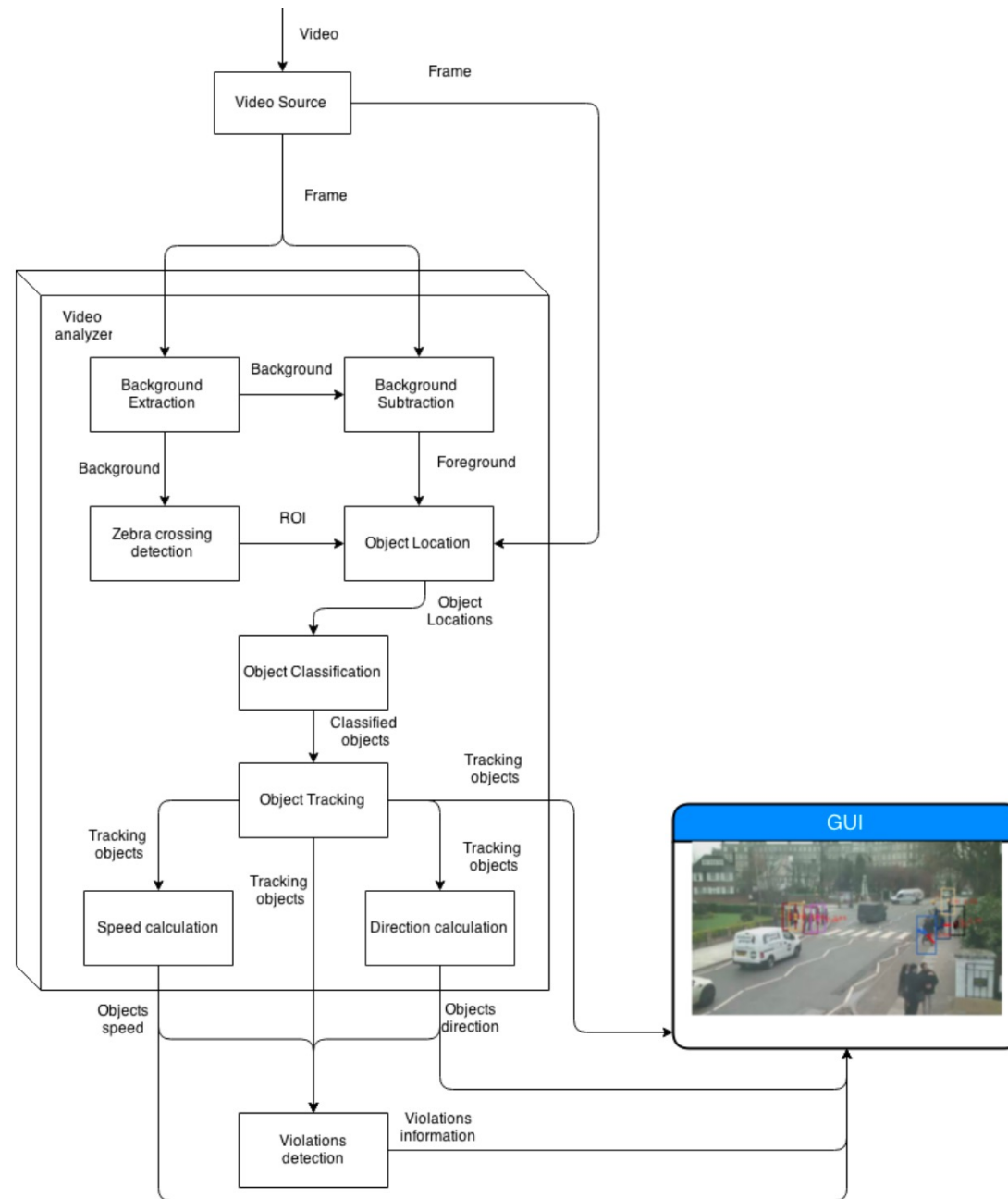
- What data flows into and out of each functional block  
(into and out of the leaf nodes in the functional decomposition tree)
- Data-flow diagrams
  - Organized in several levels: DFD level 0, DFD level 1, ...
  - Level 0 DFD: system architecture diagram

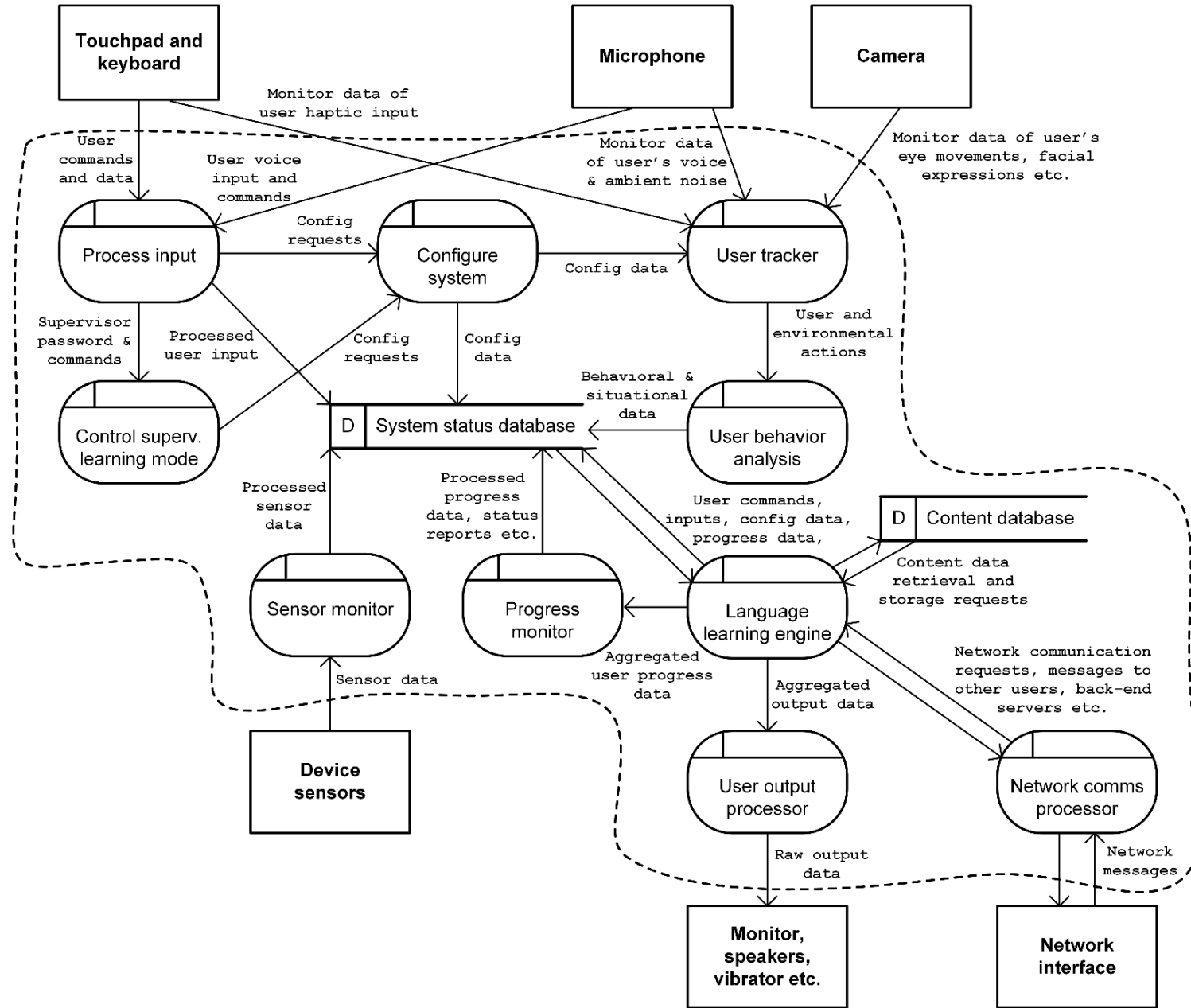
# Software Development Life Cycle

## 4. System analysis & specification

### Process-flow model

- DFDs model the transformation of inputs into outputs
- **Processes/Functions** represent individual functions that the system carries out and transform inputs to outputs
- **Flows** represent connections between processes and the flow of information and data between processes
- **Data Stores** show collections or aggregations of data
- **I/O Entities** show external entities with which the system communicates
  - They are the sources and consumers of data
  - They can be users, groups, organizations, systems,...





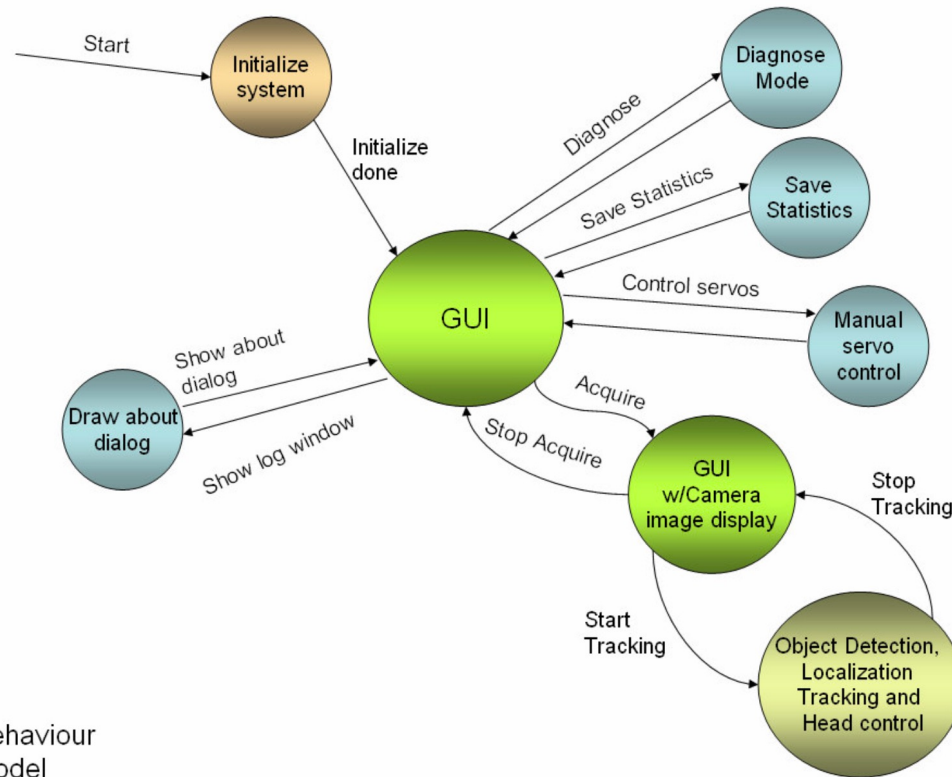
# Software Development Life Cycle

## 4. System analysis & specification

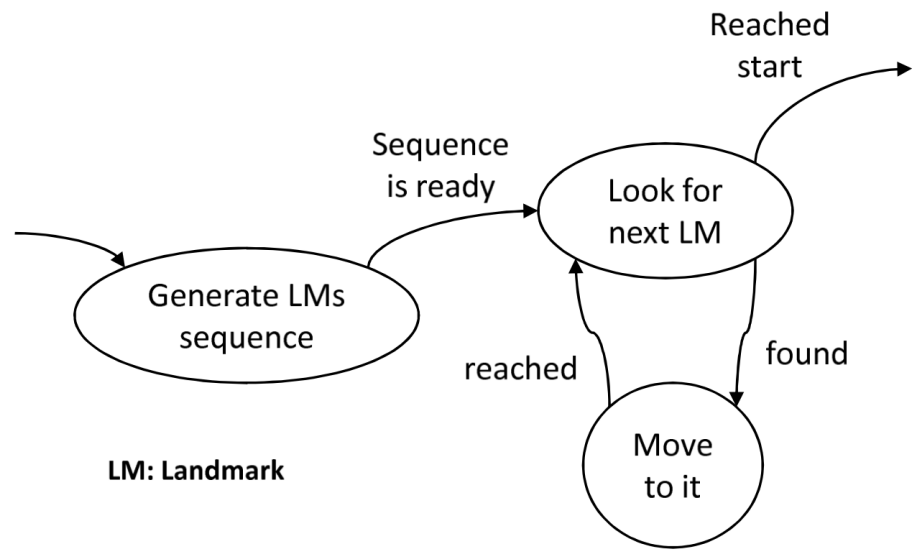
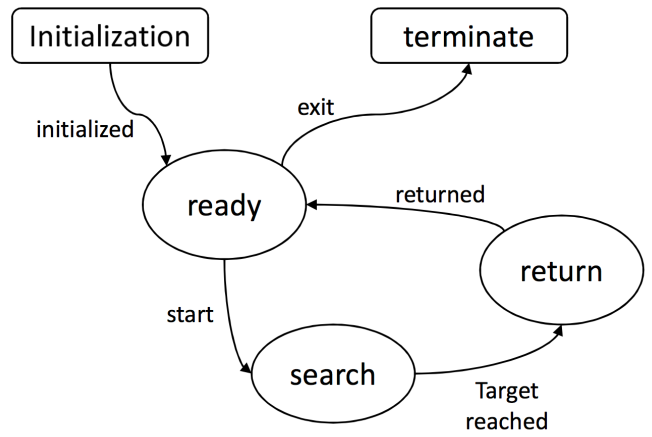
### Behavioural model

- Behaviour over time
- System states
- Triggers that cause transition  
(from state to state)
- Functional block associated with each state
- State transition diagram
  - Finite state machine
  - Finite automaton
- Control-flow diagram  
(version of DFD with events and triggers on each process)





Behaviour Model



# Software Development Life Cycle

## 4. System analysis & specification

Definition of all the **user and system interfaces**

- User manual
- User interface storyboard

# Software Development Life Cycle

## 4. System analysis & specification

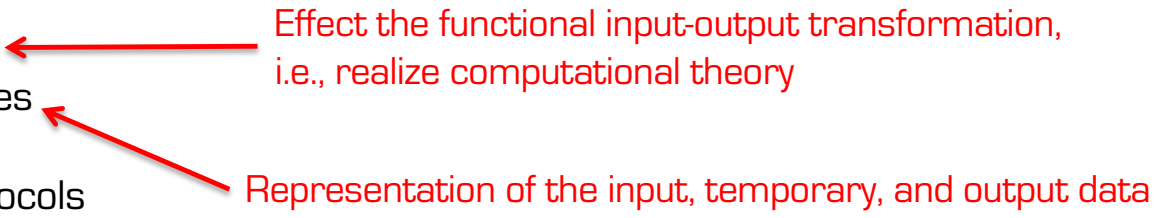
Specification of **non-functional characteristics**

- Dependability
- Security
- Composability
- Portability
- Reusability
- Interoperability

Often reflect the quality of the system

# Software Development Life Cycle

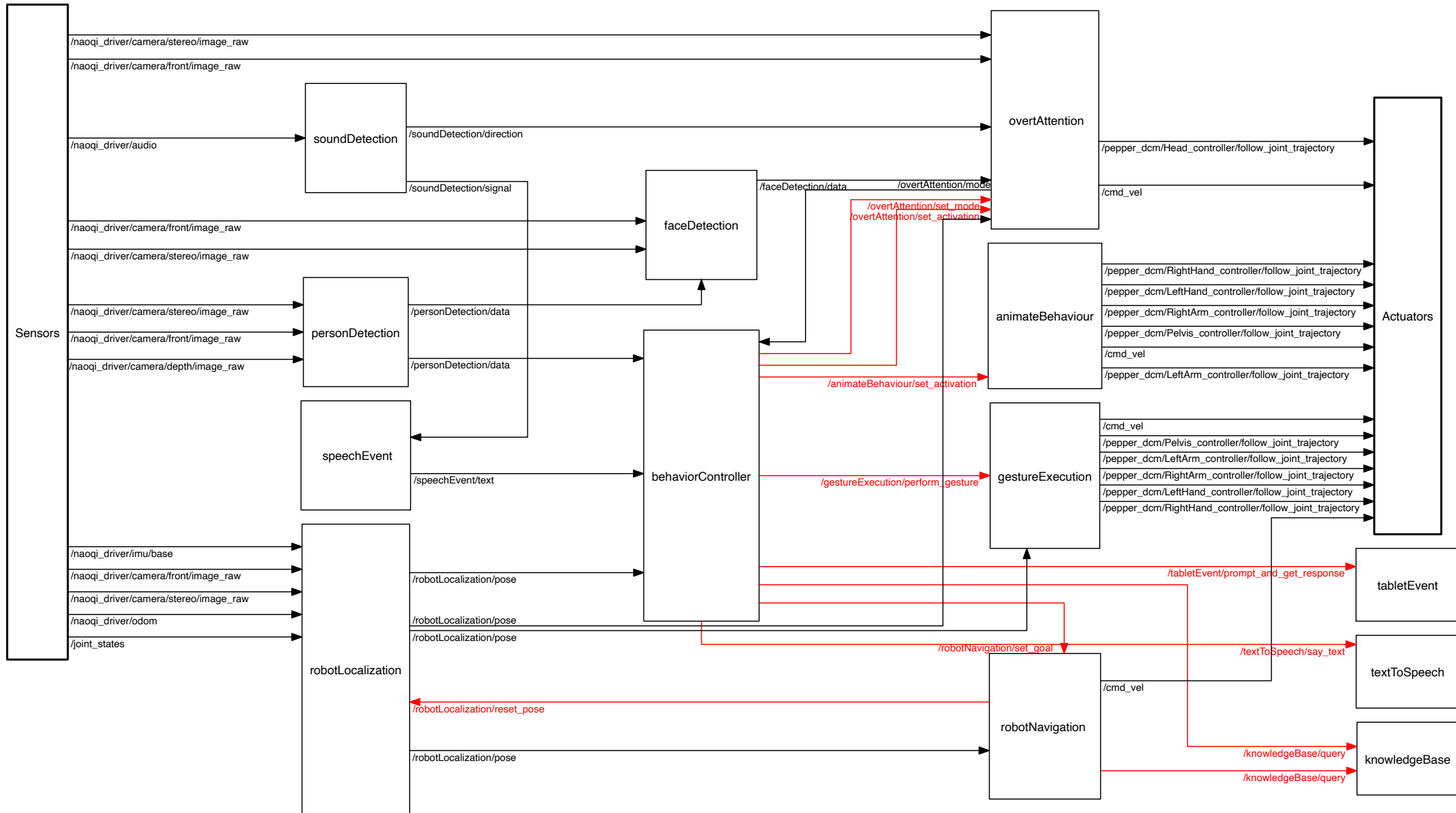
## 5. Software design

- For each module (i.e., leaf node in the hierarchical decomposition tree / system architecture diagram / lowest level DFD)
  - Identify several design options & compare them
    - Algorithms
    - Data-structures
    - Files
    - Interface protocols
  - Choose the **best** design
    - You have to define what 'best' means for your particular project
    - Use criteria derived from the functional and non-functional requirements
- 
- Effect the functional input-output transformation, i.e., realize computational theory
- Representation of the input, temporary, and output data

# Software Development Life Cycle

## 6. Module implementation and system integration

- Use a modular construction approach
- Don't attempt the so-called Big Bang approach
- Build (and test) each component or modular sub-system individually
  - **Driver** (dummy calling routine) ... test harness
  - **Stub** (dummy called routine)
- Link or connect them together, one component at a time.



# Software Development Life Cycle

## 6. Module implementation and system integration

### You Must Validate Data

- Validate input
- Validate parameters
- ‘Constraints on data and computation usually take the form of wrappers – access routines (or methods) that prevent bad data from being stored or used and ensure that all programs modify data through a single, common interface’

J. A. Whittaker and S. Atkin, “Software Engineering Is Not Enough”, IEEE Software, July/August 2002, pp. 108-115.



# Software Development Life Cycle

## 7. Unit, integration, & acceptance test and evaluation

- NOT about showing the system works
- Showing it meets specifications
- Showing it meets requirements
- Showing the system doesn't fail (stress testing)
- Three goals of testing
  1. Verification
  2. Validation
  3. Evaluation

# Software Development Life Cycle

## 7. System test and evaluation

### 1. Verification

- Has the system been built correctly?
- Is it computing the right answer (producing correct data)?
- Extensive test data sets
- Exercise each module or computation
  - Independently
  - As a whole system
- Live data (not just data in test files)

# Software Development Life Cycle

## 7. System test and evaluation

### 2. Validation

- Does it meet the client's requirements?
- Can the user adjust all the main parameters on which operation depends? (List them!)

# Software Development Life Cycle

## 7. System test and evaluation

### 3. Evaluation

- How good is the system?
- **Hallmark of good engineering: assess performance and benchmark against other systems**
- Identify quantitative metrics
- Identify qualitative metrics
- Vary parameters and collect statistics
- Evaluate against **ground-truth** data (data for which you know the correct result)
- Evaluate against **other systems** (benchmarking)

# Software Development Life Cycle

## 7. System test and evaluation

- Tests need to be automated (run several times as the system is tuned)
- Regression testing
- Types of test
  - Unit Tests ... individual modules / components
  - Integration Tests ... sub-systems and system
  - Acceptance Tests ... system

# Software Development Life Cycle

## 8. Documentation

- Internal documentation
  - Documentation comments
    - Intended to be extracted automatically by, e.g., Doxygen tool
    - Describe the functionality from an implementation-free perspective
    - Purpose is to explain how to use the component through its application programming interface (API), rather than understand its implementation
  - Implementation comments
    - Overviews of code
    - Provide additional information that is not readily available in the code itself
    - Comments should contain only information that is relevant to reading and understanding the program
  - **Use standards**

# Software Development Life Cycle

## 8. Documentation

“There is rarely such a thing as too much documentation ...

Documentation – often exceeding the source code in size – is a requirement, not an option.”

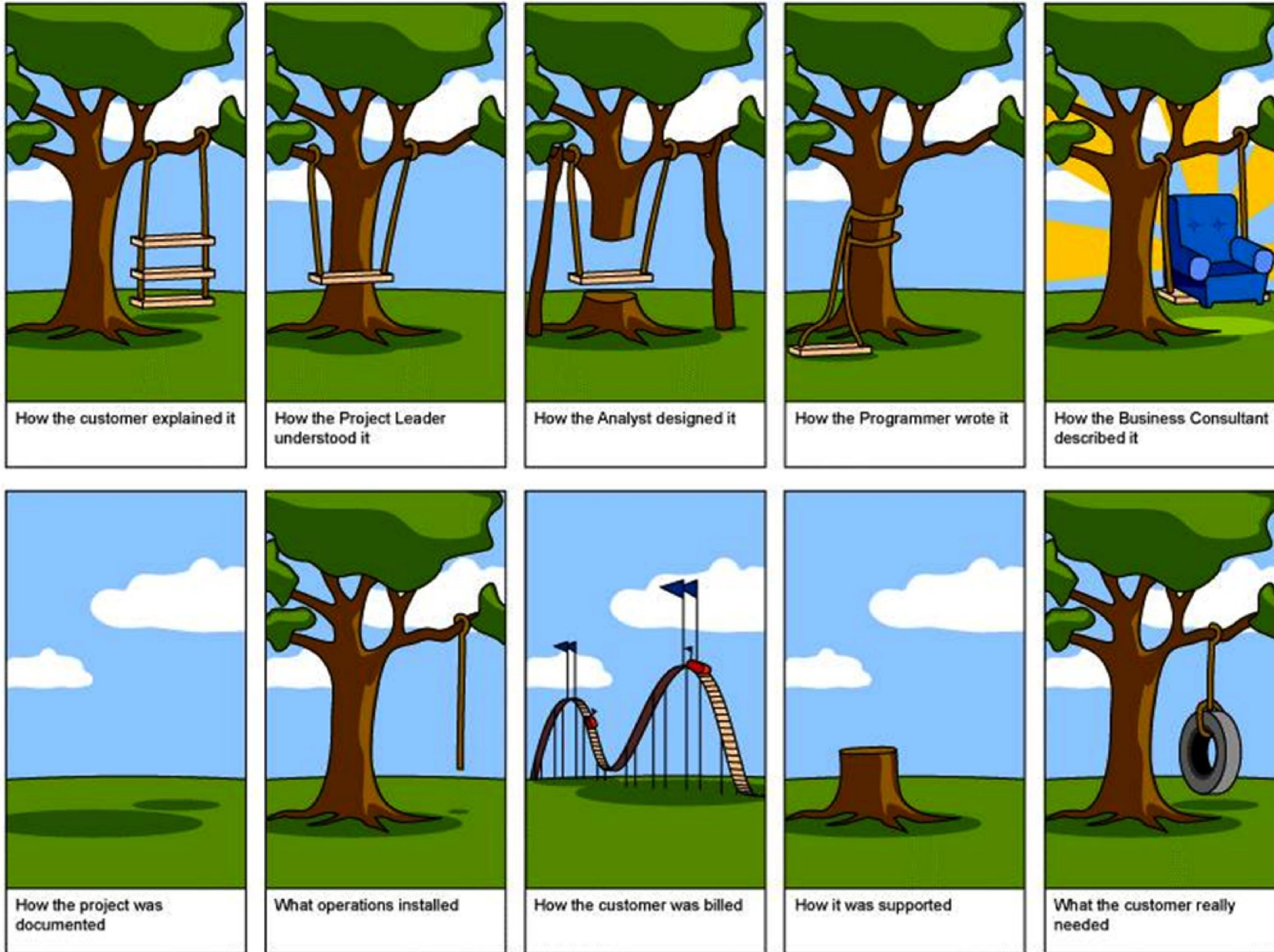
J. A. Whittaker and S. Atkin, “Software Engineering Is Not Enough”, IEEE Software, July/August 2002, pp. 108-115.

# Software Development Life Cycle

## 8. Documentation

- External documentation
  - User manual
  - Reference manual
  - Design documents

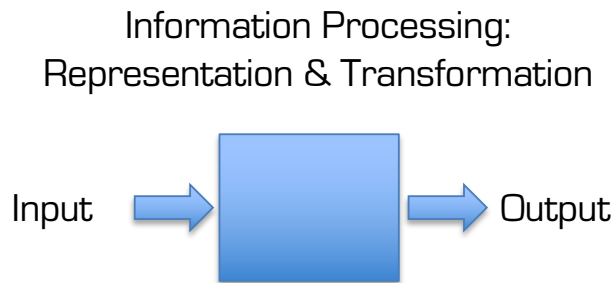




# Formalisms for Representing Algorithms

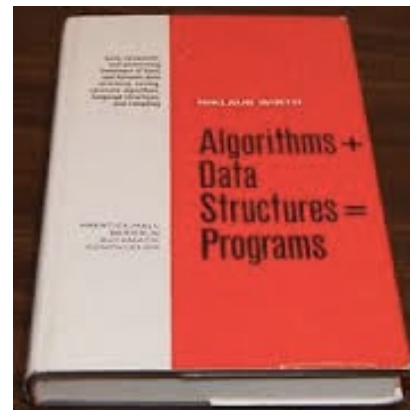
## Informal definition

An algorithm is a systematic procedure for transforming information from one (input) state to another (output) state



# Definition of an Algorithm

Typically, there is a strong link between an algorithm and the information representation, i.e., the data structure



# Formalisms for Representing Algorithms

## Required characteristics

- Simple, clear, and intuitive (as far as possible)
- As rigorous as practical – but keeping the math as simple as possible
- Language neutral
- Factor out the hardware and operating systems
- Focus on algorithmic essence
- Properly scoped (not too big, not too trivial or obvious)

# Practical Representations

- Some candidate representations
  - Pseudo Code
  - Flow Charts
  - State Diagrams
  - Formalisms
  - Modeling Methodologies (e.g., UML)
- Many engineers use these, but some use them
  - At the wrong time
  - To model the wrong kinds of things (poor scoping)
  - Incorrectly
  - Mix “what is needed” with “how we will build it”

# Pseudo Code

- Pseudo code is an informal abstraction of an algorithm that:
  - uses the structural conventions of a programming language
  - is simplified for human reading rather than machine compilation
  - omits details that are not essential for algorithmic analysis
  - shows the temporal relation of instruction execution (**sequencing**)
- Despite many attempts, no standard for pseudo code syntax currently exists

# Pseudo Code

## Declaration

```
type variable;  
integer A; string name;
```

## Assignment

```
variable = value;  
a = 45; x = y
```

## Basic mathematical operators

```
result = variable_value operator variable_value  
y = a+b; z = 5.0/e; j = k*1; r = 2*(22/7)*(r^2)
```

## Basic functions, subroutines, methods

```
read(), write(), print(),...
```

Assumed functions should be clearly defined prior to use; more on functions, subroutines, and methods later

# Pseudo Code

## Control Structures

- Direct sequence  
**do X, then do Y**
- Conditional branching  
**if Q then do X, else do Y**
- Bounded iteration  
**do Z exactly X times**
- Conditional or unbounded iteration  
**do Z until Q becomes true**  
**while Q is true do Z**



# Pseudo Code

Example: algorithm to find the greatest common denominator (GCD)

- How the **read()** function work is not important for our analysis
- We focus on the essence of the algorithm, not on checking input, formatting output, error handling, and so forth
- Now that the algorithm has been distilled to its essence we can analyze: **how do we know we solved the problem? how quickly does it compute the answer?**

```
a = read()
b = read()
if a = 0
    return b
while b ≠ 0
    if a > b
        a := a - b
    else
        b := b - a
return a
```

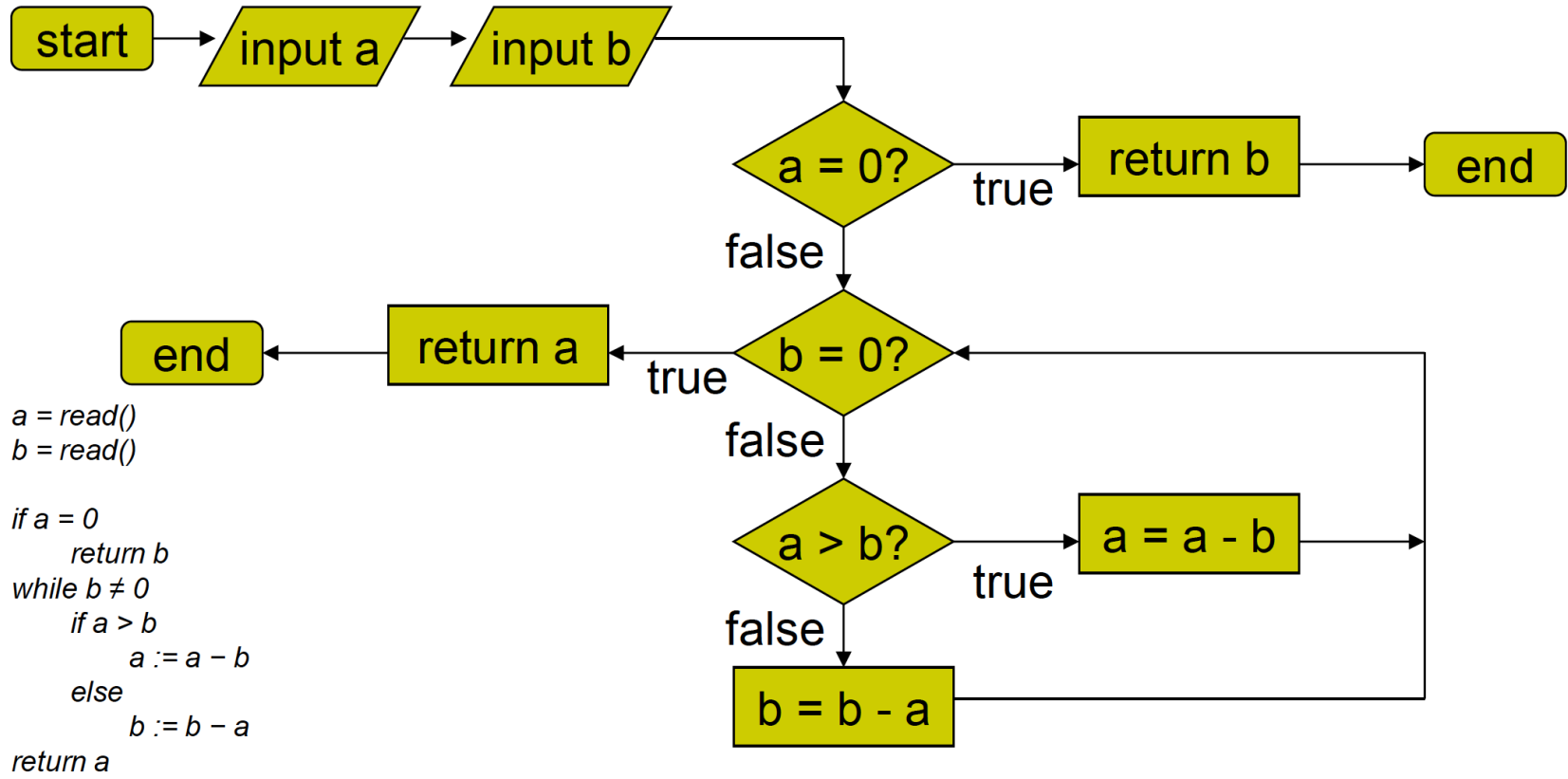
# Pseudo Code

- Pseudo code is attractive because
  - It looks like the computer-interpretable code
  - It is complete in terms of describing computer algorithms
- In practice, pseudo code is sometimes extended and violates notions of minimalism
  - Pseudo code should only support what is necessary to describe the algorithm – and no more!
  - Sometimes, pseudo code is used to describe entire applications, and becomes too cumbersome to support analysis of algorithms

# Flowcharts

- Graphical representation of the behavior of an algorithm
  - Represents the steps of an algorithm by geometric shapes
  - Temporal relationships are shown by connections
- Developed in the early 20th century for use in industrial engineering
  - Used in many domains for the last 100 years
  - John von Neumann developed the flow chart while working at IBM as a means to describe how programs operated
  - Flowcharts are still used to describe computer algorithms- UML activity diagrams are an extension of the flowchart
- There are many flowchart notation standards

# Flowcharts



# Flowcharts

## Strengths and weaknesses

- The set of defined constructs is both minimal and complete
- The resulting algorithms can be hard to understand and analyze
- Graphical methods do not scale well – very difficult to represent large or complex algorithms
- Hard to distribute, share, and reuse

# Finite State Machines (FSM)

- Behavioral models composed of a finite number of **states**, **transitions** between those states, and **actions**
- FSMs are represented by state diagrams
- State diagrams have been used for 50+ years in software, hardware, and system design and there are a variety of notations and approaches
  - Traditional Mealy-Moore state machines
  - Harel state machines
  - UML state machines

# Finite State Machines (FSM)

A traditional (e.g. Mealy-Moore) type of FSM is a quintuple  $(\Sigma, S, s_0, \delta, F)$

$\Sigma$  is the input alphabet where  $\Sigma$  is finite  $\wedge \Sigma \neq \emptyset$

$S$  is a set of states where  $S$  is finite  $\wedge S \neq \emptyset$

$s_0$  is an initial state where,  $s_0 \in S$

$\delta(q, x)$  is the state transition function where  $q \in S \wedge x \in \Sigma$

(If the FSM is nondeterministic, then  $\delta$  could be a set of states)

$F$  is the set of final states where  $F \subseteq S \cup \{\emptyset\}$

# Finite State Machines (FSM)

$\delta(q, x)$  may be a partial function:

$\delta(q, x)$  does not have to be defined for every combination of  $q$  and  $x$

If it is not defined, then the FSM can enter an error state or reject the input

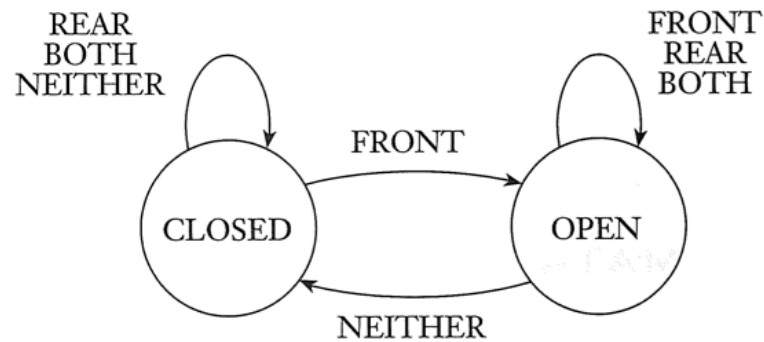
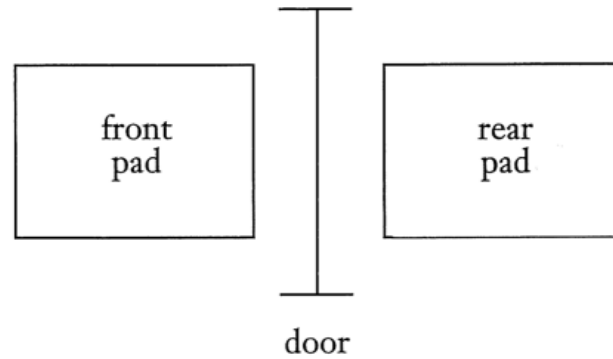


# Finite State Machines (FSM)

The following (limiting) assumptions are made regarding traditional (deterministic) Mealy-Moore FSAs

- an FSA can only be in one state at a time, and must be in exactly one state at all times
- States of one FSA are independent from the states of all other FSAs
- Transitions between states are not interruptible
- Actions are atomic and run to completion
- Actions may be executed on entry into a state, on exit from a state, or during the transition from one state to another

# Finite State Machines (FSM)



state

input signal

	NEITHER	FRONT	REAR	BOTH
CLOSED	CLOSED	OPEN	CLOSED	CLOSED
OPEN	CLOSED	OPEN	OPEN	OPEN

# Finite State Machines (FSM)

**Transitions** indicate state change from one state to another that are described by

- a **condition** that needs to be fulfilled to enable a transition
- an **action** which is an activity that is to be performed at some point in the transition
  - **Entry action**: which is performed when entering the state
  - **Exit action**: which is performed when exiting the state
  - **Input action**: which is performed depending on present state and input conditions
  - **Transition action**: which is performed when performing a certain transition

# Finite State Machines (FSM)

- Popular form of FSM are the Harel State Diagrams
- A variant which was adopted for Unified Modeling Language (UML) State Machines
- There are two types of UML State Machines
  - Behavioral State Machines (BSM)  
Model the behaviour of objects
  - Protocol State Machines (PSM)  
Model protocols of interfaces and ports
- Most use users of UML don't differentiate

# Finite State Machines (FSM)

FSAs are limited and it is difficult to model concurrency, complex object states, threads, multi-tasking

UML state machines extend the traditional automata theory in several ways that include

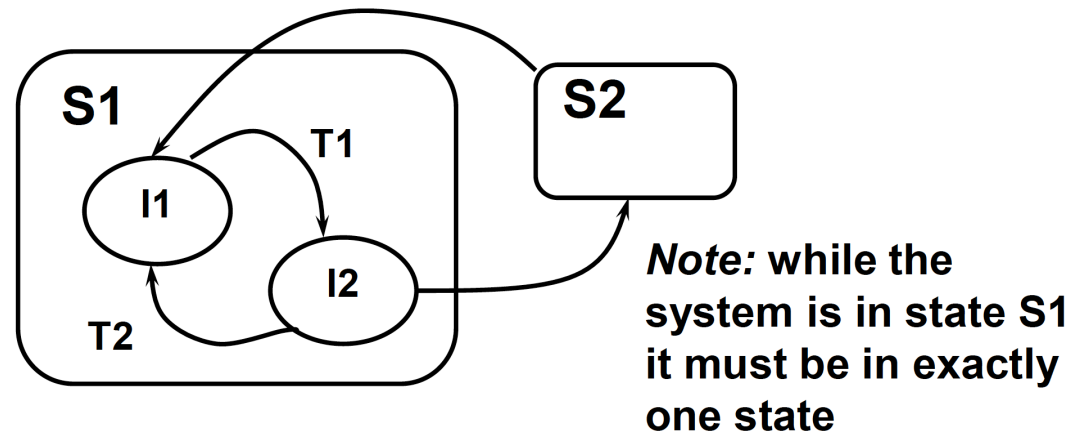
- nested state
- guards
- actions
- activities
- orthogonal components
- concurrent state models

# Finite State Machines (FSM)

UML State Machines – **Nested States**

Outer state is called the **superstate**

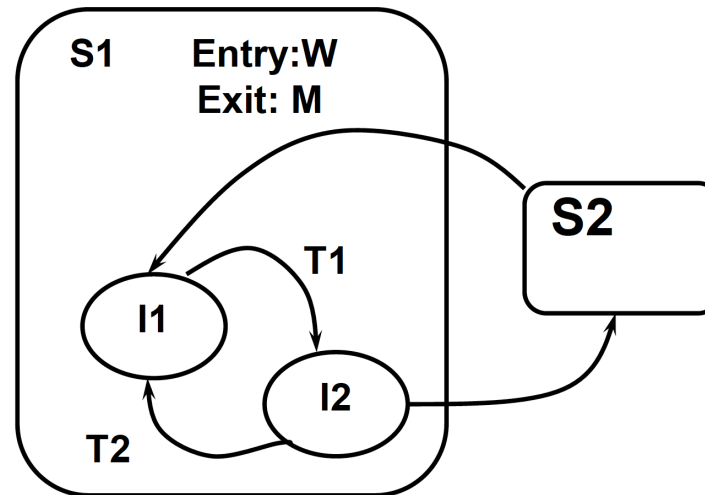
Inner states are called **substates**



# Finite State Machines (FSM)

## UML State Machines – Actions

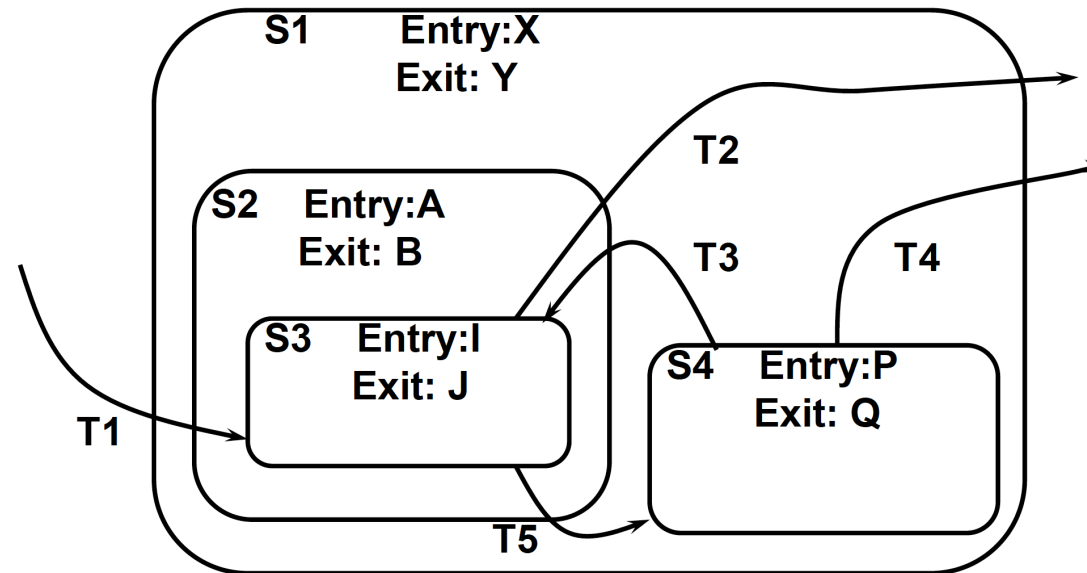
You can specify state entry and exit actions



# Finite State Machines (FSM)

UML State Machines – **Actions**

You can nest entry and exit actions

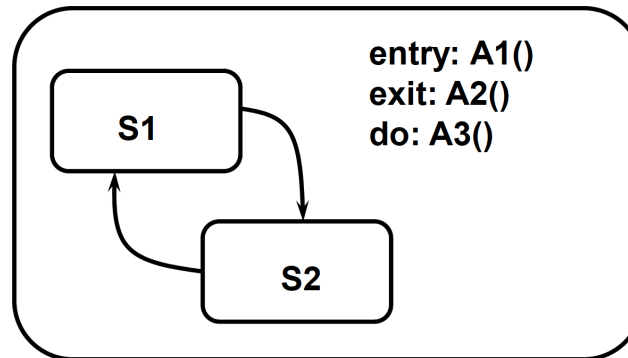




# Finite State Machines (FSM)

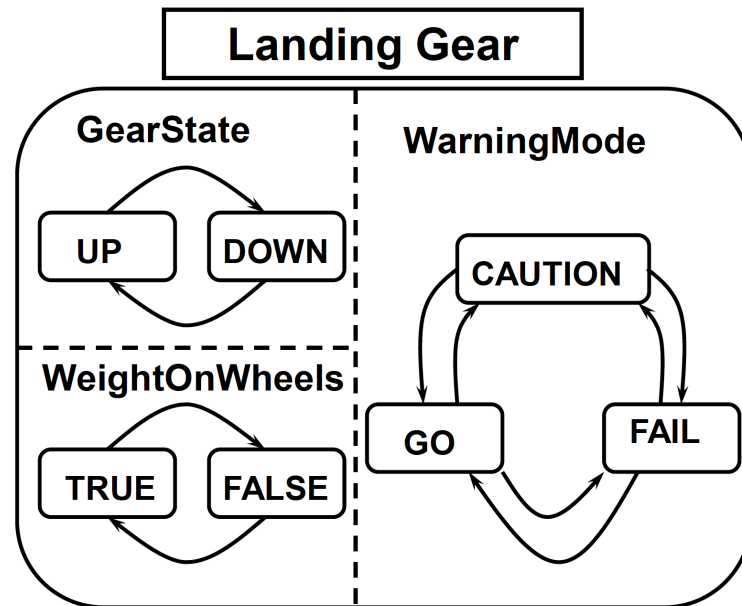
## UML State Machines – Activities

- Like actions except they are performed as long as the state is active
- Activities are indicated with a **do:** statement



# Finite State Machines (FSM)

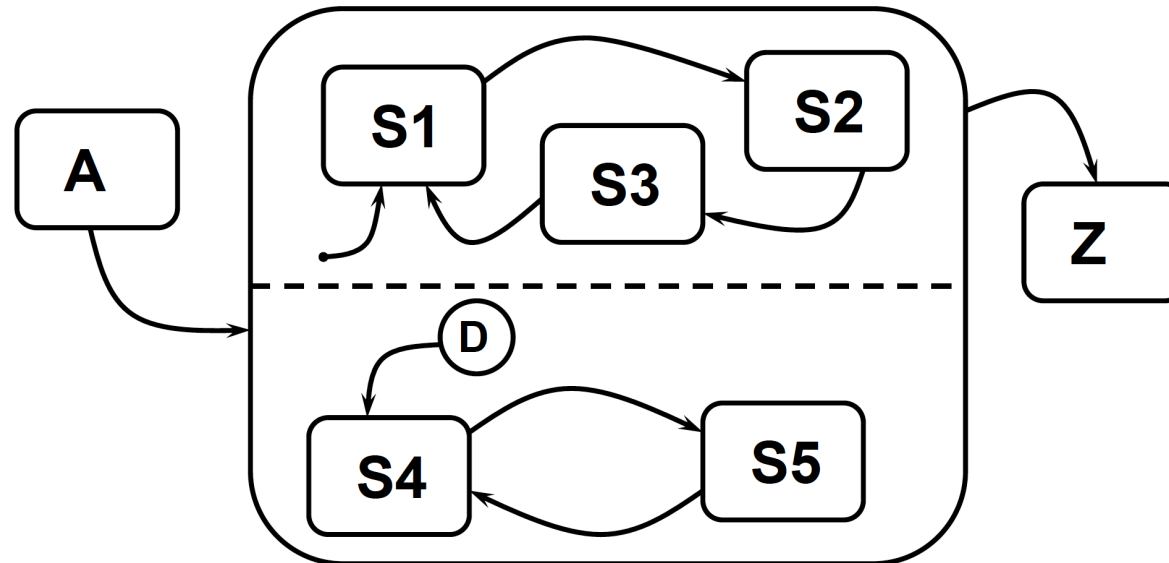
## UML State Machines – Orthogonal Components



# Finite State Machines (FSM)

UML State Machines – **Concurrent State Models**

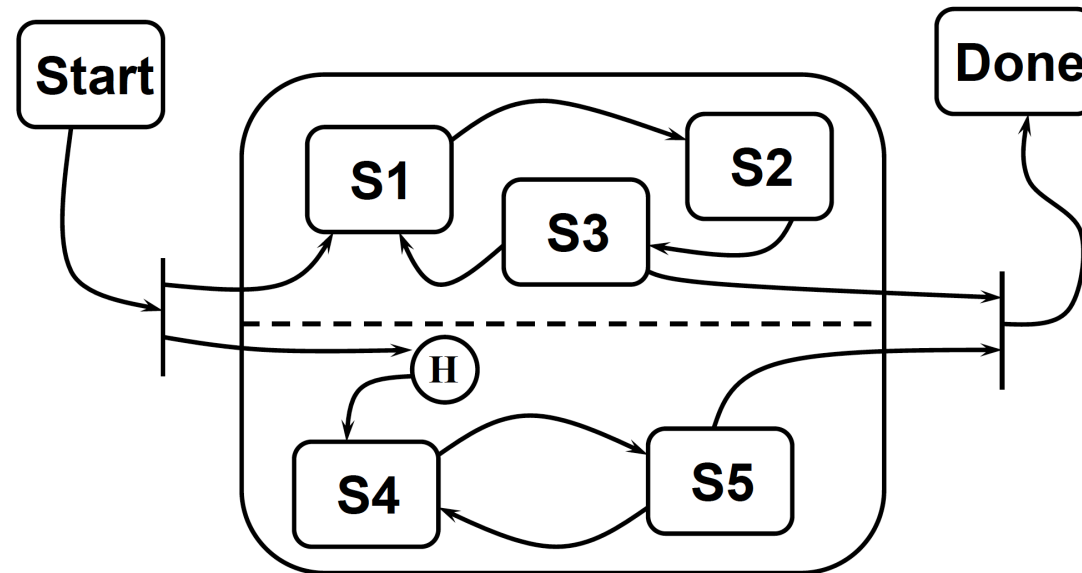
**Concurrent threading** can be modelled



# Finite State Machines (FSM)

## UML State Machines – Concurrent State Models

Forking / Joining can be modelled



# Finite State Machines (FSM)

- In traditional FSA, transitions carry little or no information
- UML state machine transitions carry a lot of information:
  - Event Name - Name of triggering event
  - Parameters - data passed with event
  - Guard - condition that must be true for the transition to occur
  - Action List - list of actions executed
  - Event List - list of events executed

# Finite State Machines (FSM)

- The key problem with FSM technologies is that they simply do not scale up well
  - State explosion is a common problem
  - Care must be taken to restrict the scope of what is being modeled
- FSMs often abstract away the very algorithms we want to model
  - Care must be taken to maintain a proper and consistent level of abstraction
  - Can violate notions of completeness

# Finite State Machines (FSM)

UML state machines are really more like a notation than traditional FSMs

- Violates minimalism
- Any benefit gain in applying mathematical rigor may be lost