Data Structures and Algorithms for Engineers

Module 5: Lists

Lecture 1: Containers. Dictionaries. List ADT: array implementation

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Containers and Dictionaries

- Containers are data structures that permit storage and retrieval of data items independent of content

- Dictionaries are data structures that retrieve data based on key values (i.e., content)

Containers and Dictionaries

- Containers are distinguished by the particular retrieval order they support
- In the case of stacks and queues, the retrieval order depends on the insertion order

Containers and Dictionaries

Stack: supports retrieval by last-in, first-out (LIFO) order

- Push(x, S) Insert item x at the **top** of a stack S
- Pop (S) Return (and remove) the **top** item of a stack S

Containers and Dictionaries

Queue: support retrieval by first-in, first-out (FIFO) order

- Enqueue(x, Q) Insert item x at the **back** of a queue Q
- Dequeue (Q) Return (and remove) the the **front** item from a queue Q

Containers and Dictionaries

Dictionaries permits access to data items by content

- You put an item into a dictionary so that you can find it when you need it

Containers and Dictionaries

Main dictionary operations are

– Search(D, k) Given a search key k, return a pointer to the element in dictionary D whose key value is k, if one exists

– Insert(D, x) Given a data item x, add it to the dictionary D

– Delete(D, x) Given a pointer to a given data item x in the dictionary D, remove it from D

Containers and Dictionaries

Some dictionary data structures also **efficiently** support other useful operations

- Max(D) Retrieve the item with the largest key from D
- Min(D) Retrieve the item with the smallest key from D

These operations allows the dictionary to serve as a priority queue; more on this later

Containers and Dictionaries

Some dictionary data structures also **efficiently** support other useful operations

- Predecessor*(D, x)* — Retrieve the item from *D* whose key is immediately before *x* in sorted order
- Successor*(D, x)* — Retrieve the item from *D* whose key is immediately after *x* in sorted order

These operations enable us to iterate through the elements of the data structure

Containers and Dictionaries

- We have defined these container and dictionary operations in an **abstract** manner, without reference to their implementation or the implementation of the structure itself
- There are many implementation options
  - Unsorted arrays
  - Sorted arrays
  - Singly-linked lists
  - Doubly-linked lists
  - Binary search trees
  - Balanced binary search trees
  - Hash tables
  - Heaps
  - ...

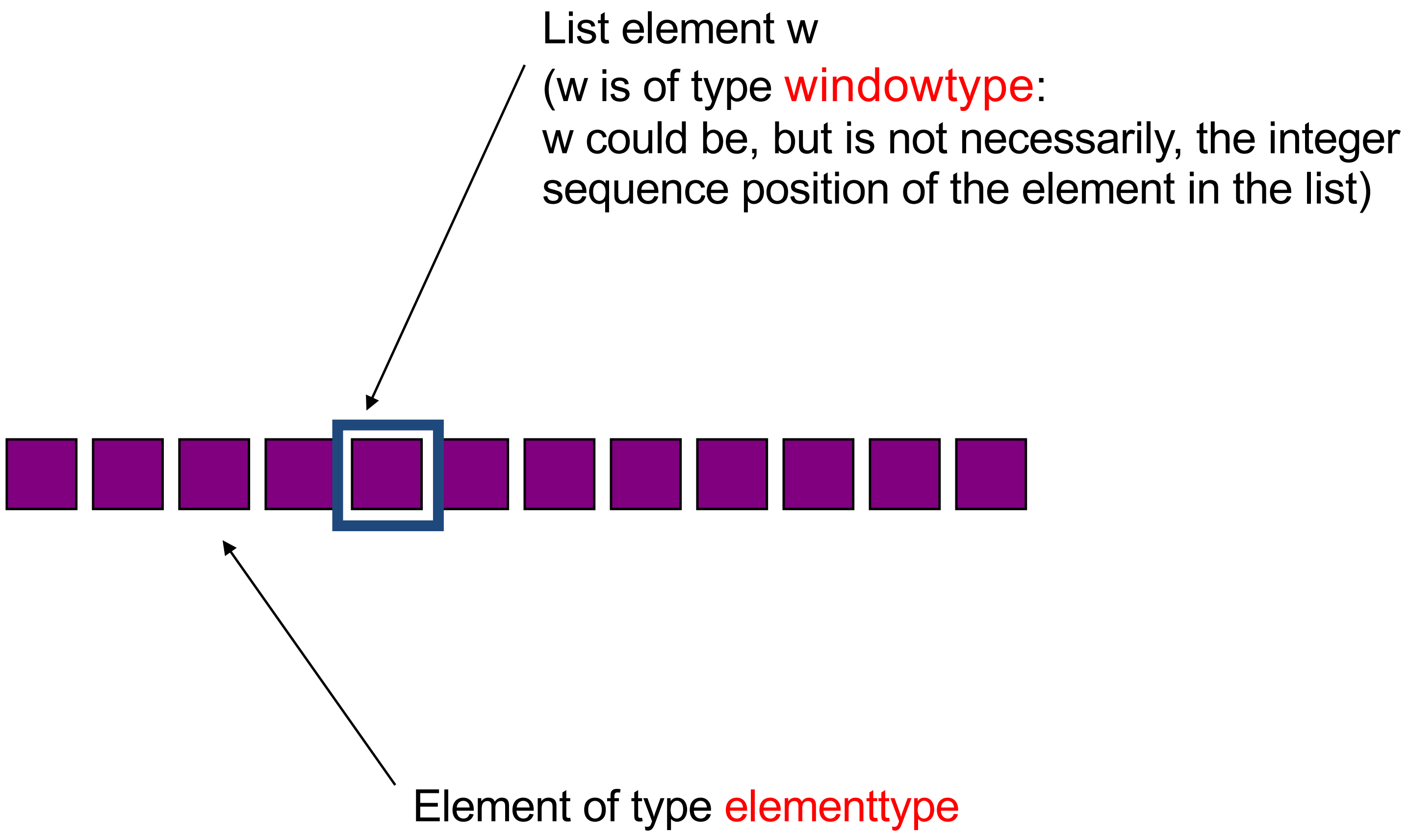
Lists

A list is an ordered sequence of zero or more elements of a given type

$a_1, a_2, a_3, \ldots a_n$

- a_i is of type elementtype
- a_i precedes a_{i+1}
- a_{i+1} succeeds or follows a_i
- If n=0 the list is empty: a null list
- The position of a_i is i

Lists



List element w
(w is of type windowtype:
w could be, but is not necessarily, the integer
sequence position of the element in the list)
Element of type elementtype

LIST: An ADT specification of a list type

- Let L denote all possible values of type LIST (i.e., lists of elements of type elementtype)
- Let E denote all possible values of type elementtype
- Let B denote the set of Boolean values true and false
- Let W denote the set of values of type windowtype

LIST Operations

Syntax of ADT Definition:

Operation:

What_You_Pass_It $\rightarrow$ What_It_Returns :

LIST Operations

Declare: $\rightarrow$ L :

The function value of Declare(L) is an empty list

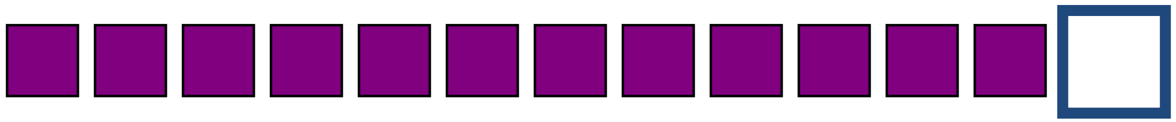
- alternative syntax: LIST L

LIST Operations

End: $L \rightarrow W$:

The function End(L) returns the position after the last element in the list

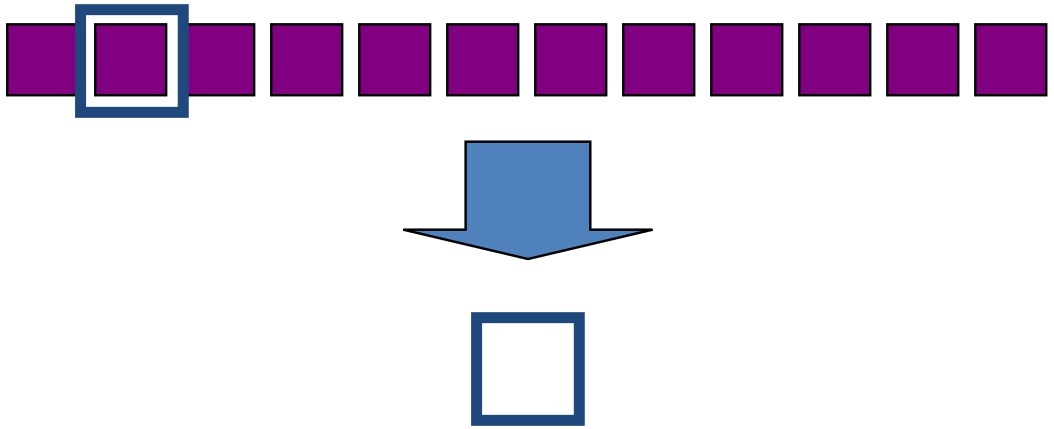
(i.e., the value of the function is the window position after the last element in the list)



LIST Operations

Empty: L $\rightarrow$ L x W :

The function Empty causes the list to be emptied, and it returns position End(L)



LIST Operations

IsEmpty: L $\rightarrow$ B :

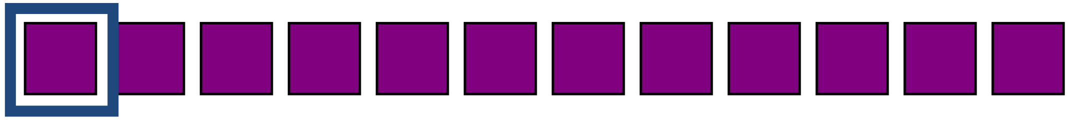
The function value IsEmpty(L) is true if *L* is empty; otherwise, it is false

LIST Operations

First: L $\rightarrow$ W :

The function value First(L) is the window position of the first element in the list;

if the list is empty, it has the value End(L)



LIST Operations

Next: L x W → W :

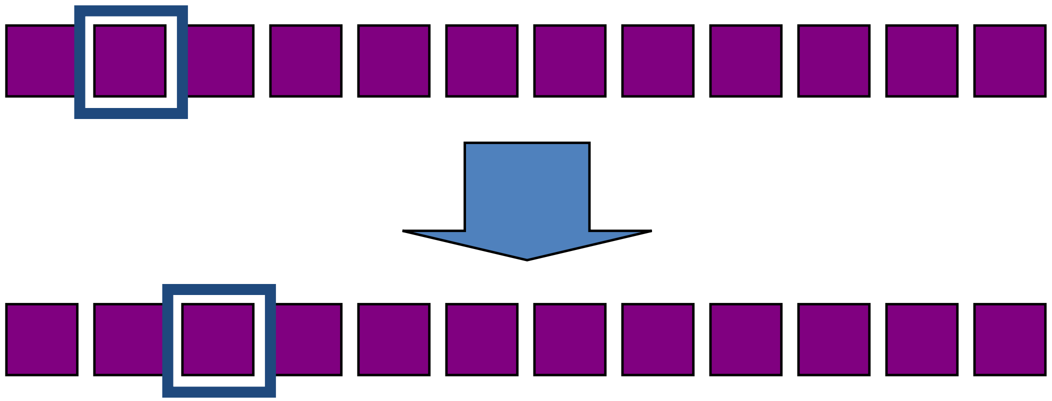
The function value Next(w, L) is the window position of the next successive element in the list;

if we are already at the last element of the list then the value of Next(w, L) is End(L);

if the value of w is End(L), then the operation is undefined

LIST Operations

Next(w,L)



LIST Operations

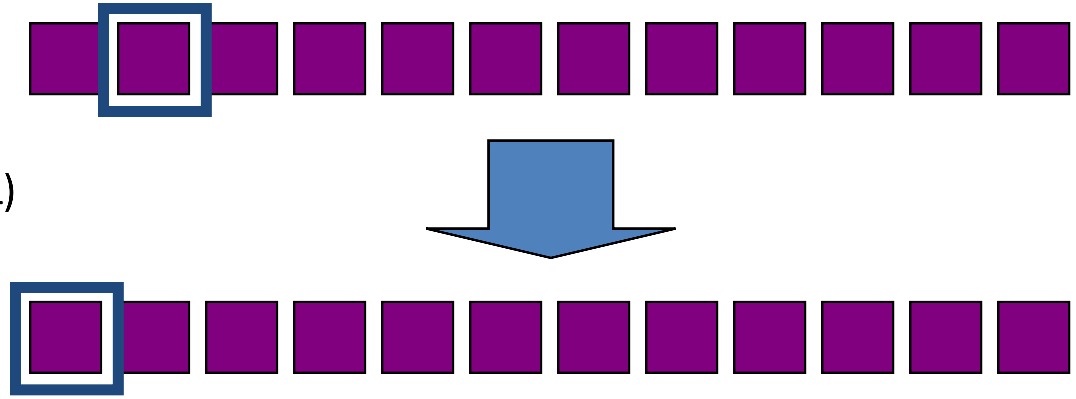
Previous: L x W $\rightarrow$ W :

The function value Previous(w, L) is the window position of the previous element in the list;

if we are already at the beginning of the list (w=First(L)), then the value is undefined

LIST Operations

Previous(w, L)

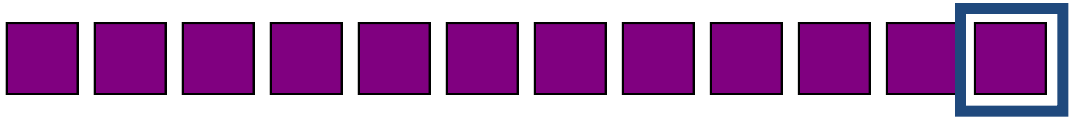


LIST Operations

Last: L $\rightarrow$ W :

The function value Last(L) is the window position of the last element in the list;

if the list is empty, it has the value End(L)



LIST Operations

Insert: $E \times L \times W \rightarrow L \times W$:

Insert(e, w, L)

Insert an element e at position w in the list L, moving elements at w and following positions to the next higher position

$a_1, a_2, \dots a_n \rightarrow a_1, a_2, \dots, a_{w-1}, e, a_w, \dots, a_n$

LIST Operations

If w = End(L) then

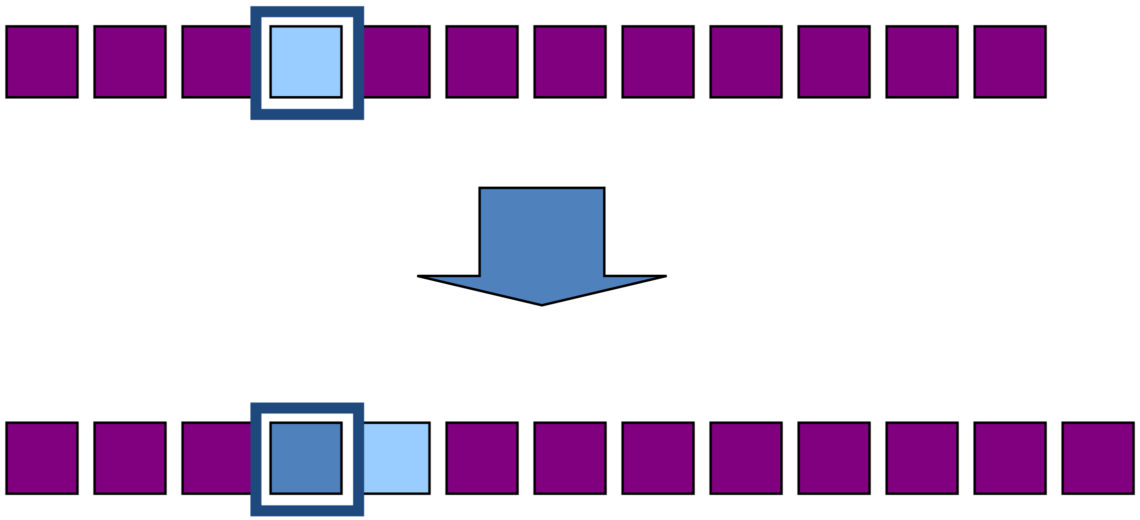
$a_1, a_2, \ldots a_n \rightarrow a_1, a_2, \ldots, a_n, e$

The window w is moved over the new element e

The function value is the list with the element inserted

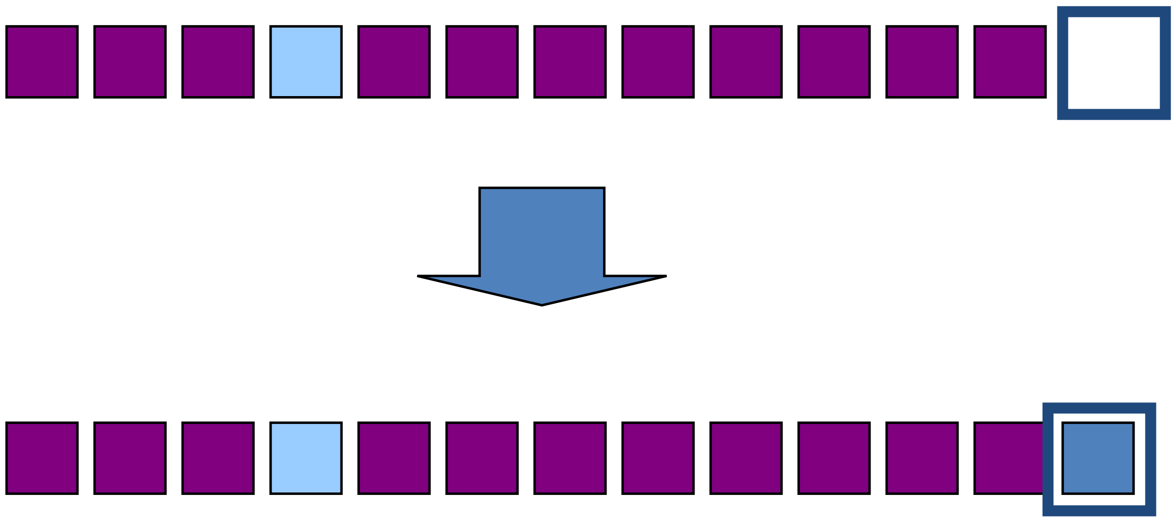
LIST Operations

Insert(e,w,L)



LIST Operations

Insert(e,w,L)



LIST Operations

Delete: $L \times W \rightarrow L \times W$:

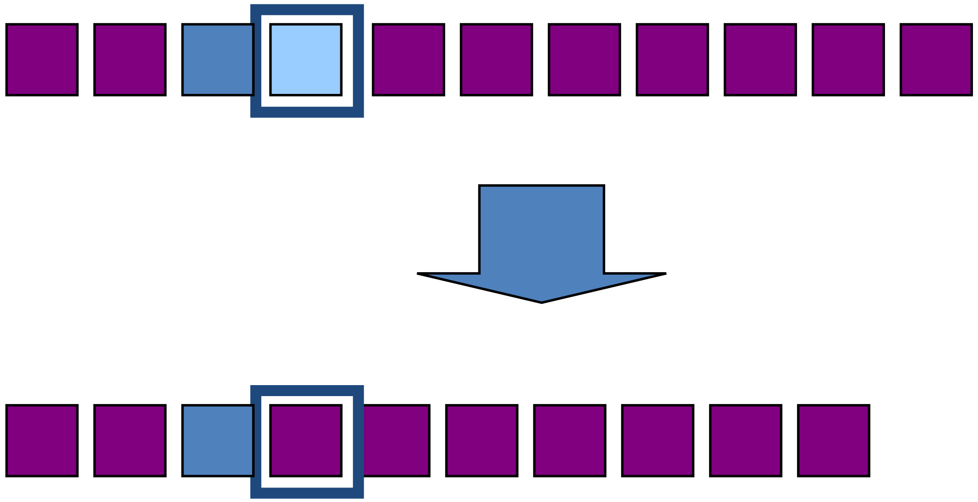
Delete(w, L)
Delete the element at position w in the list L

$a_1, a_2, \ldots a_n \rightarrow a_1, a_2, \ldots, a_{w-1}, a_{w+1}, \ldots, a_n$

- If w = End(L) then the operation is undefined
- The function value is the list with the element deleted

LIST Operations

Delete(w,L)



LIST Operations

Examine: $L \times W \rightarrow E$:

The function value Examine(w, L) is the value of the element at position w in the list;

if we are already at the end of the list (i.e., w = End(L)), then the value is undefined

LIST Operations

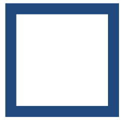
Declare(L)	returns	listtype
End(L)	returns	windowtype
Empty(L)	returns	windowtype
IsEmpty(L)	returns	Boolean
First(L)	returns	windowtype
Next(w,L)	returns	windowtype
Previous(w,L)	returns	windowtype
Last(L)	returns	windowtype
Insert(e,w,L)	returns	listtype
Delete(w,L)	returns	listtype
Examine(w,L)	returns	elementtype

LIST Operations

Example of List manipulation

Declare(L)

w = End(L)



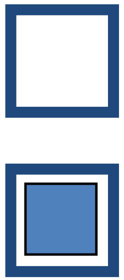
empty list

LIST Operations

Example of List manipulation

w = End(L)

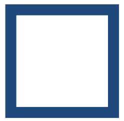
Insert(e, w, L)



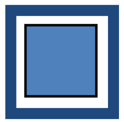
LIST Operations

Example of List manipulation

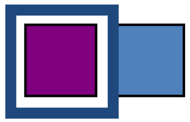
w = End(L)



Insert(e, w, L)



Insert(e, w, L)



LIST Operations

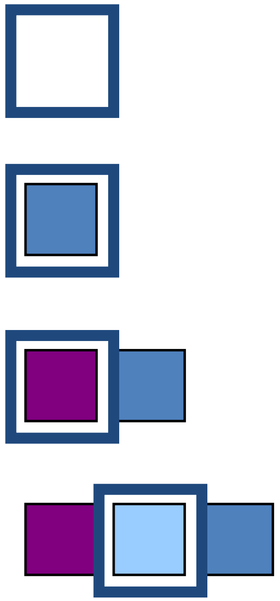
Example of List manipulation

w = End(L)

Insert(e, w, L)

Insert(e, w, L)

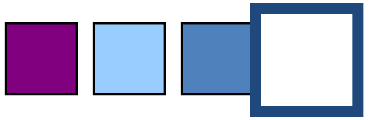
Insert(e, Last(L), L)



LIST Operations

Example of List manipulation

w = Next(Last(L), L)

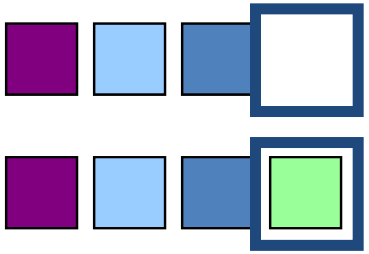


LIST Operations

Example of List manipulation

w = Next(Last(L), L)

Insert(e, w, L)



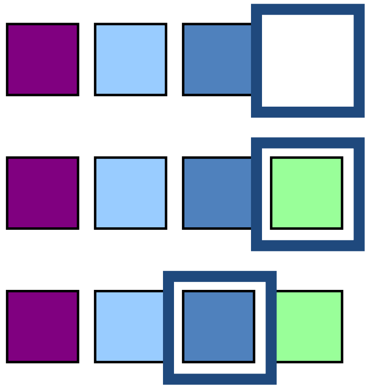
LIST Operations

Example of List manipulation

w = Next(Last(L), L)

Insert(e, w, L)

w = Previous(w, L)



LIST Operations

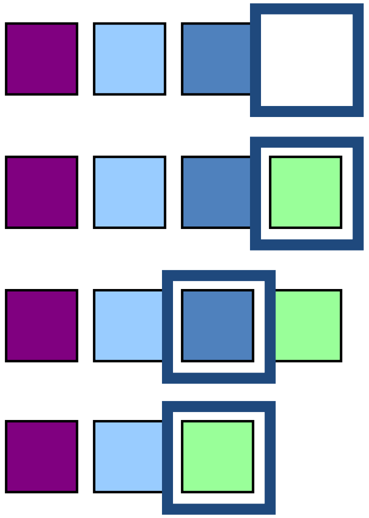
Example of List manipulation

w = Next(Last(L),L)

Insert(e,w,L)

w = Previous(w,L)

Delete(w,L)



ADT Specification

- The key idea is that we have not specified how the lists are to be implemented, merely their values and the operations of which they can be operands

- This ‘old’ idea of data abstraction is one of the key features of object-oriented programming

- C++ is a particular implementation of this object-oriented methodology

ADT Implementation

- Of course, we still have to implement this ADT specification
- The choice of implementation will depend on the requirements of the application

ADT Implementation

We will look at two implementations

- Array implementation
  - uses a static data-structure
  - reasonable if we know in advance the maximum number of elements in the list

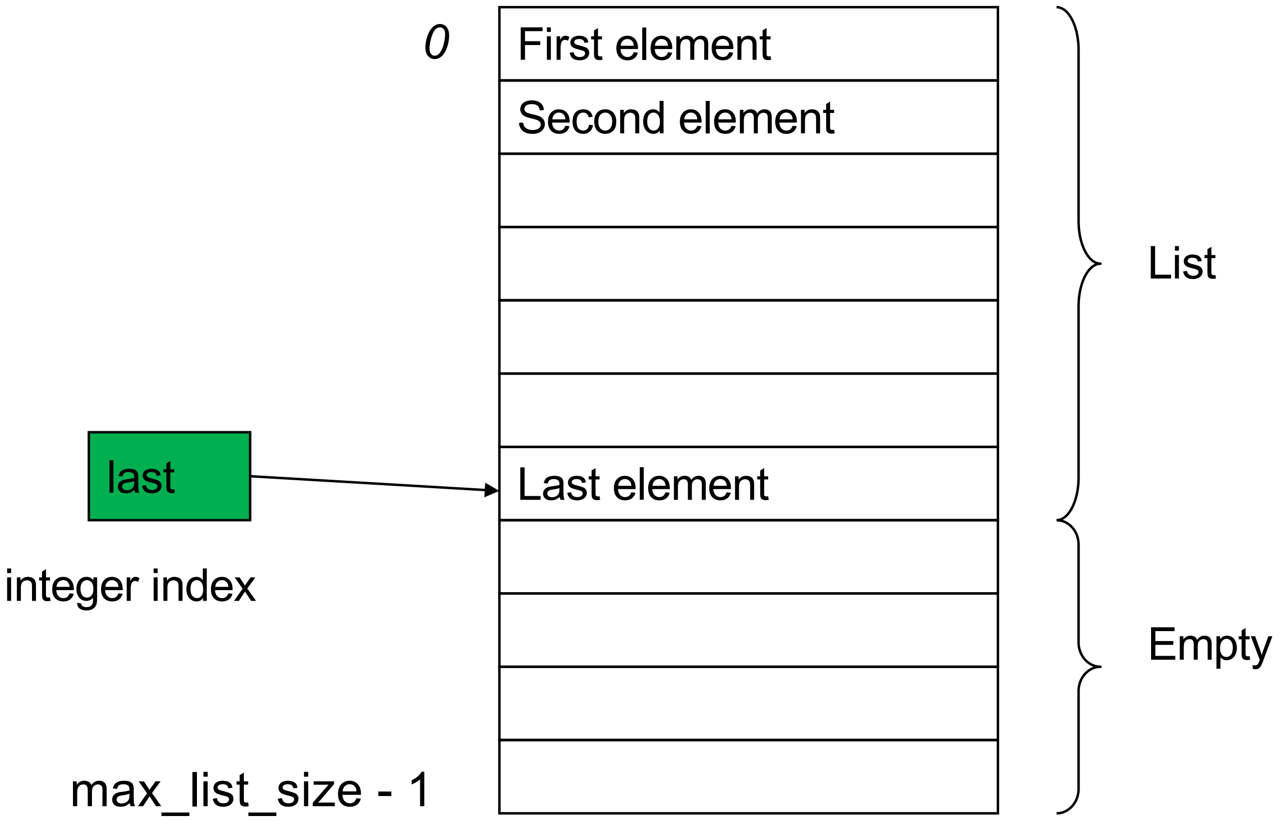
- Pointer implementation
  - Also known as a linked-list implementation
  - uses dynamic data-structure
  - best if we don't know in advance the number of elements in the list (or if it varies significantly)
  - overhead in space: the pointer fields

LIST: Array Implementation

We will do this in two steps:

- the implementation (or representation) of the four constituents datatypes of the ADT:
    - list
    - elementtype
    - Boolean
    - Windowtype
- the implementation of each of the ADT operations

LIST: Array Implementation



LIST: Array Implementation

type elementtype

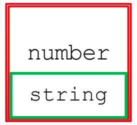
type LIST

type Boolean

type windowtype

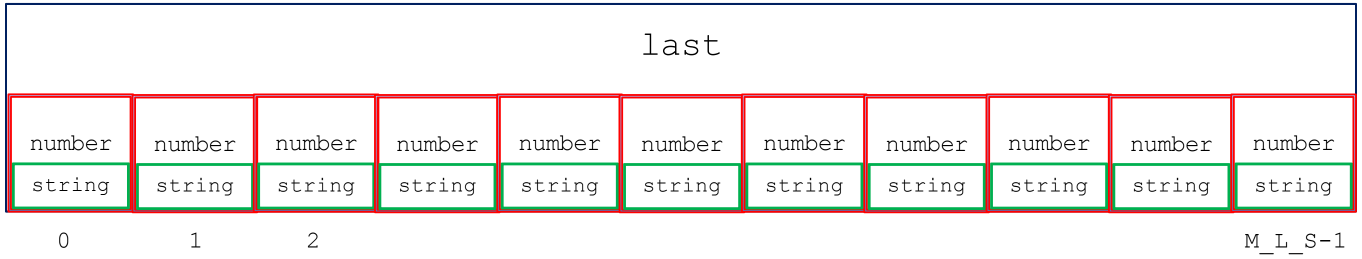
LIST: Array Implementation

```
/* array implementation of LIST ADT */

#include <stdio.h>
#include <math.h>
#include <string.h>

#define MAX_LIST_SIZE 100
#define FALSE 0
#define TRUE 1

typedef struct {
           int number;
           char *string;
        } ELEMENT_TYPE;
```



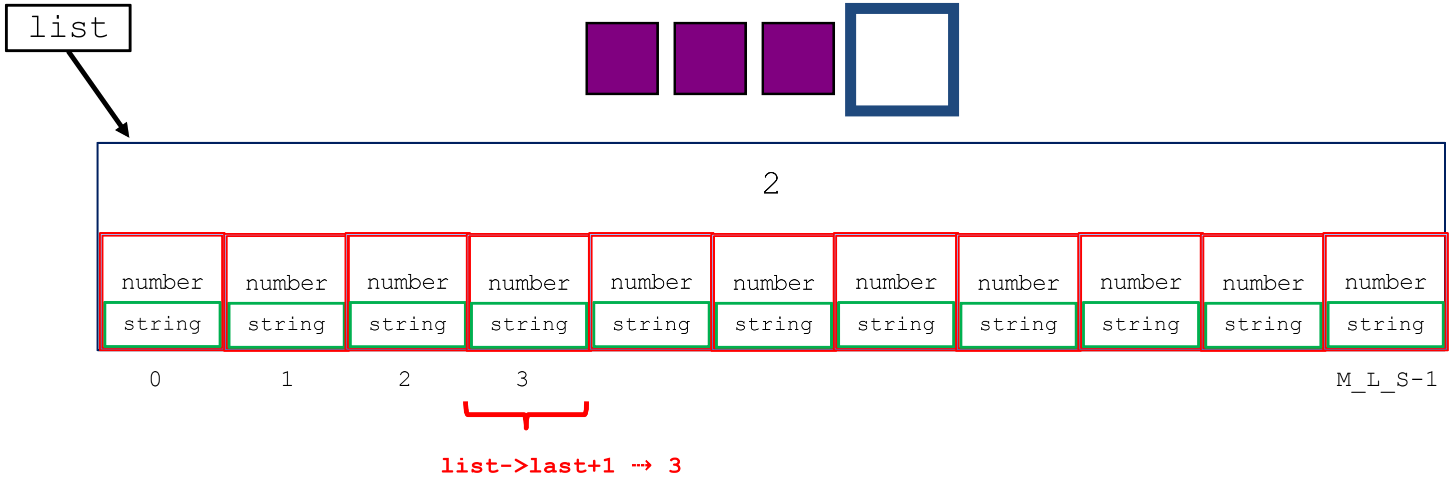
LIST: Array Implementation

```
typedef struct {
            int last;
            ELEMENT_TYPE a[MAX_LIST_SIZE];
        } LIST_TYPE;

typedef int WINDOW_TYPE;
```



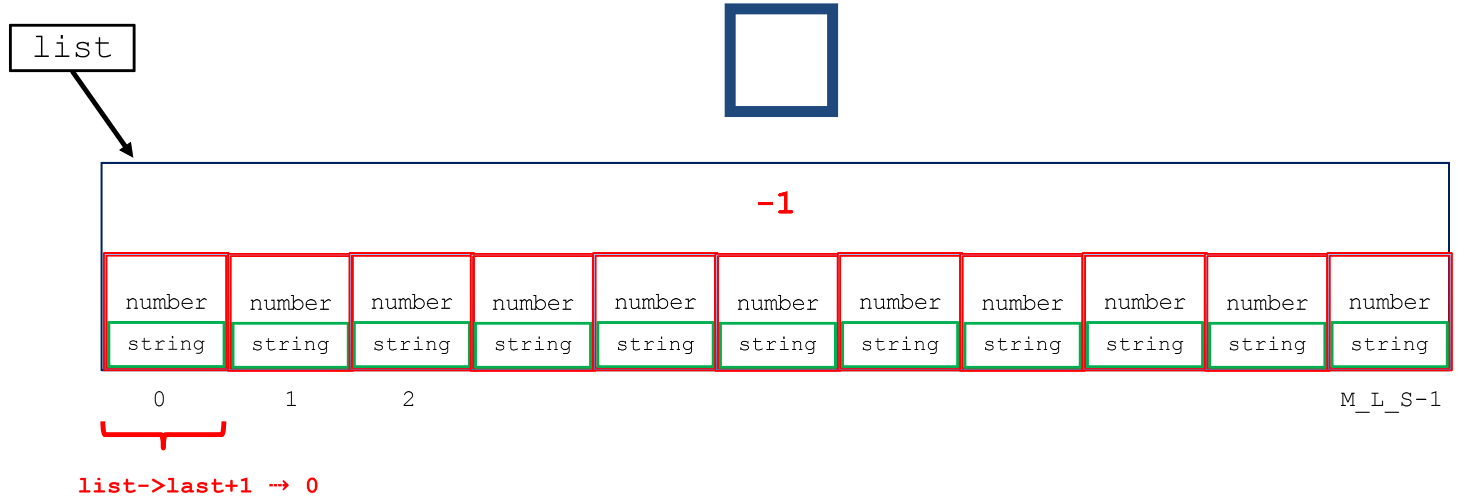
LIST: Array Implementation

```
/*** position following last element in a list ***/

WINDOW_TYPE end(LIST_TYPE *list) {
   return(list->last+1); // return((*list).last+1);
}
```



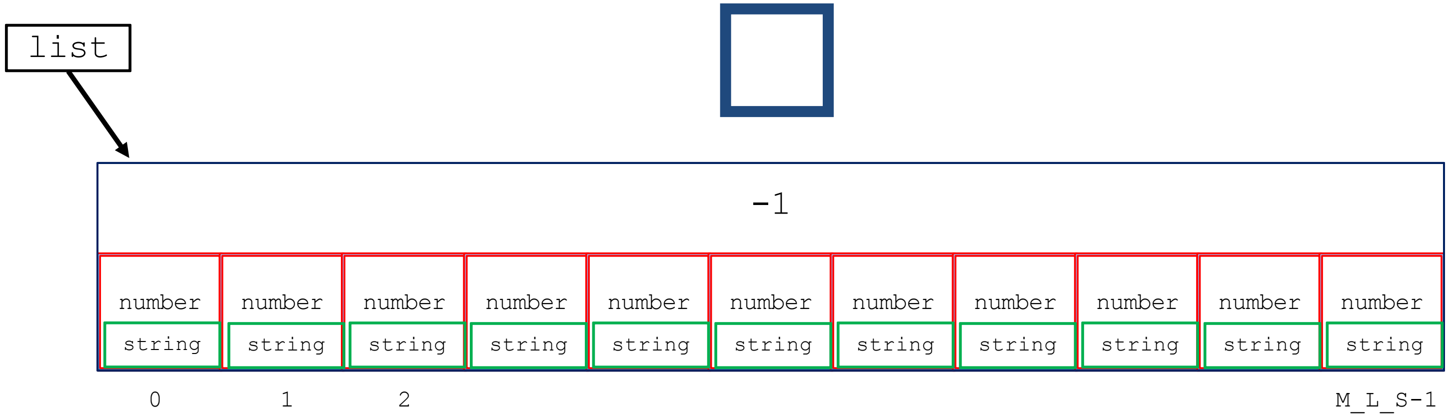
LIST: Array Implementation

```
/*** empty a list ***/

WINDOW_TYPE empty(LIST_TYPE *list) {
    list->last = -1;
    return(end(list));
}
```



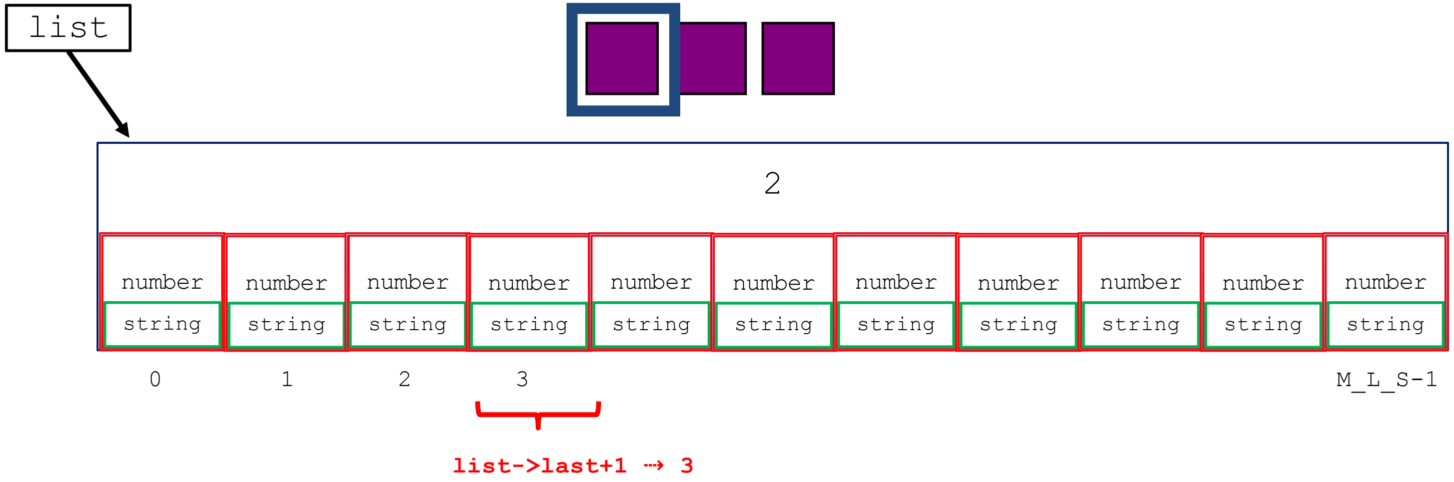
LIST: Array Implementation

```
/*** test to see if a list is empty ***/

int is_empty(LIST_TYPE *list) {
   if (list->last == -1)
      return(TRUE);
   else
      return(FALSE)
}
```



LIST: Array Implementation

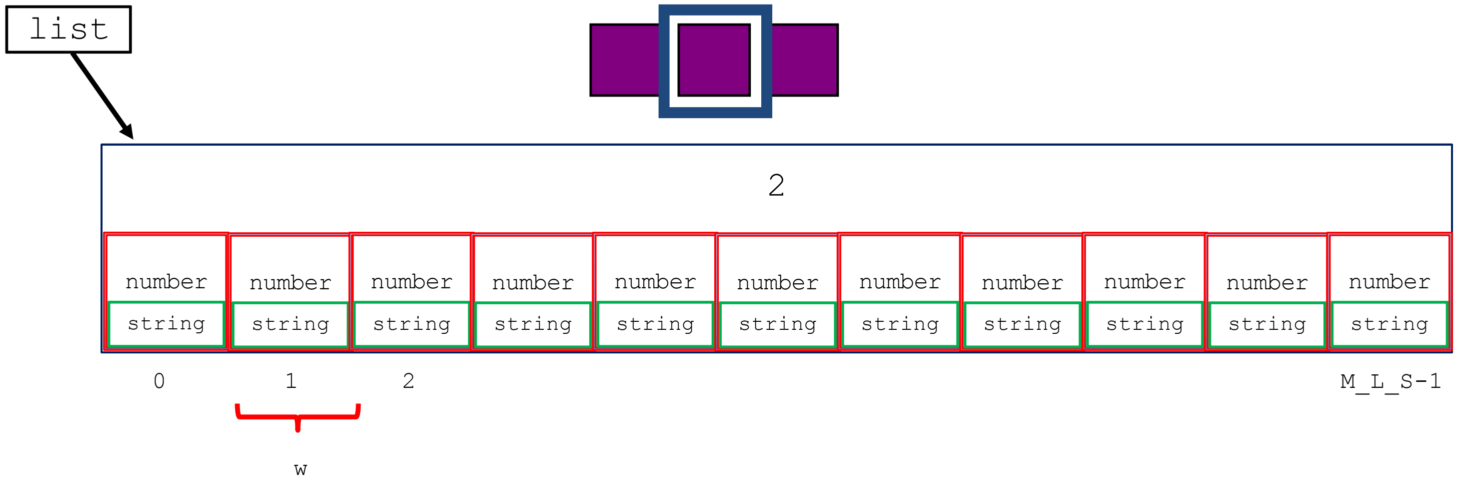
```c
/*** position at first element in a list ***/

WINDOW_TYPE first(LIST_TYPE *list) {
   if (is_empty(list) == FALSE) {
      return(0);
   else
      return(end(list));
}
```

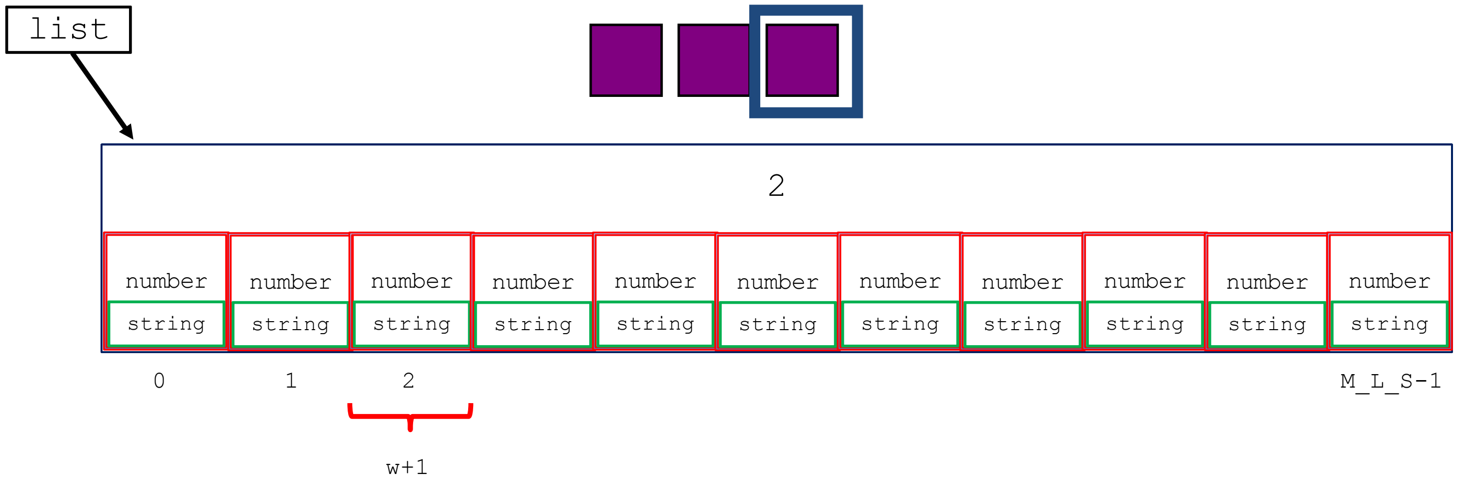


LIST: Array Implementation

```
/*** position at next element in a list ***/

WINDOW_TYPE next(WINDOW_TYPE w, LIST_TYPE *list) {
   if (w == last(list)) {
      return(end(list));
   else if (w == end(list)) {
      error("can't find next after end of list");
   }
   else {
      return(w+1);
   }
}
```

LIST: Array Implementation

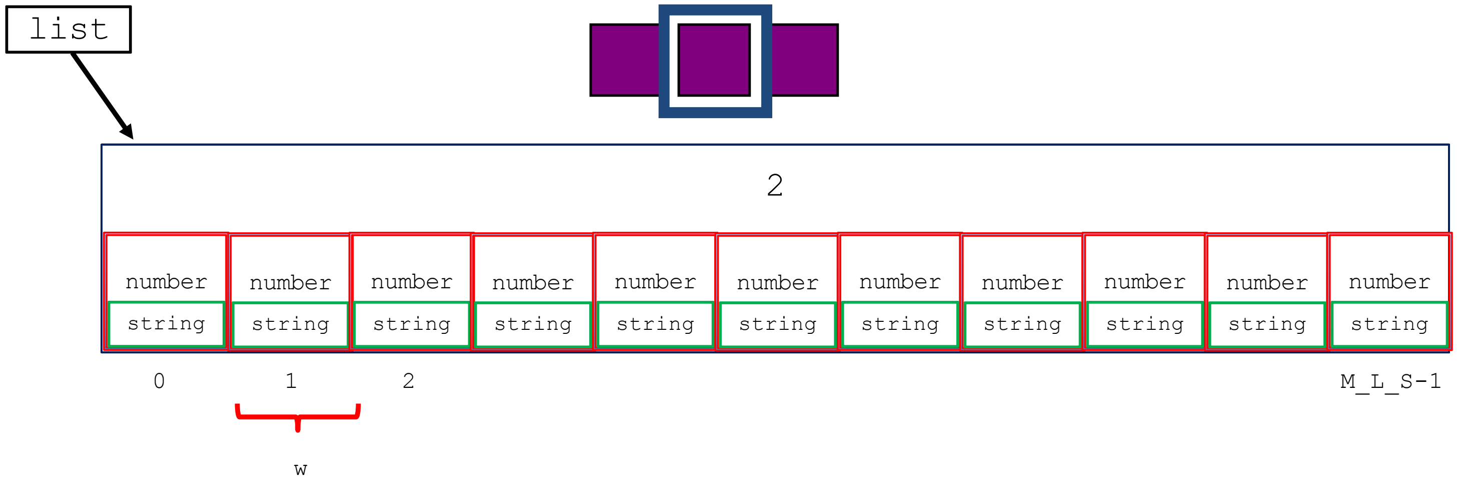


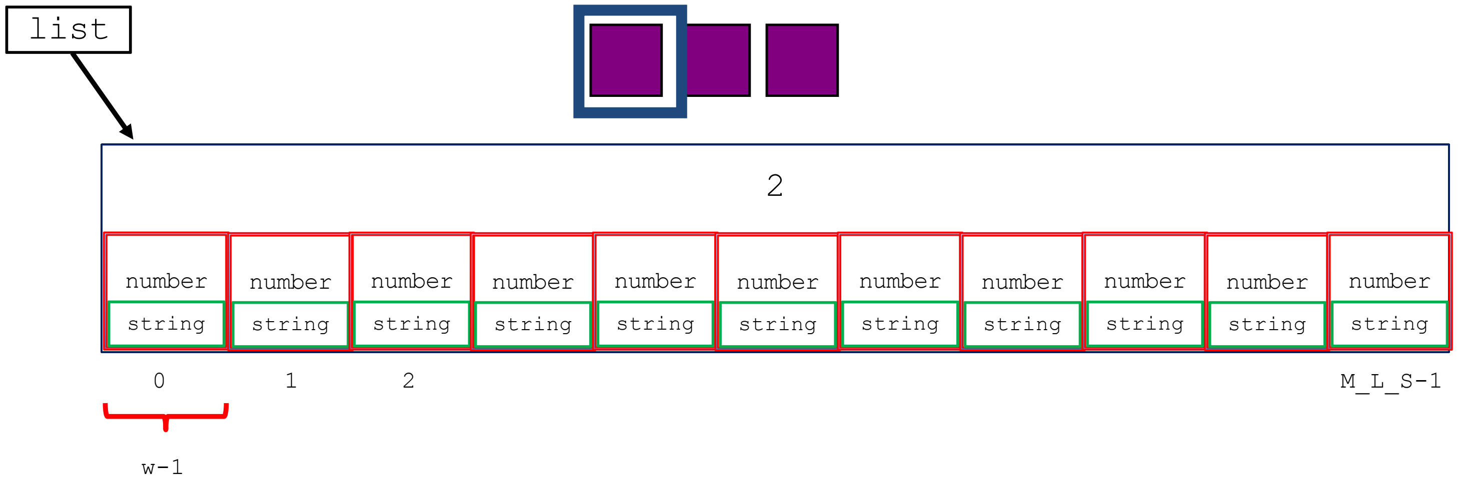


LIST: Array Implementation

```
/*** position at previous element in a list ***/

WINDOW_TYPE previous(WINDOW_TYPE w, LIST_TYPE *list) {
   if (w != first(list)) {
      return(w-1);
   else {
      error("can't find previous before first element of list");
      return(w);
   }
}
```

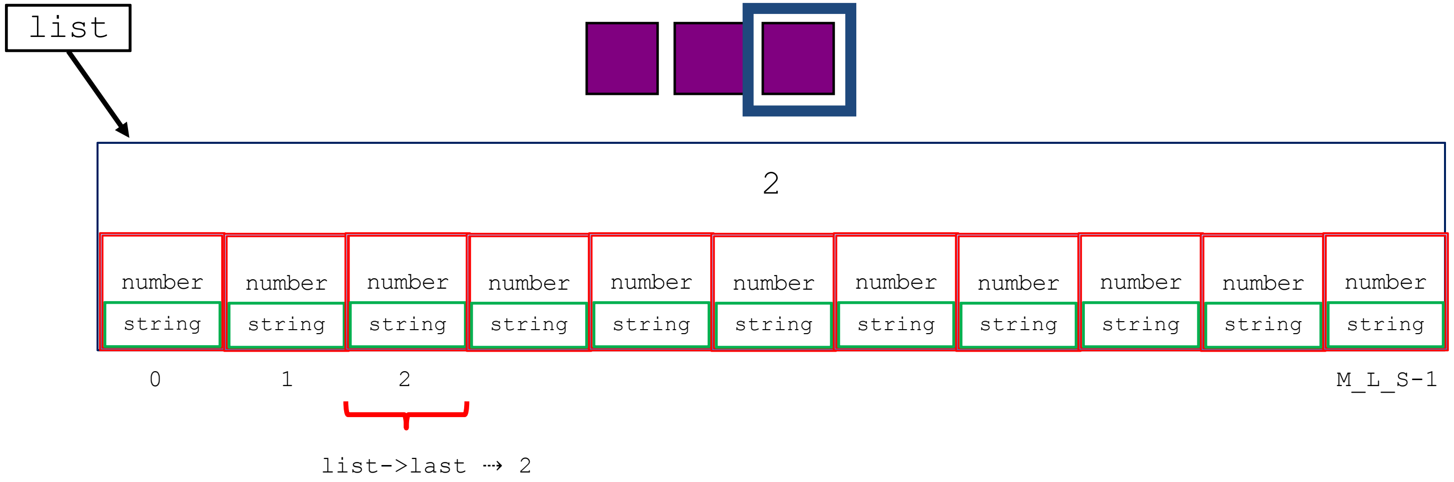
LIST: Array Implementation





LIST: Array Implementation

```
/*** position at last element in a list ***/

WINDOW_TYPE last(LIST_TYPE *list) {
   return(list->last);
}
```



LIST: Array Implementation

```
/*** insert an element in a list ***/

LIST_TYPE *insert(ELEMENT_TYPE e, WINDOW_TYPE w,
                  LIST_TYPE *list) {
   int i;
   if (list->last >= MAX_LIST_SIZE-1) {
      error("Can't insert - list is full");
   }
   else if ((w > list->last + 1) ¦¦ (w < 0)) {
      error("Position does not exist");
   }
   else {
      /* insert it … shift all from w to the right */
```

Insert: E x L x W → L x W :

Insert(e, w, L)

Insert an element e at position w in the list L, moving elements at w and following positions to the next higher position

$a_1, a_2, \ldots a_n \rightarrow a_1, a_2, \ldots, a_{w-1}, e, a_w, \ldots, a_n$

LIST: Array Implementation

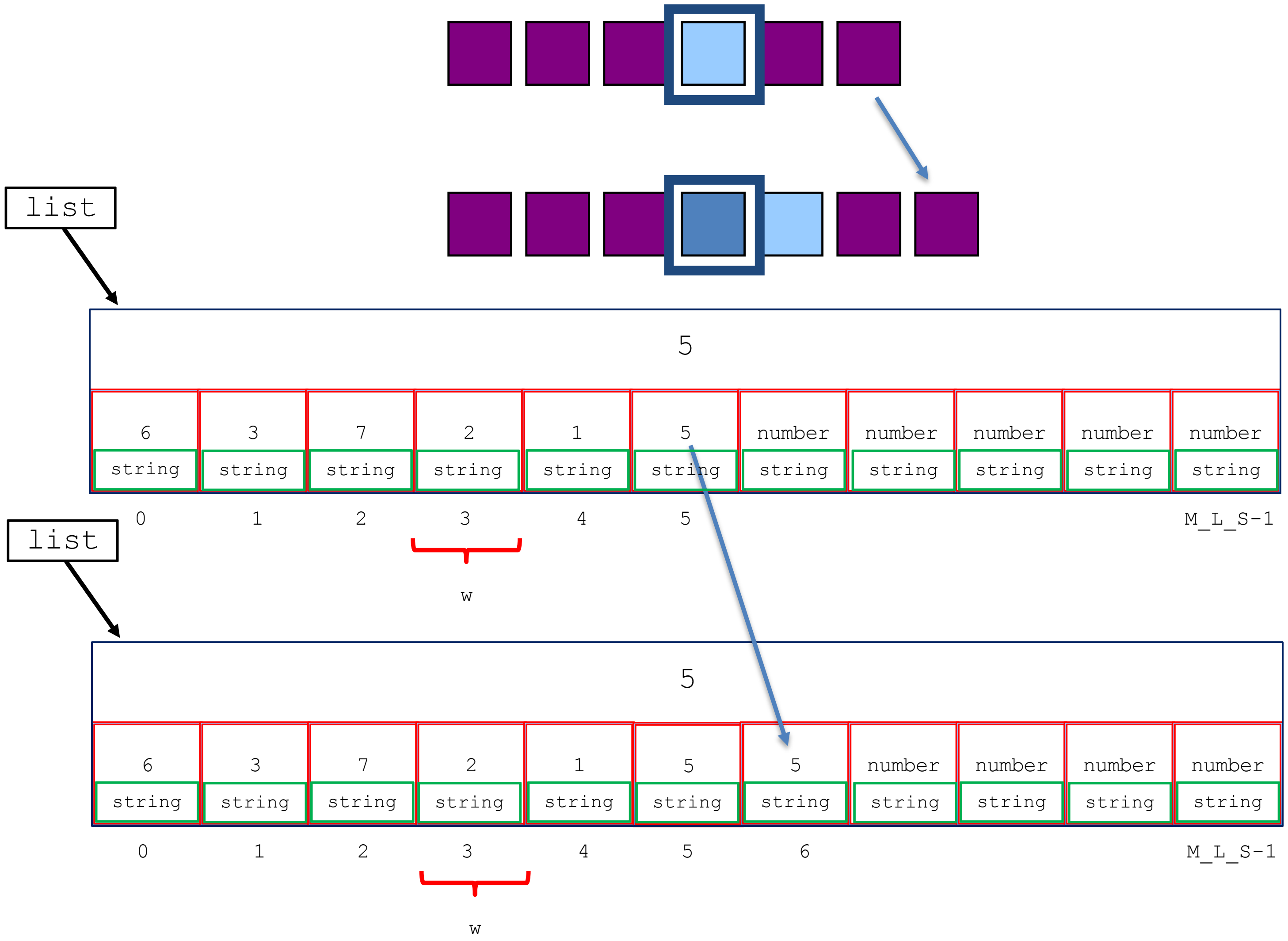
```
        for (i=list->last; i>= w; i--) {
            list->a[i+1] = list->a[i];
        }

        list->a[w] = e;
        list->last = list->last + 1;

        return(list);
    }
}
```

Insert: E x L x W → L x W :

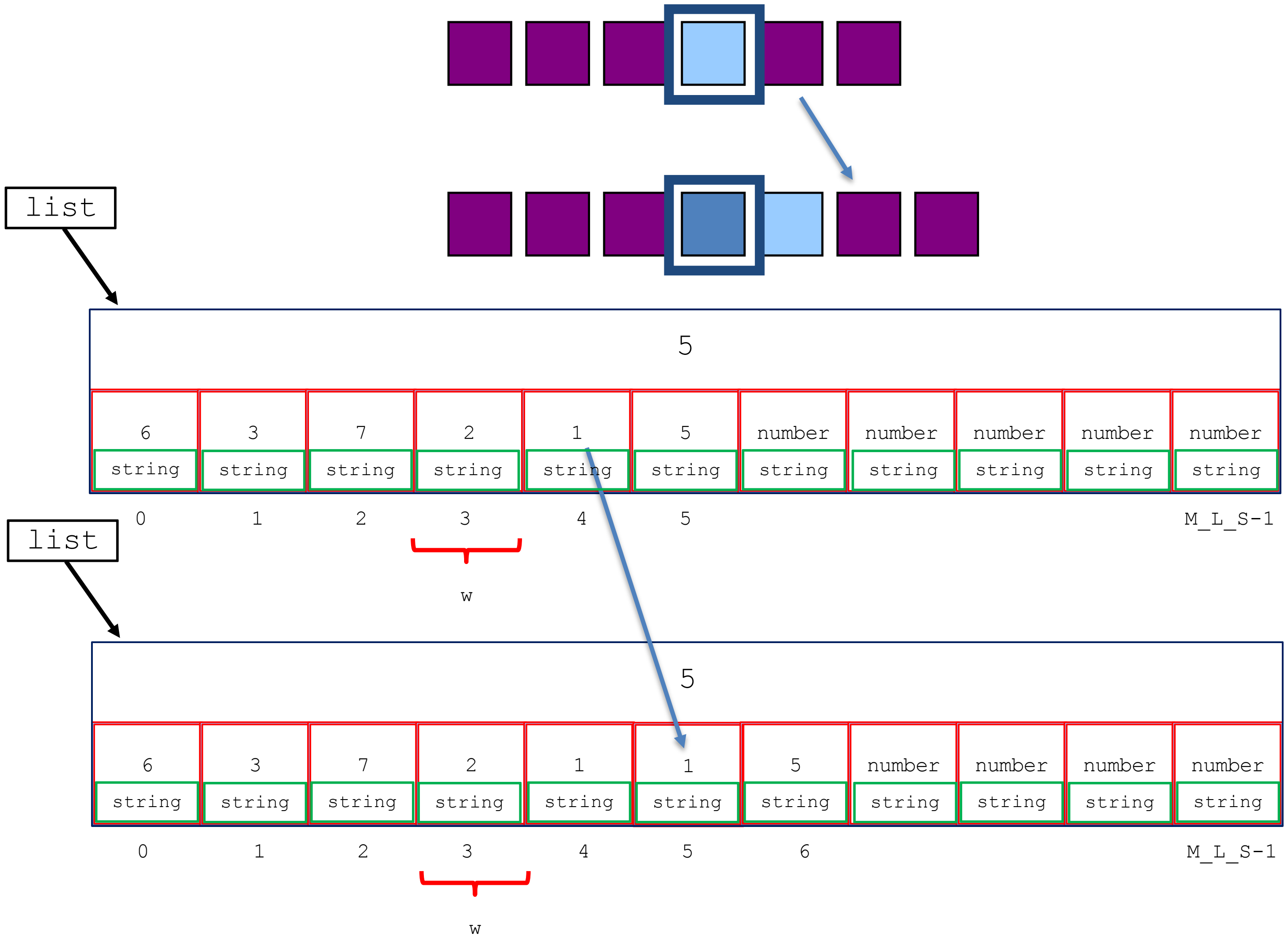
Insert(e, w, L)
Insert an element e at position w in the list L, moving elements at w and following positions to the next higher position

$a_1, a_2, \ldots a_n \rightarrow a_1, a_2, \ldots, a_{w-1}, e, a_w, \ldots, a_n$

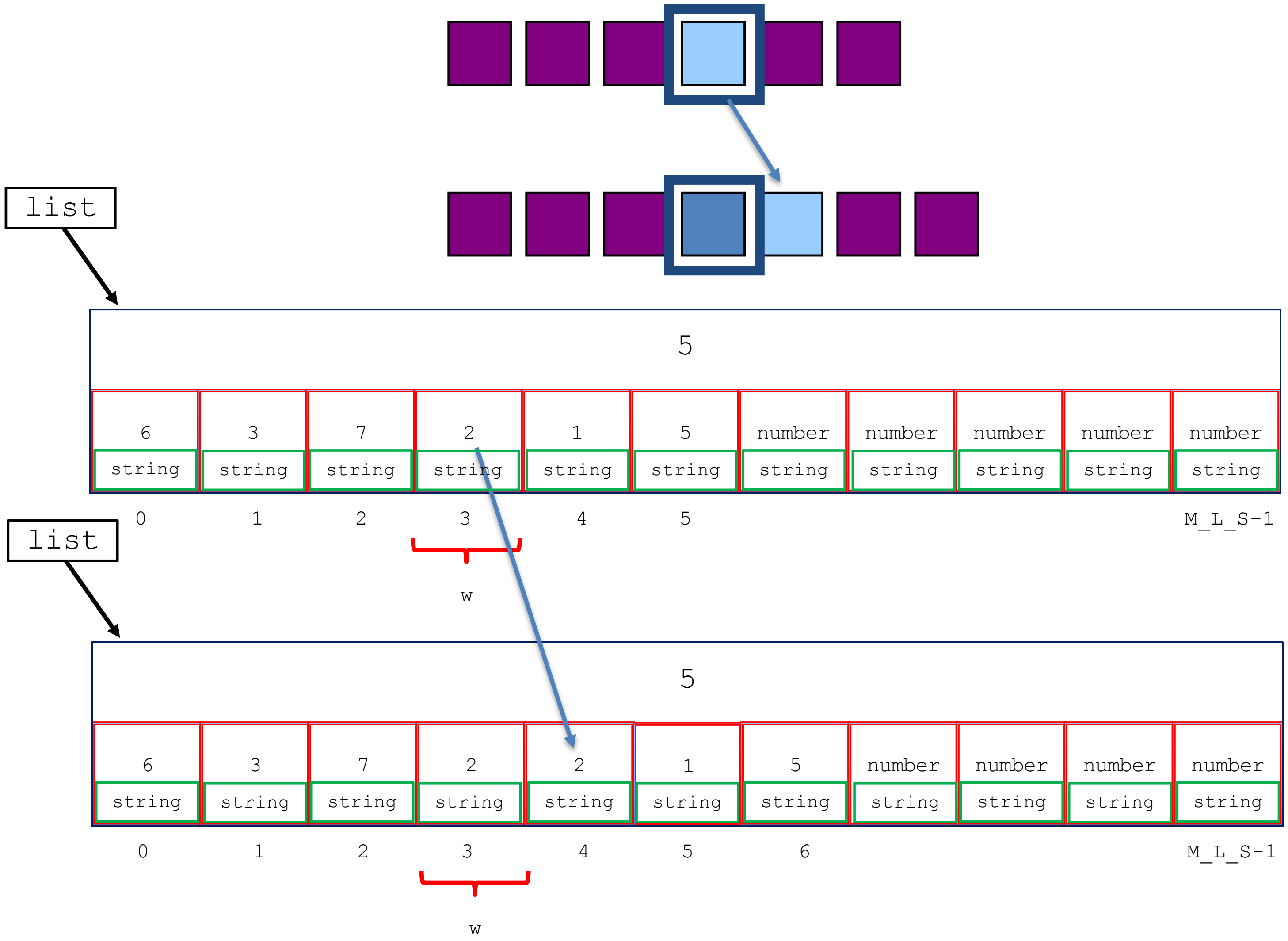
LIST: Array Implementation



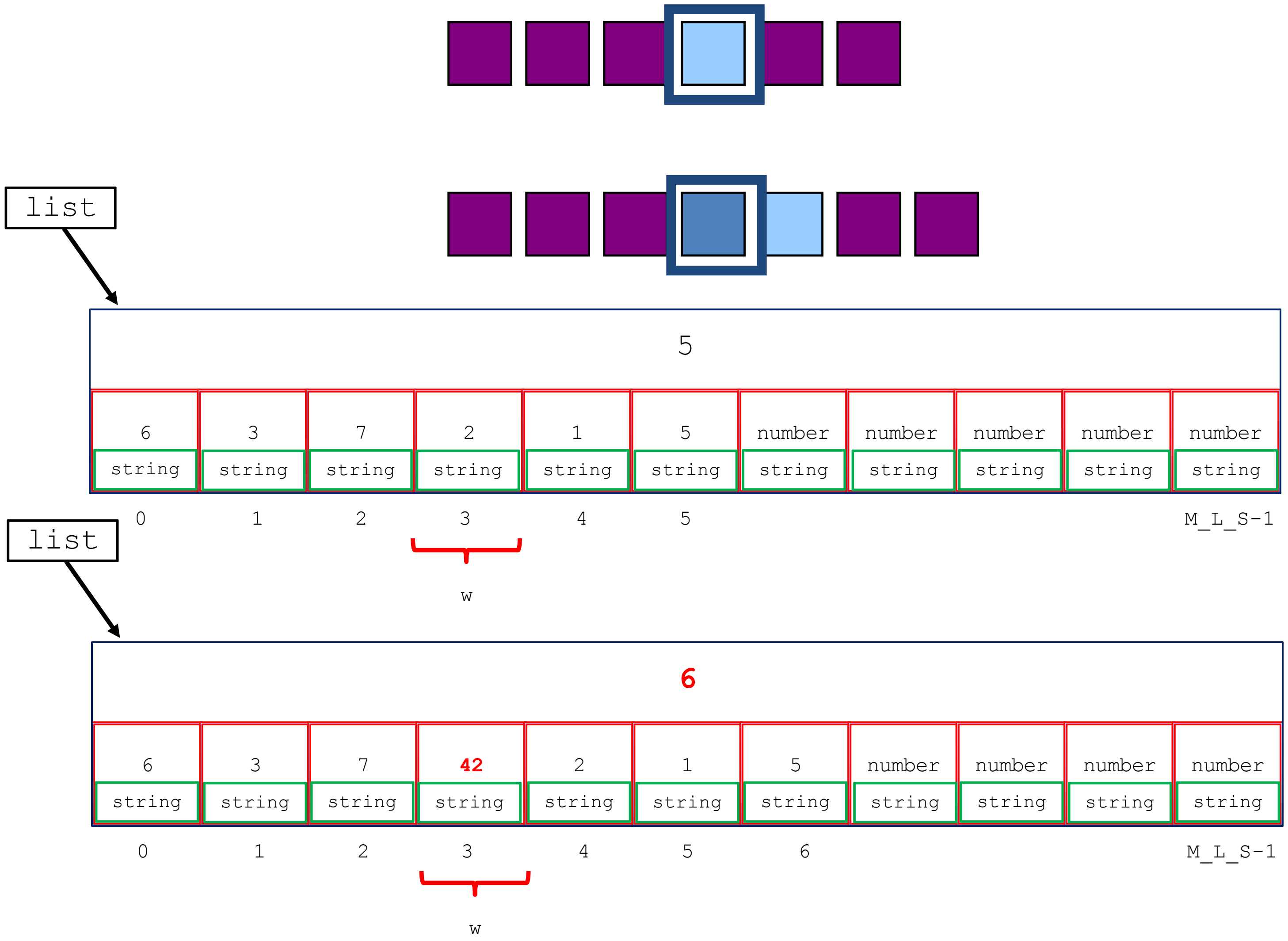
LIST: Array Implementation



LIST: Array Implementation



LIST: Array Implementation



LIST: Array Implementation

```
/*** delete an element from a list ***/

LIST_TYPE *delete(WINDOW_TYPE w, LIST_TYPE *list) {
   int i;
   if ((w > list->last) ¦¦ (w < 0)) {
      error("Position does not exist");
   }
   else {
      /* delete it … shift all after w to the left */
      list->last = list->last - 1;
      for (i=w; i < list->last; i++) {
         list->a[i] = list->a[i+1];
      }
      return(list);
   }
}
```

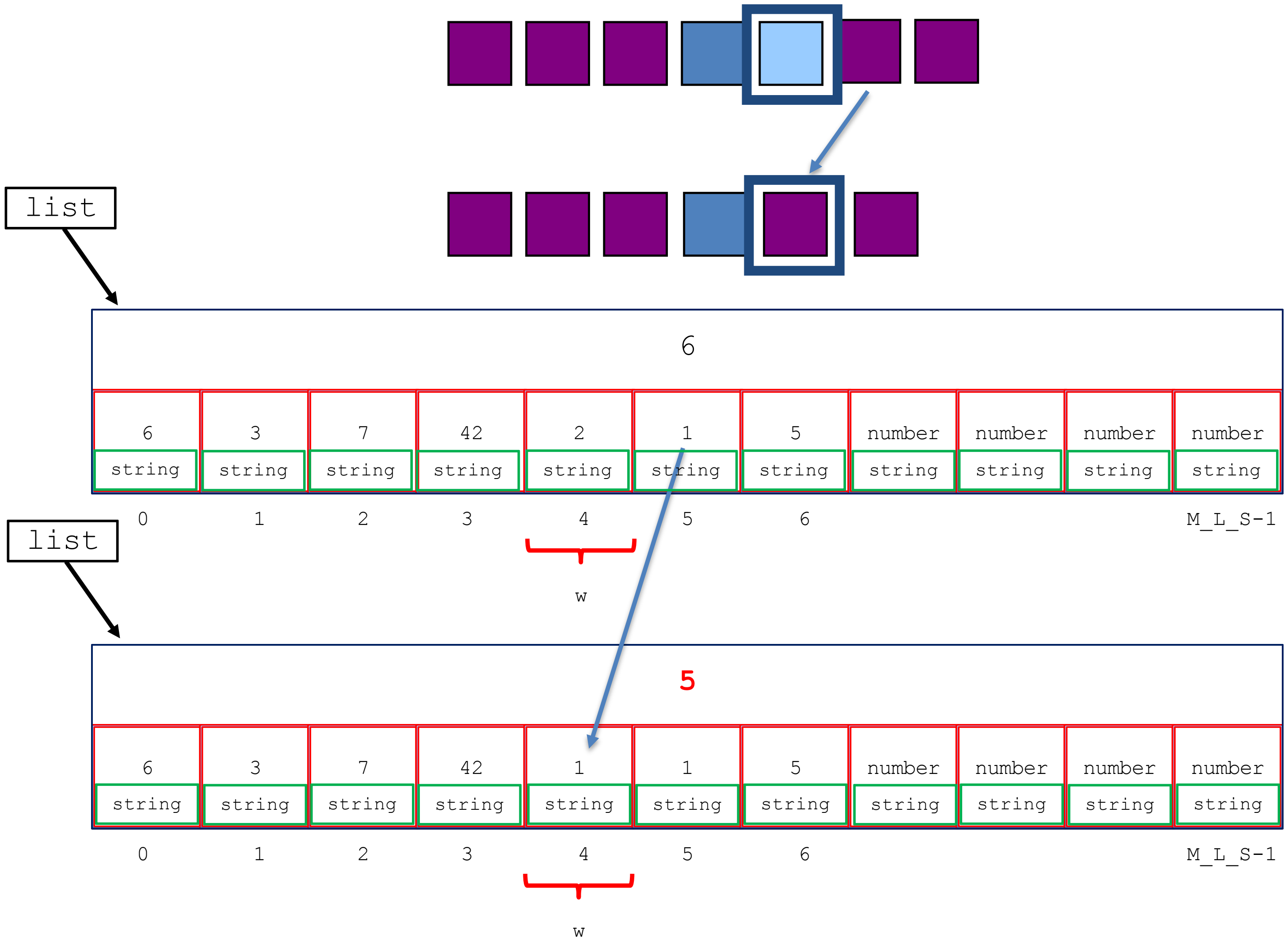
Delete: L x W → L x W :

Delete(w, L)
Delete the element at position w in the list L

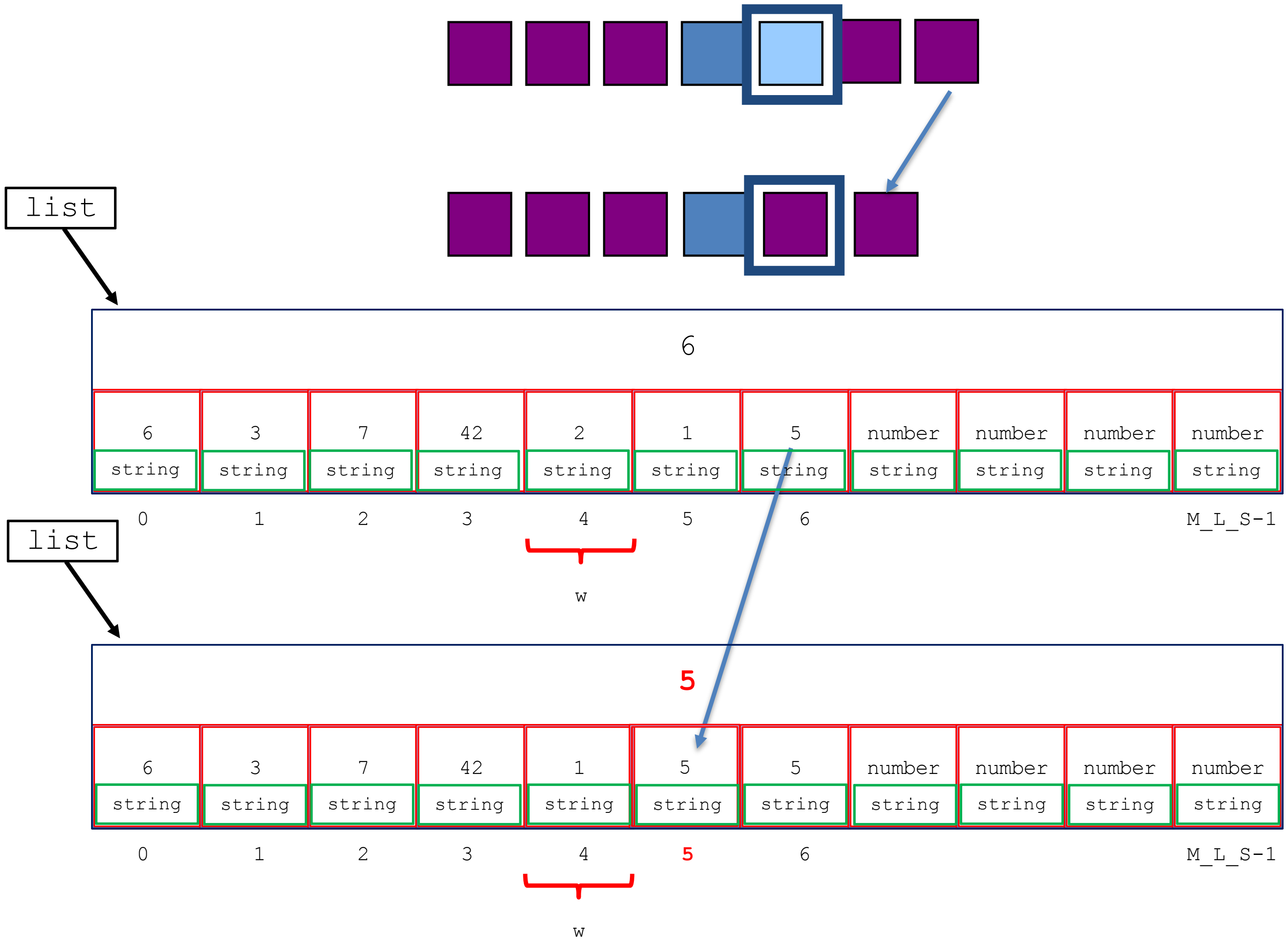
$a_1, a_2, \ldots a_n \rightarrow a_1, a_2, \ldots, a_{w-1}, a_{w+1}, \ldots, a_n$

- If w = End(L) then the operation is undefined
- The function value is the list with the element deleted

LIST: Array Implementation



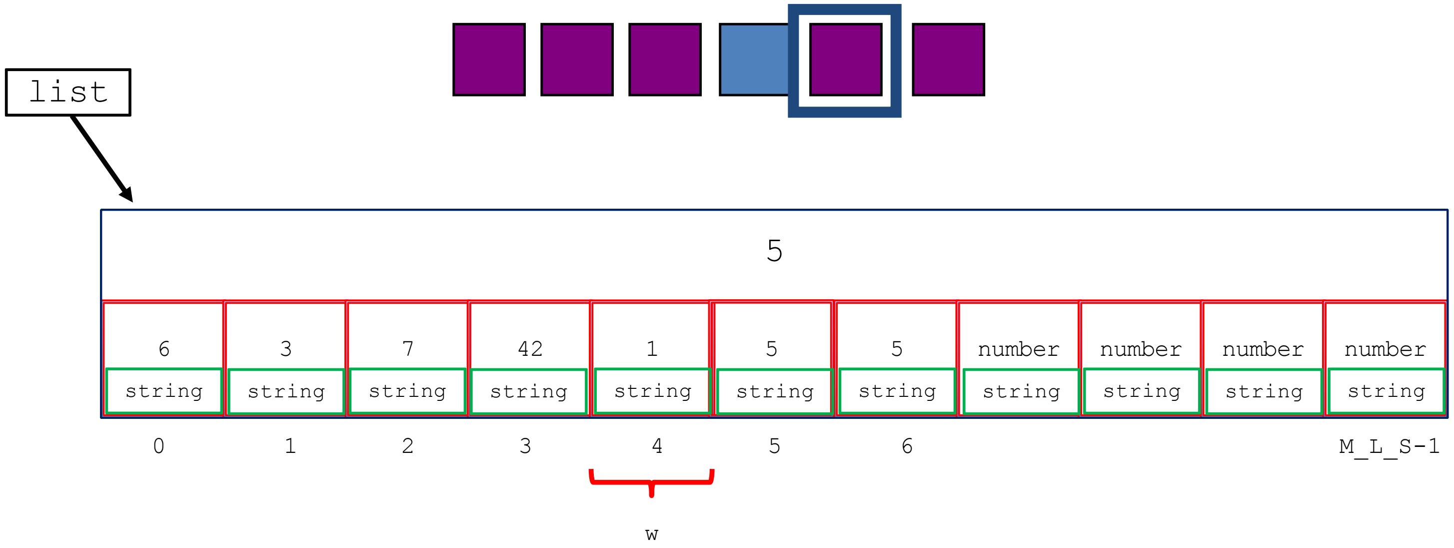
LIST: Array Implementation



LIST: Array Implementation

```
/*** retrieve an element from a list ***/

ELEMENT_TYPE retrieve(WINDOW_TYPE w, LIST_TYPE *list) {
   if ( (w < 0)) ¦¦  (w > list->last)) {

      /* list is empty */

      error("Position does not exist");
   }
   else {
      return(list->a[w]);
   }
}
```

LIST: Array Implementation



list->a[4].number … 1

LIST: Array Implementation

```
/*** assign values to an element ***/

int assign_element_values(ELEMENT_TYPE *e, int number, char s[]){
   e->string = (char *) malloc(sizeof(char)* (strlen(s)+1));
   strcpy(e->string, s);
   e->number = number;
}
```

LIST: Array Implementation

```
/*** print all elements in a list ***/

int print(LIST_TYPE *list) { // rewrite as application code
   WINDOW_TYPE w;
   ELEMENT_TYPE e;
   printf("Contents of list: \n");
   w = first(list);
   while (w != end(list)) {
      e = retrieve(w, list);
      printf("%d %s\n", e.number, e.string);
      w = next(w, list);
   }
   printf("---\n");
   return(0);
}
```

LIST: Array Implementation

```
/*** error handler: print message passed as argument and
     take appropriate action                          ***/

int error(char *s) {
   printf(“Error: %s\n”, s);
   exit(0);
}
```

LIST: Array Implementation

```
/*** main application routine ***/

WINDOW_TYPE w;
ELEMENT_TYPE e;
LIST_TYPE list;
int i;

empty(&list);
print(&list);

assign_element_values(&e, 1, "String A");
w = first(&list);
insert(e, w, &list);
print(&list);
```

LIST: Array Implementation

```
assign_element_values(&e, 2, “String B”);
insert(e, w, &list);
print(&list);

assign_element_values(&e, 3, “String C”);
insert(e, last(&list), &list);
print(&list);

assign_element_values(&e, 4, “String D”);
w = next(last(&list), &list);
insert(e, w, &list);
print(&list);
```

LIST: Array Implementation

```
    w = previous(w, &list);
    delete(w, &list);
    print(&list);

}
```

LIST: Array Implementation

Key points:

- we have implemented all list manipulation operations with dedicated access functions
- we never directly access the data-structure when using it, but we always use the access functions
- Why?

LIST: Array Implementation

Key points:

- greater security: localized control and more resilient software maintenance
- data hiding: the implementation of the data-structure is hidden from the user and so we can change the implementation and the user will never know

LIST: Array Implementation

Possible problems with the implementation:

- have to shift elements when inserting and deleting (i.e., insert and delete are O(n))
- have to specify the maximum size of the list at compile time