Data Structures and Algorithms for Engineers

Module 7: Graphs

Lecture 2: Breadth-First Search (BFS) traversal & application of BFS

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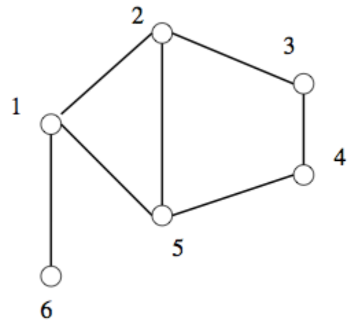
Traversing a Graph

- Visit every vertex and edge in a systematic way

- Key idea: mark each vertex when we first visit it & keep track of what we have not yet completely explored



- Each vertex will exist in one of three states
  1. Undiscovered – the vertex is in its initial untouched state
  2. Discovered – the vertex has been found, but we have not yet processed all its edges
  3. Processed – the vertex after we have visited all its edges

Traversing a Graph

- Keep a record of all the vertices discovered but not yet completely processed

- Begin with a starting vertex

You have to decide where to start
or be told where to start

- Explore each vertex
  - Evaluate each edge leaving it
  - If the edge goes to an undiscovered vertex
    - Mark it discovered
    - Add it to the list of work to do
  - If the edge goes to a processed vertex, ignore it
  - If the edge goes to a discovered unprocessed vertex, ignore it

Traversing a Graph

- There are two primary graph traversal algorithms
  - Breadth-first search (BFS)
  - Depth-first search (DFS)
- The difference is the order in which they explore vertices

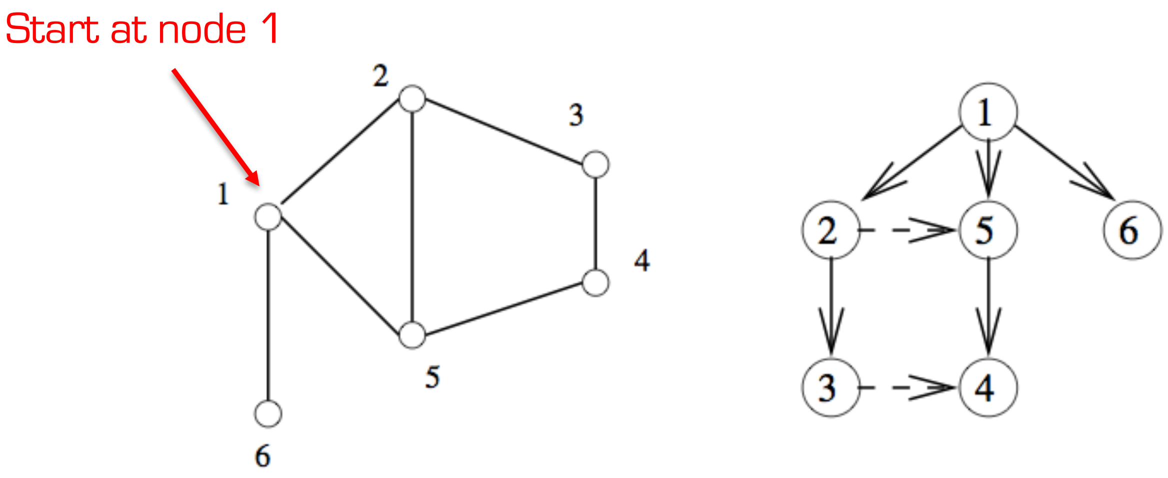
Traversing a Graph

The order depends completely on the container data structure used to store the discovered but not processed vertices

- BFS uses a **queue**
  - By storing the vertices in a FIFO queue, we explore the **oldest** unexplored vertices first
  - Thus explorations radiate out slowly from the starting vertex
- DFS uses a **stack**
  - By storing the vertices in a LIFO stack, we explore the vertices by diving down a path, visiting a new neighbour if one is available, and backing up only when we are surrounded by (i.e. connected by edges to) previously discovered vertices
  - Thus explorations quickly wander away from out starting vertex

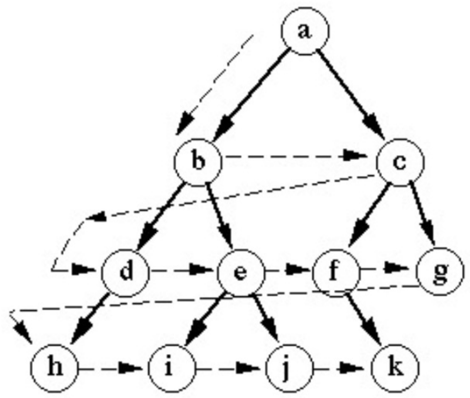
Traversing a Graph

Breadth-first search (BFS)



Traversing a Graph

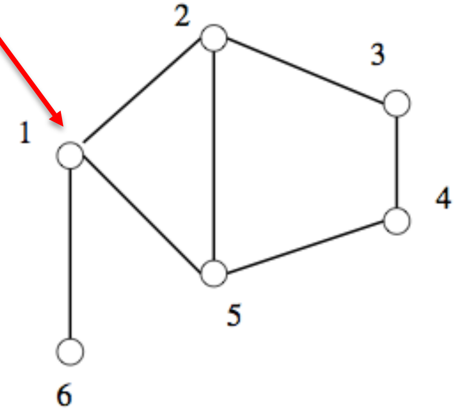
Breadth-first search (BFS)

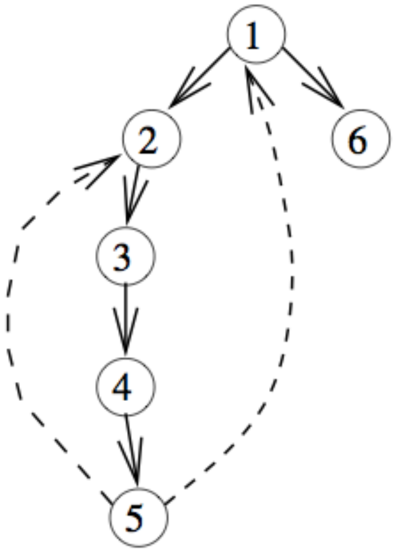


Traversing a Graph

Depth-first search (DFS)

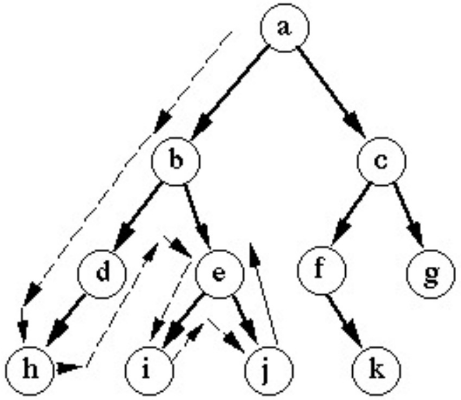
Start at node 1





Traversing a Graph

Depth-first search (DFS)



Breadth-First Search

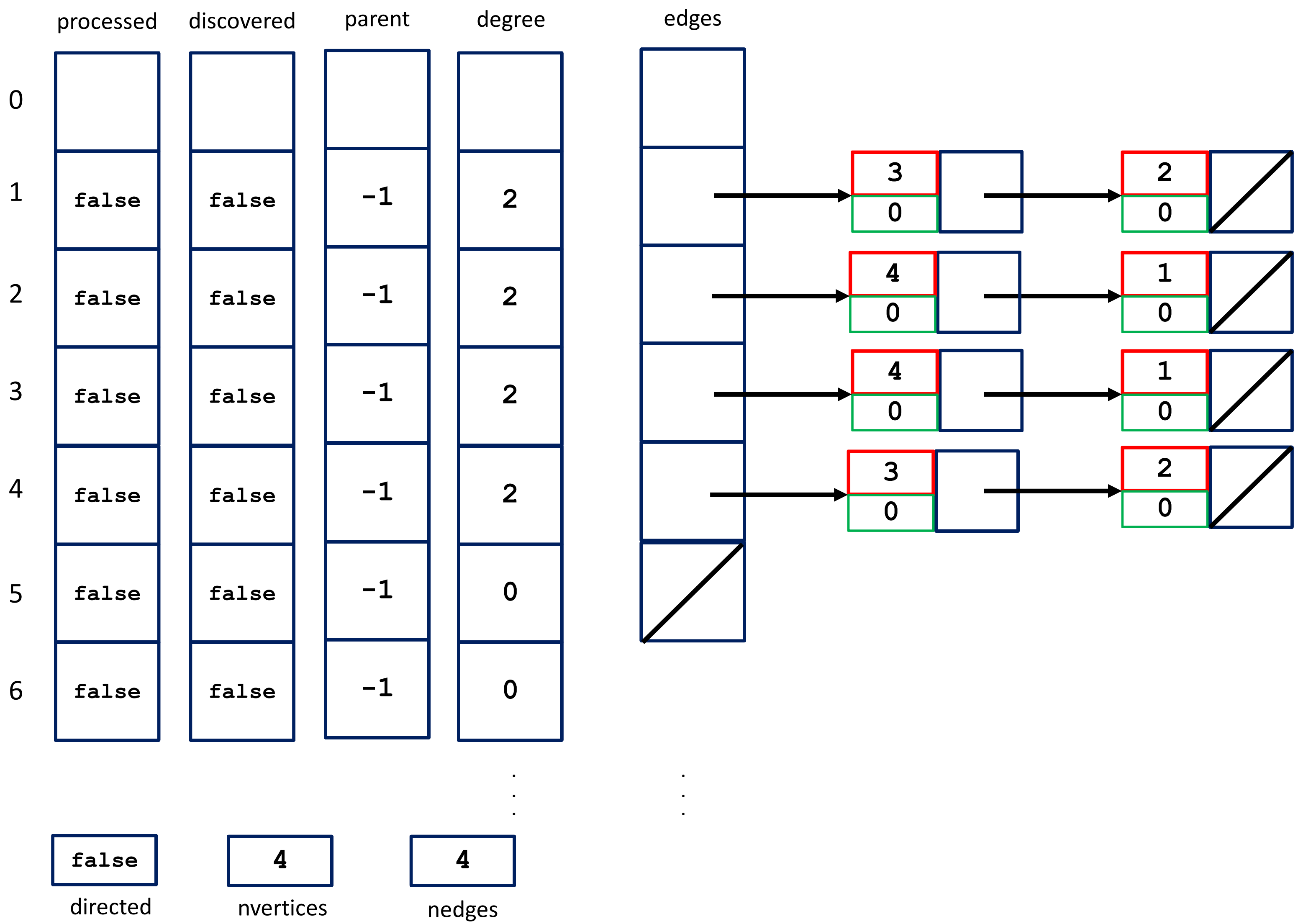
- Assign a direction to each edge, from discoverer vertex u to discovered vertex v
- Since each node has exactly one parent, except for the root (i.e., start vertex), this defines a tree on the vertices of the graph
- This tree defines the shortest path from the root to every other node in the tree
- This makes the BFS very useful for in shortest path problems
(in unweighted graphs)

Breadth-First Search

```
BFS(G, s)
    for each vertex u ∈ V[G] − {s} do
        state[u] = "undiscovered"
        p[u] = nil, i.e. no parent is in the BFS tree
    state[s] = "discovered"
    p[s] = nil
    Q = {s}
    while Q ≠ ∅ do
        u = dequeue[Q]
        process vertex u as desired
        for each v ∈ Adj[u] do
            process edge (u, v) as desired
            if state[v] = "undiscovered" then
                state[v] = "discovered"
                p[v] = u
                enqueue[Q, v]
        state[u] = "processed"
```

Breadth-First Search

```
/* Breadth-First Search                                                 */

bool processed[MAXV+1];    /* which vertices have been processed     */
bool discovered[MAXV+1];   /* which vertices have been found         */
int parent[MAXV+1];        /* discovery relation                     */


/* Each vertex is initialized as undiscovered:                          */

initialize_search(graph *g){

   int i;                          /* counter */

   for (i=1; i<=g->nvertices; i++) {
      processed[i] = discovered[i] = false;
      parent[i] = -1;
   }
}
```

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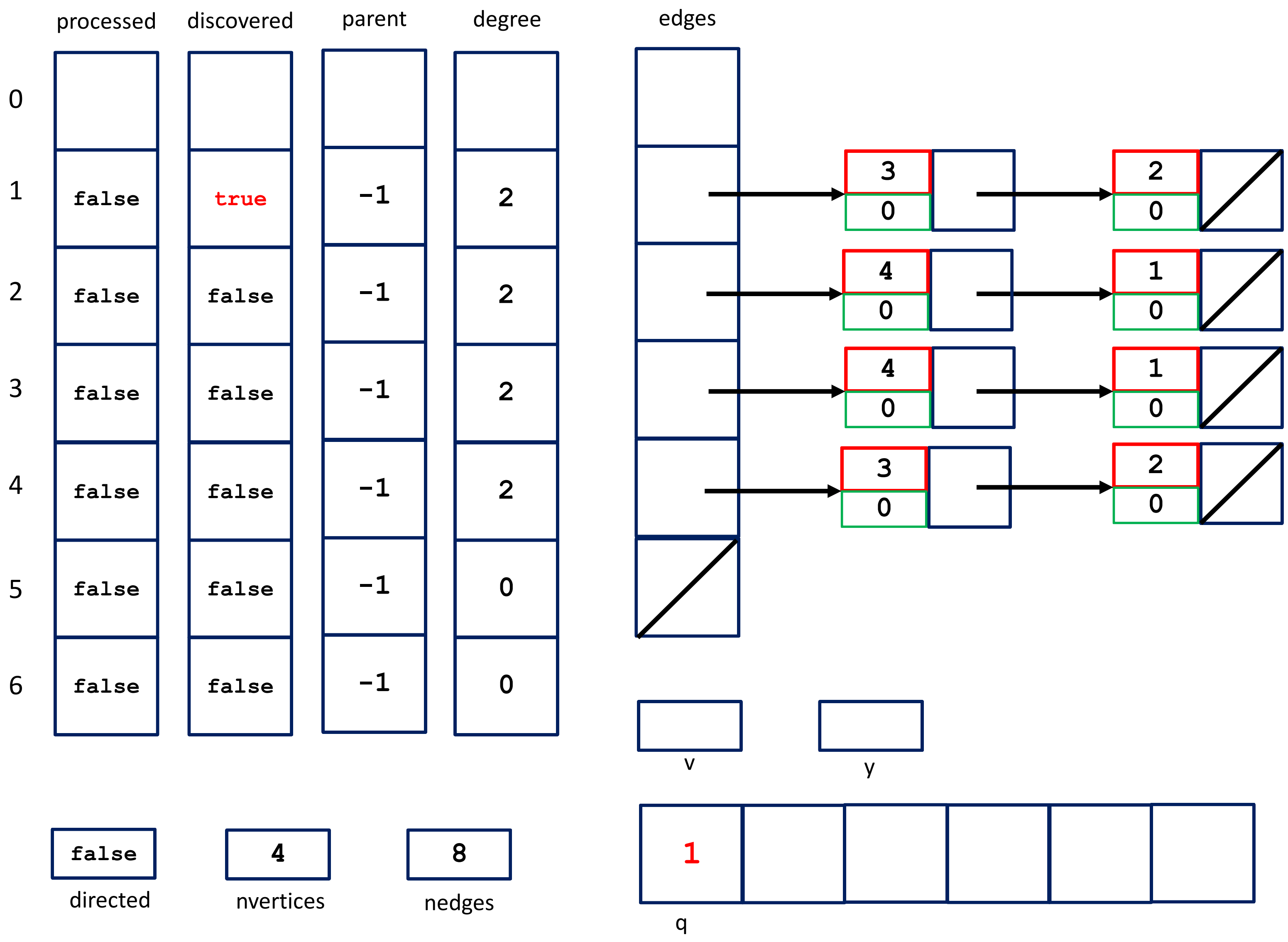
Breadth-First Search

```
/* Once a vertex is discovered, it is placed on a queue.           */
/* Since we process these vertices in first-in, first-out order,   */
/* the oldest vertices are expanded first, which are exactly those */
/* closest to the root                                             */

bfs(graph *g, int start)
{
   queue q;                    /* queue of vertices to visit */
   int v;                      /* current vertex             */
   int y;                      /* successor vertex           */
   edgenode *p;                /* temporary pointer          */

   init_queue(&q);
   enqueue(&q,start);
   discovered[start] = true;
```



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if $state[v]$ = "undiscovered" then
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enqueue[Q, v]
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Breadth-First Search

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        v = dequeue(&q);
        process_vertex_early(v);
        processed[v] = TRUE;
        p = g->edges[v];

        while (p != NULL) {
            y = p->y;
            if ((processed[y] == FALSE) || g->directed)
                process_edge(v,y);
            if (discovered[y] == FALSE) {
                enqueue(&q,y);
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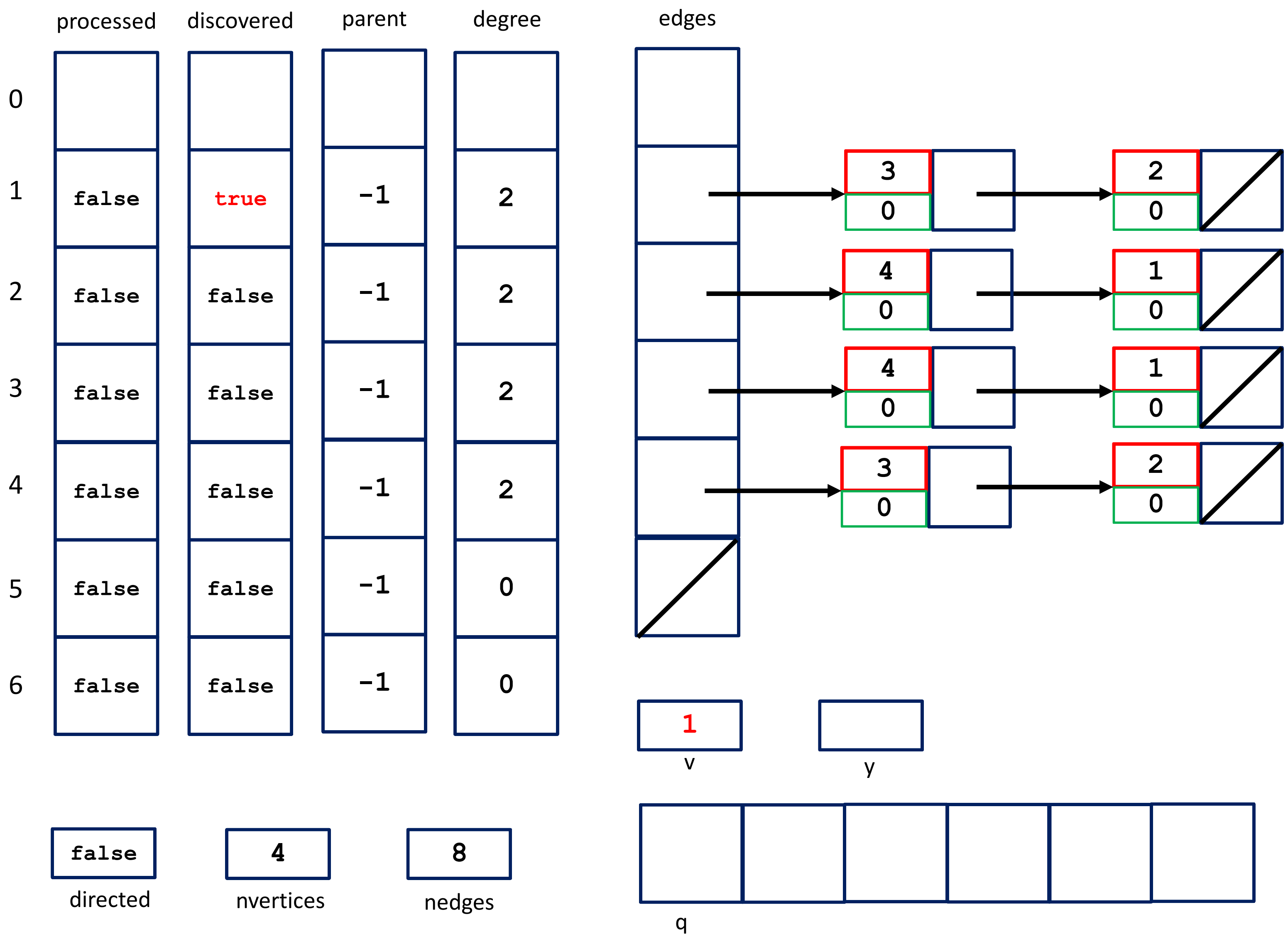
                parent[y] = v;
            }
            p = p->next;
        }
        process_vertex_late(v);
    }
}
```

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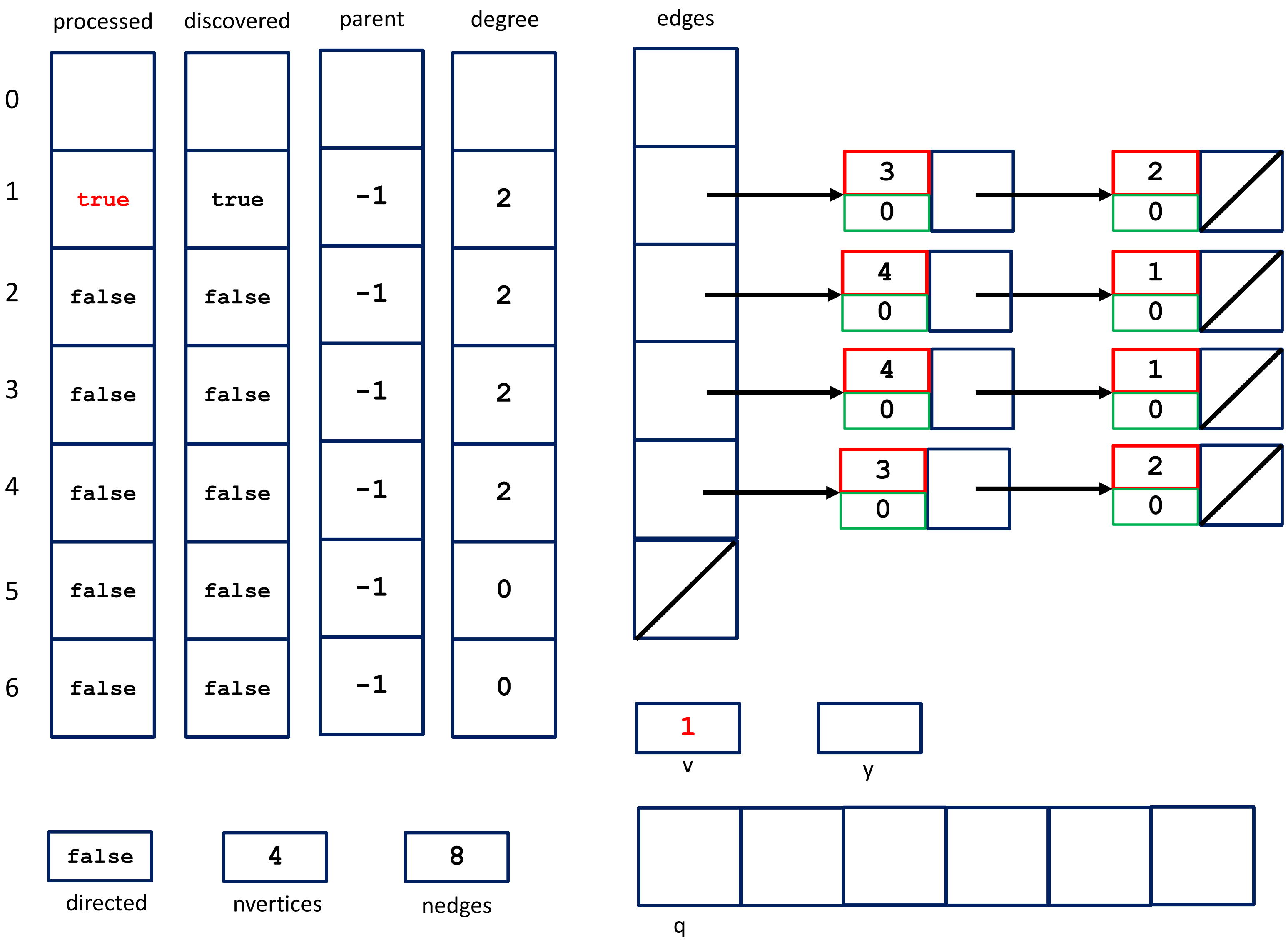


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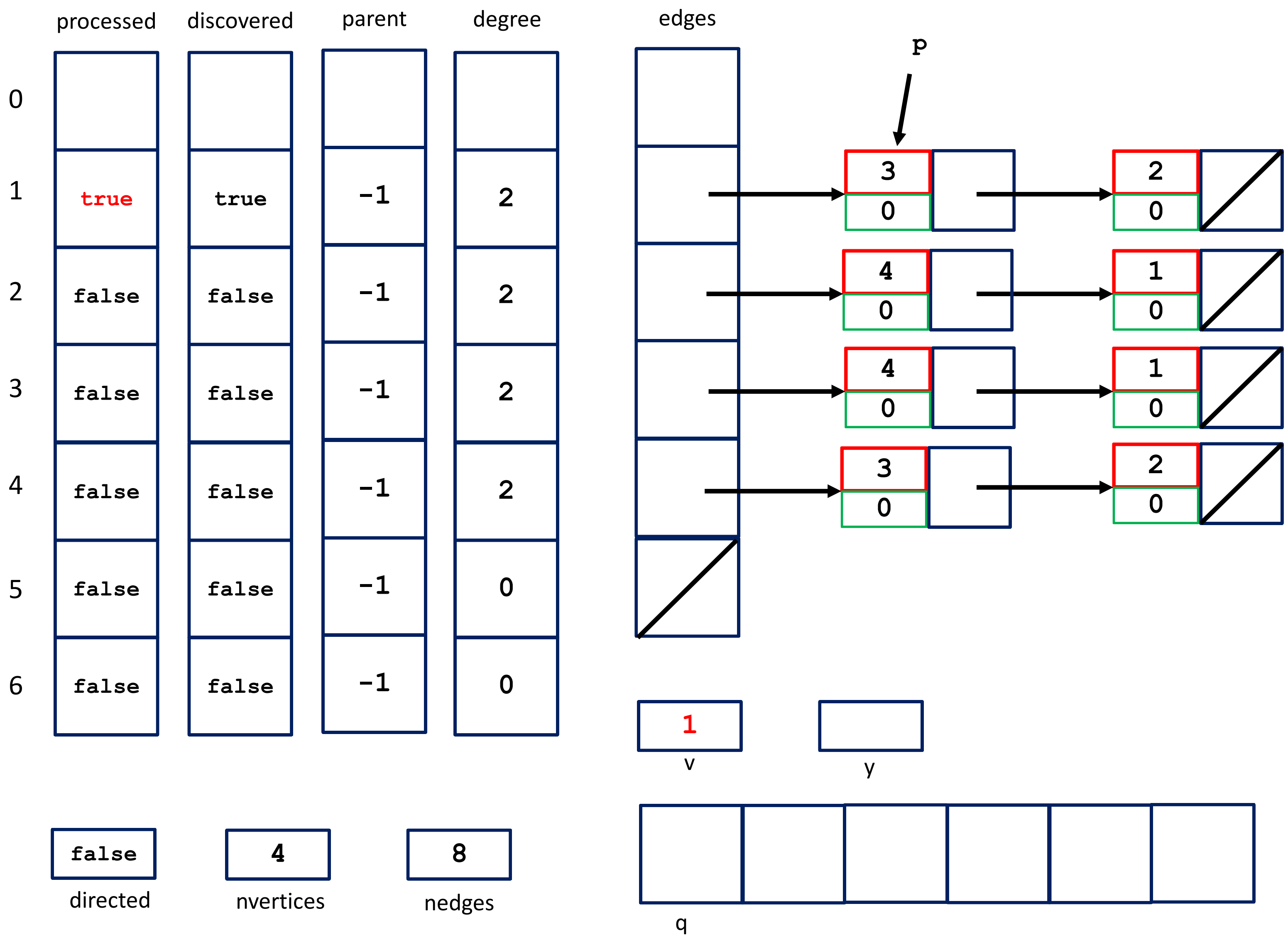

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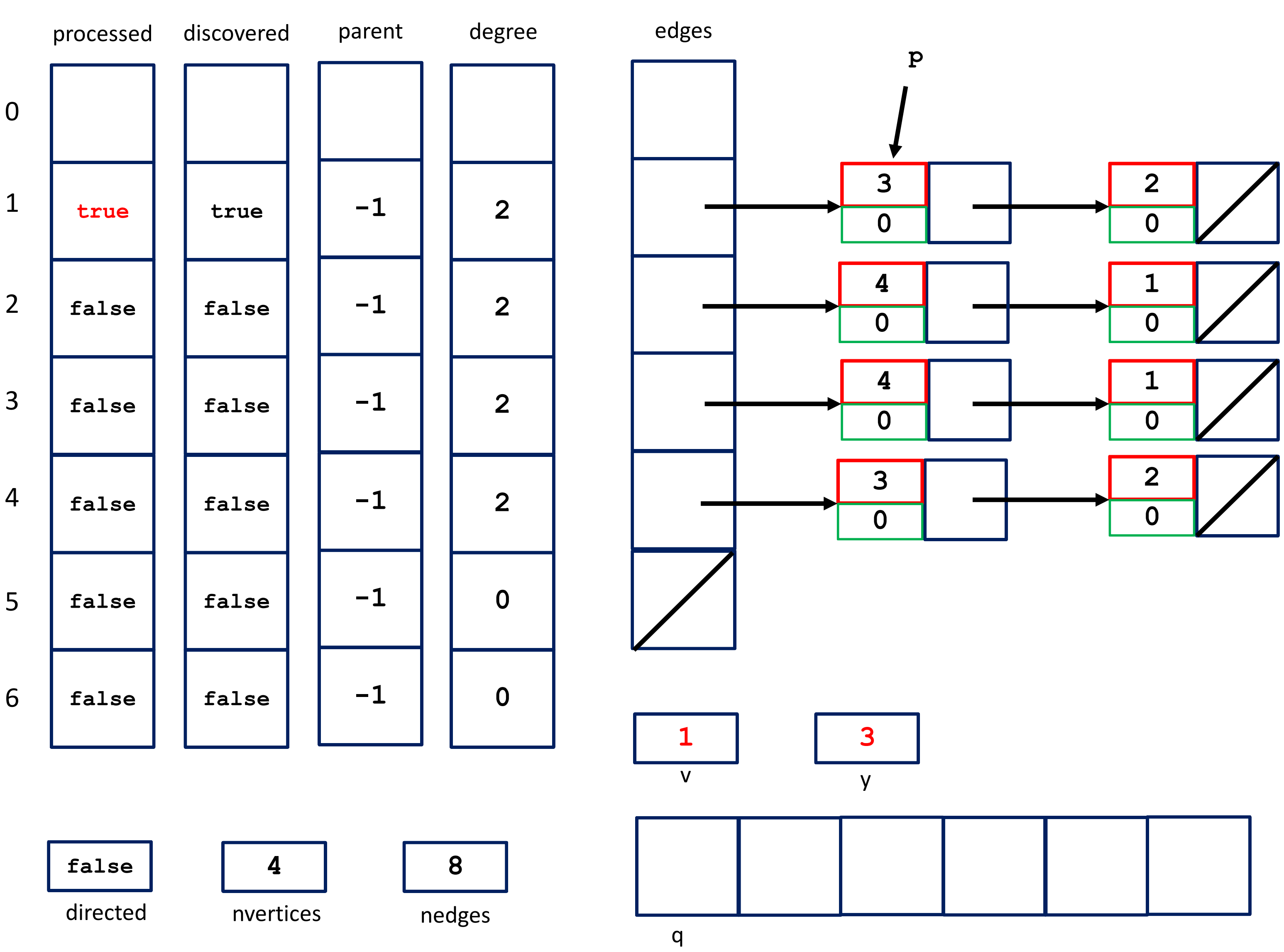


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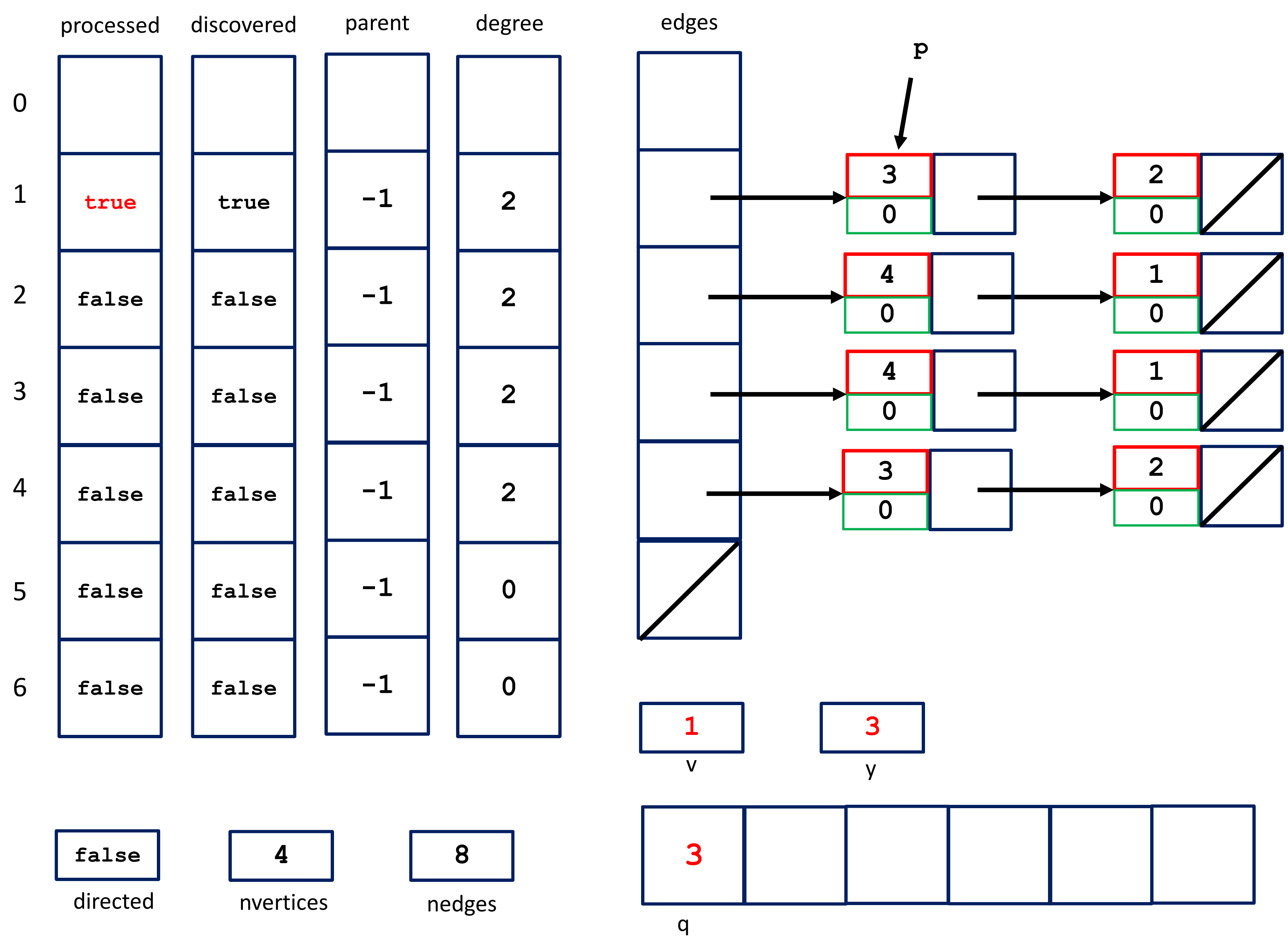


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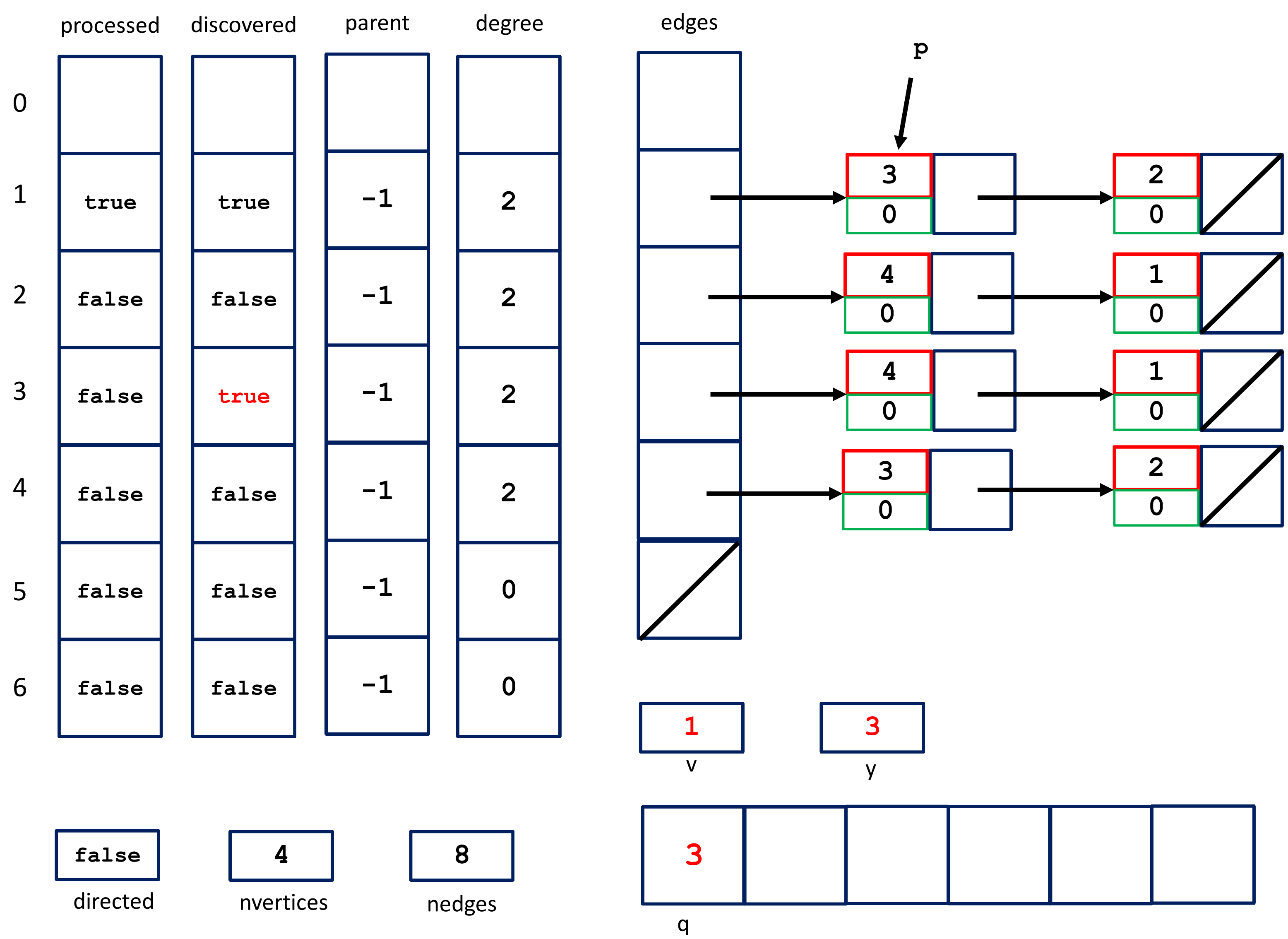


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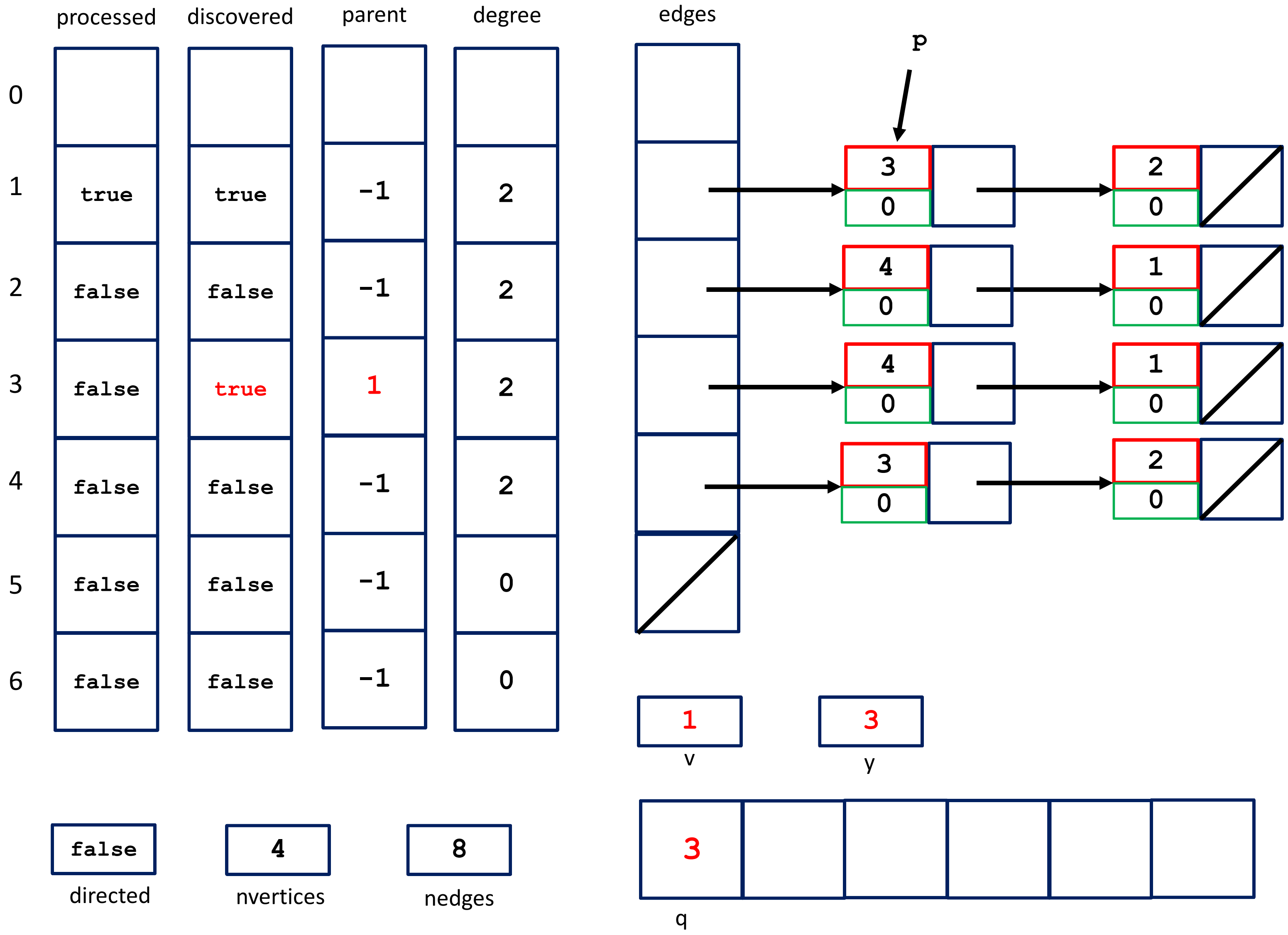


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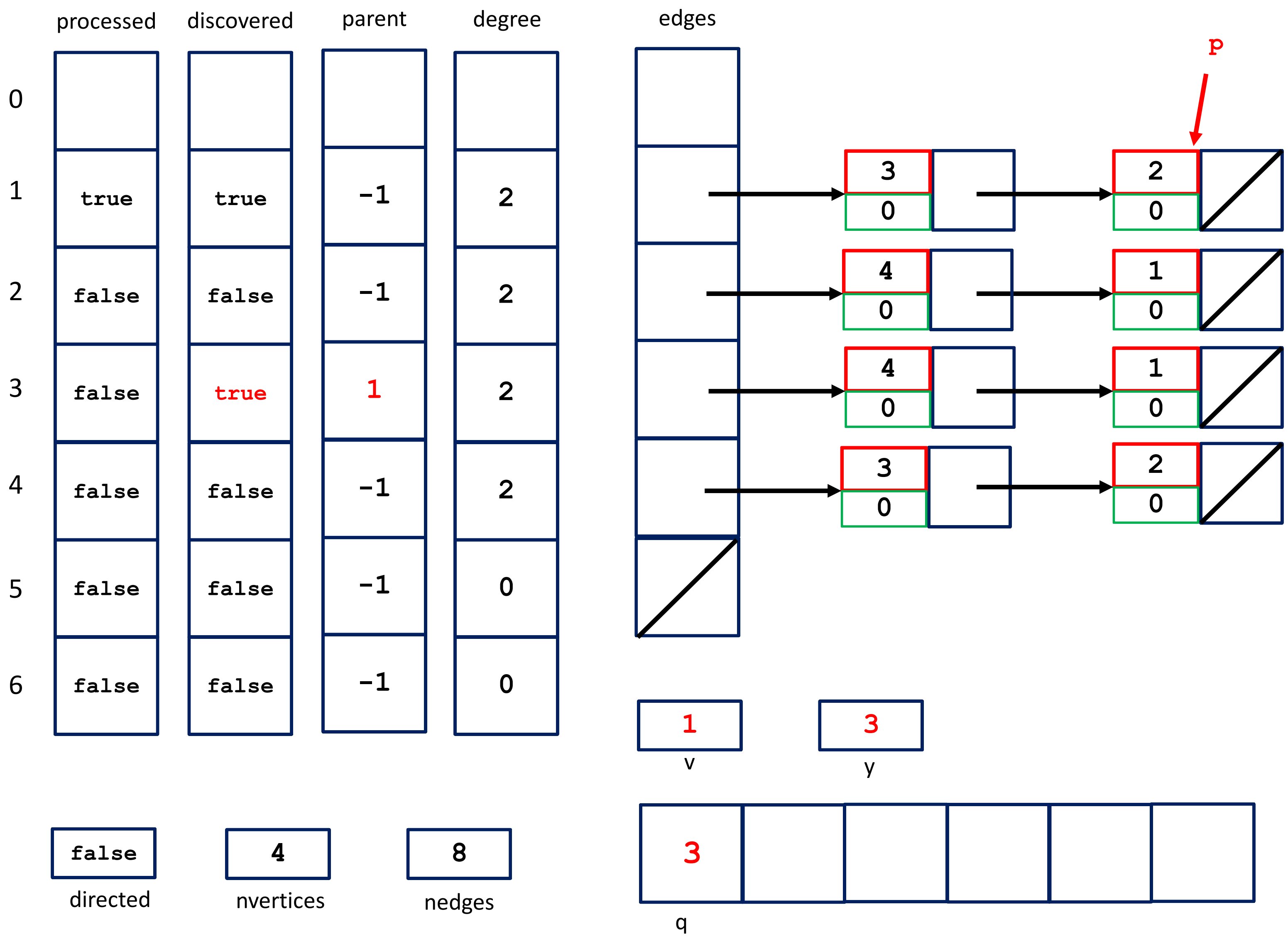

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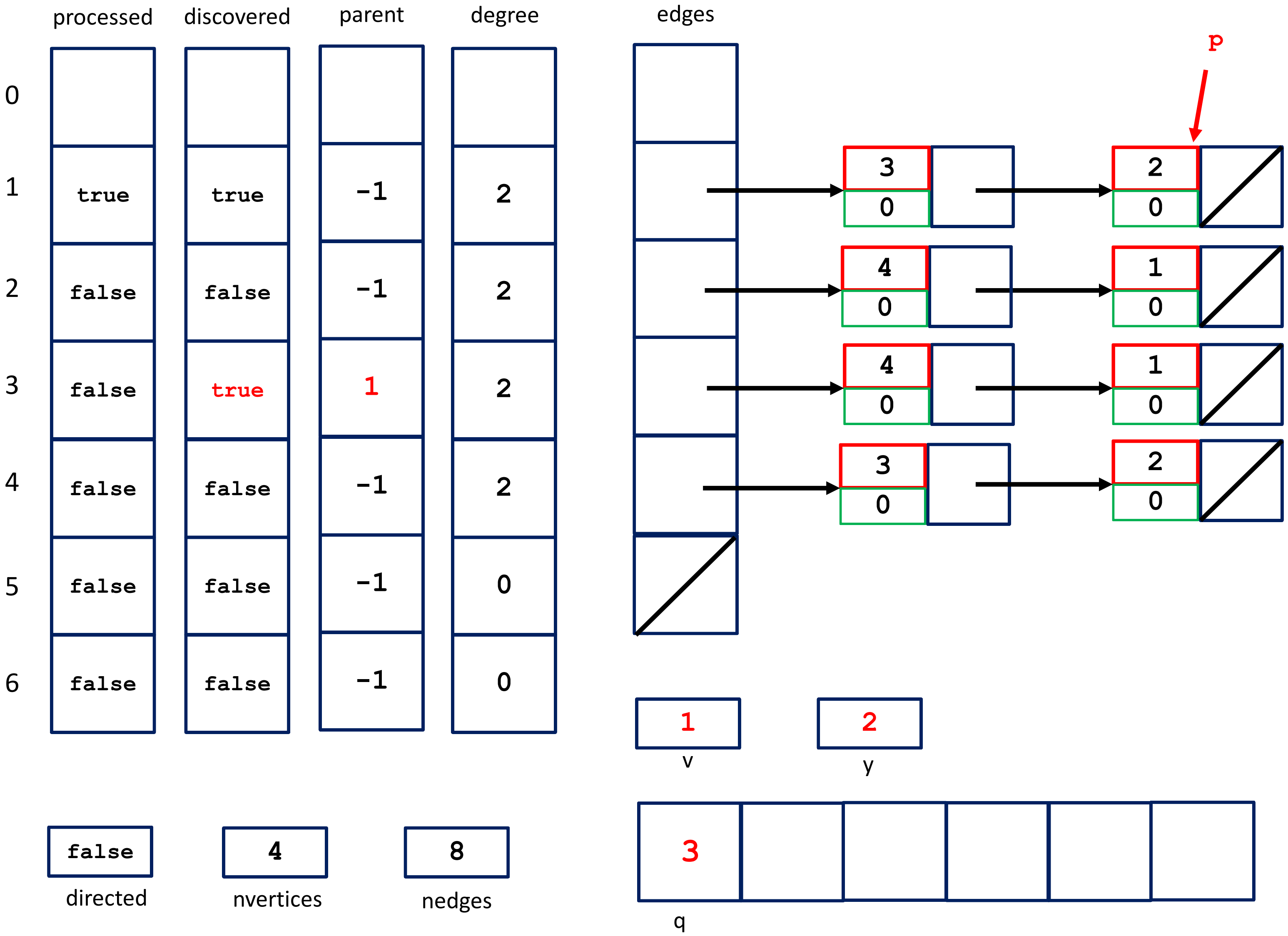

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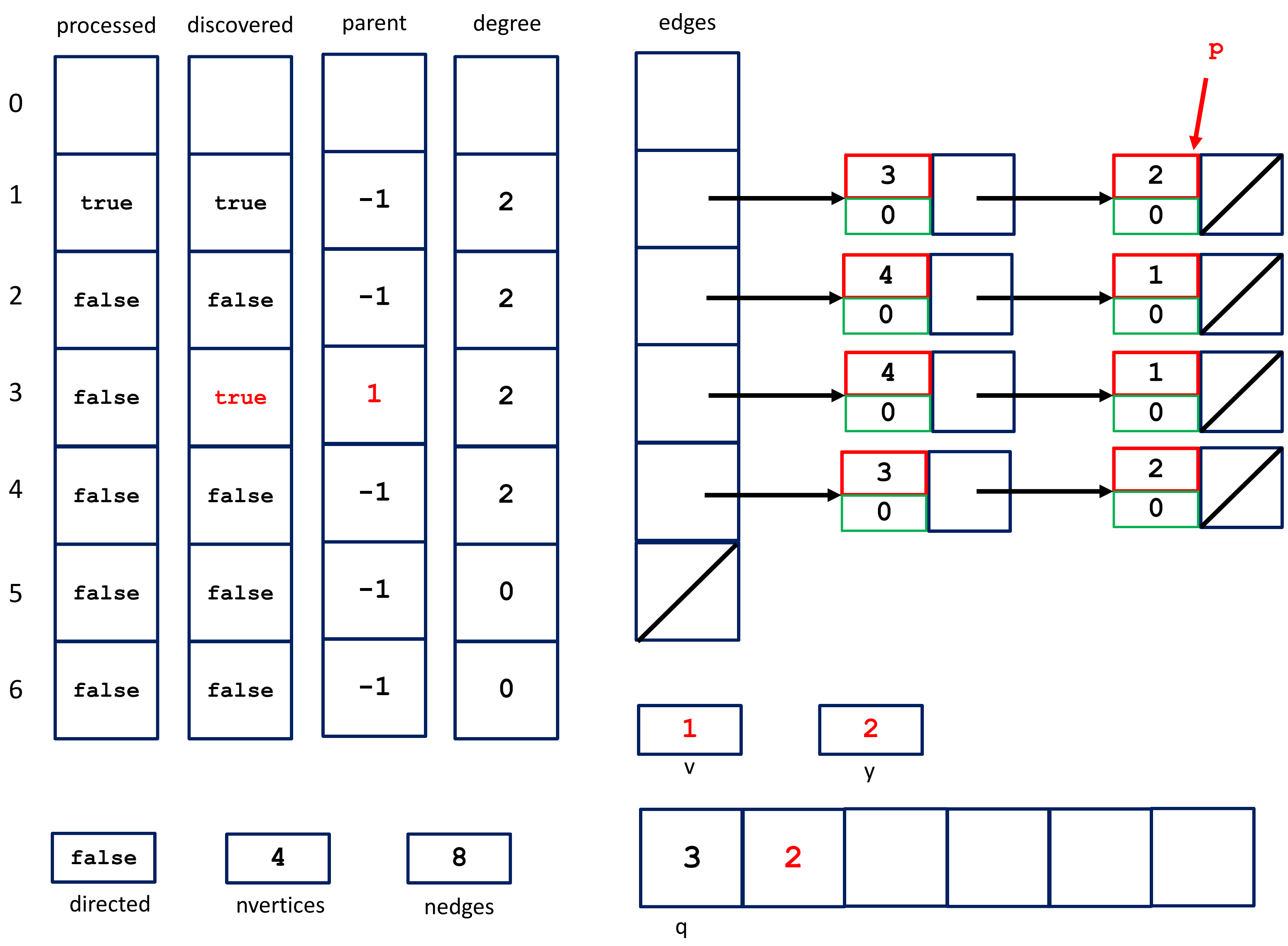

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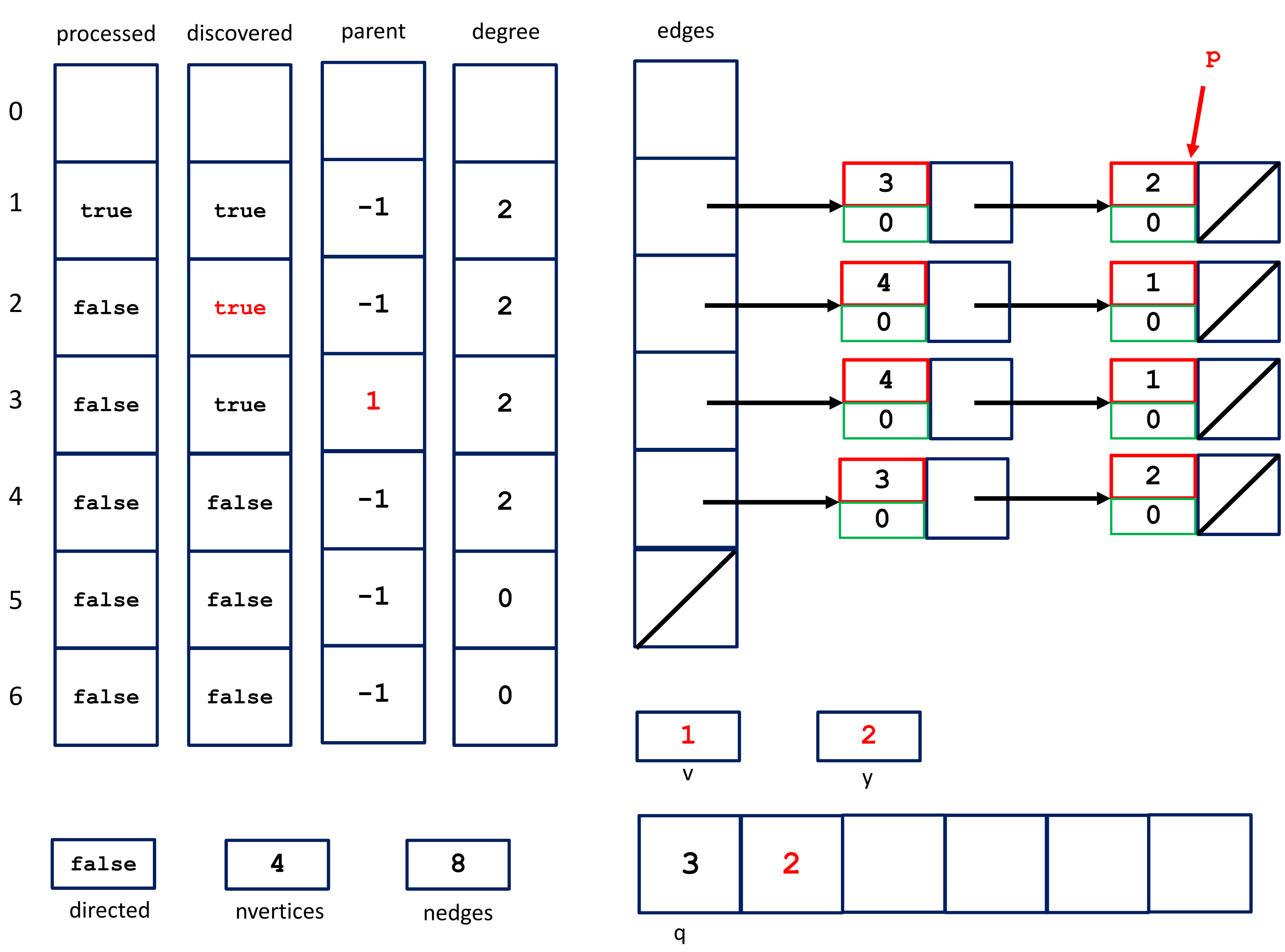
        for each $v \in Adj[u]$ do
            process edge (u, v) as desired
            if $state[v]$ = "undiscovered" then
                $state[v]$ = "discovered"
                $p[v] = u$
                enqueue$[Q, v]$
        $state[u]$ = "processed"

Breadth-First Search

```
    while (empty_queue(&q) == FALSE) {
        v = dequeue(&q);
        process_vertex_early(v);
        processed[v] = true;
        p = g->edges[v];

        while (p != NULL) {
            y = p->y;
            if ((processed[y] == false) || g->directed)
                process_edge(v,y);
            if (discovered[y] == false) {
                enqueue(&q,y);
                discovered[y] = true;
                parent[y] = v;
            }
            p = p->next;
        }
        process_vertex_late(v);
    }
}
```



BFS(G, s)
for each vertex $u \in V[G] - \{s\}$ do
$state[u]$ = "undiscovered"
$p[u] = nil$, i.e. no parent is in the BFS tree
$state[s]$ = "discovered"
$p[s] = nil$
$Q = \{s\}$
while $Q \neq \emptyset$ do
u = dequeue$[Q]$
process vertex u as desired
for each $v \in Adj[u]$ do
process edge (u, v) as desired
if $state[v]$ = "undiscovered" then
$state[v]$ = "discovered"
$p[v] = u$
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$state[u]$ = "processed"

Breadth-First Search

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        p = g->edges[v];

        while (p != NULL) {
            y = p->y;
            if ((processed[y] == false) || g->directed)
                process_edge(v,y);
            if (discovered[y] == false) {
                enqueue(&q,y);
                discovered[y] = true;
                parent[y] = v;
            }
            p = p->next;
        }
        process_vertex_late(v);
    }
}
```

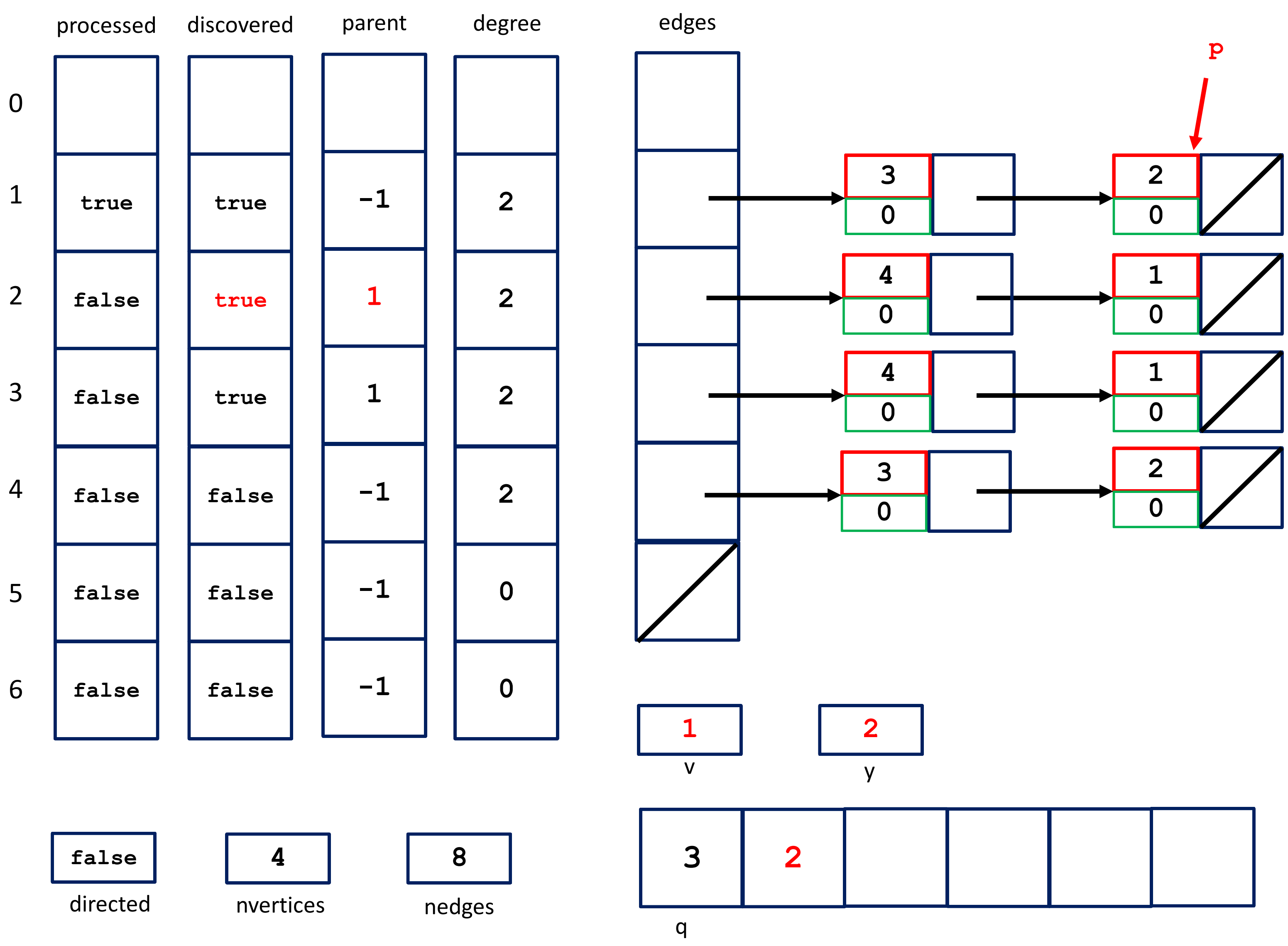

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for each $v \in Adj[u]$ do
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$p[v] = u$
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Breadth-First Search

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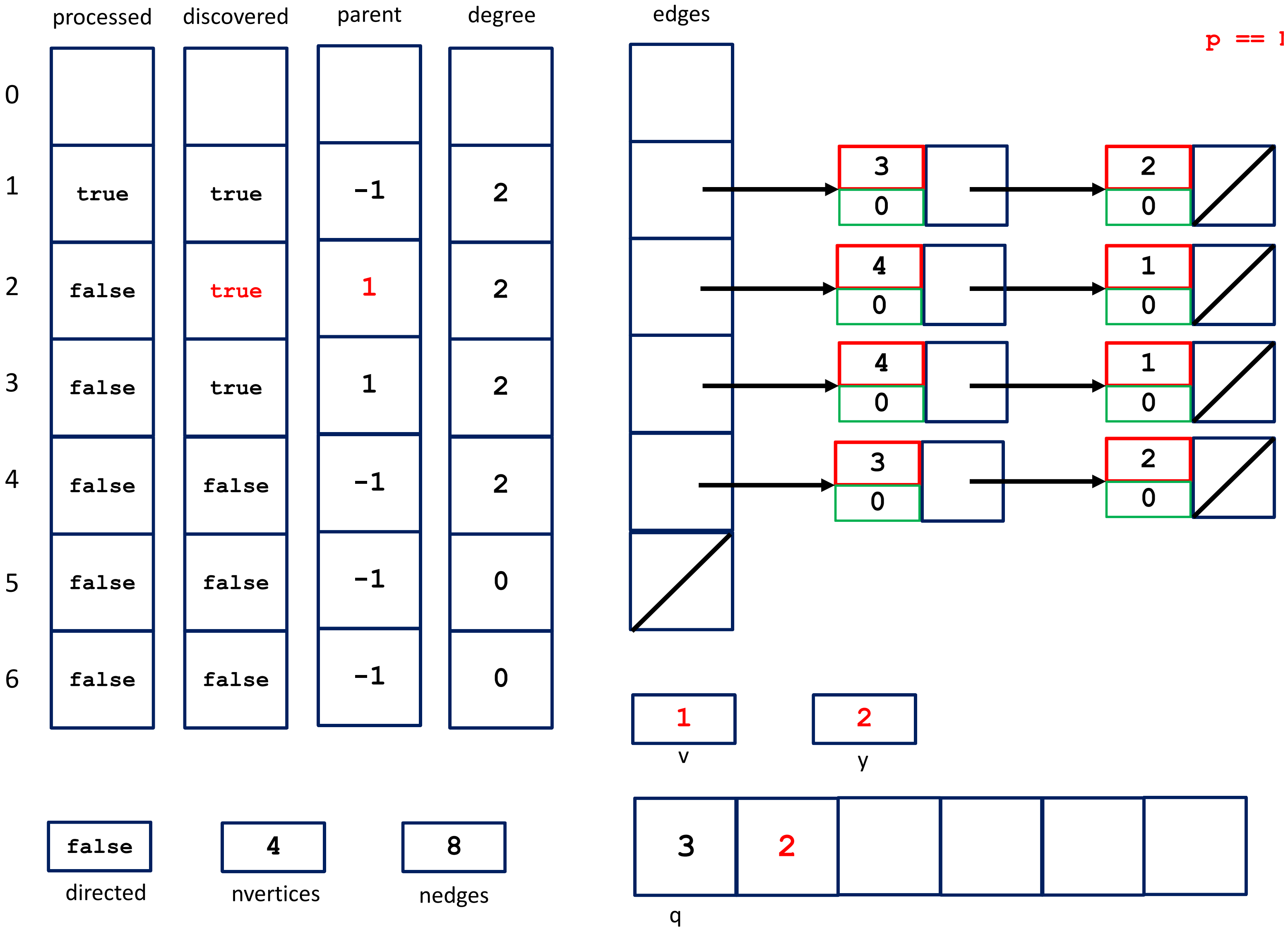
                process_edge(v,y);
            if (discovered[y] == false) {
                enqueue(&q,y);
                discovered[y] = true;
                parent[y] = v;
            }
            p = p->next;
        }
        process_vertex_late(v);
    }
}
```



BFS(G, s)
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$state[u]$ = "undiscovered"
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Breadth-First Search

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        while (p != NULL) {
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                process_edge(v,y);
            if (discovered[y] == false) {
                enqueue(&q,y);
                discovered[y] = true;
                parent[y] = v;
            }
            p = p->next;
        }
        process_vertex_late(v);
    }
}
```
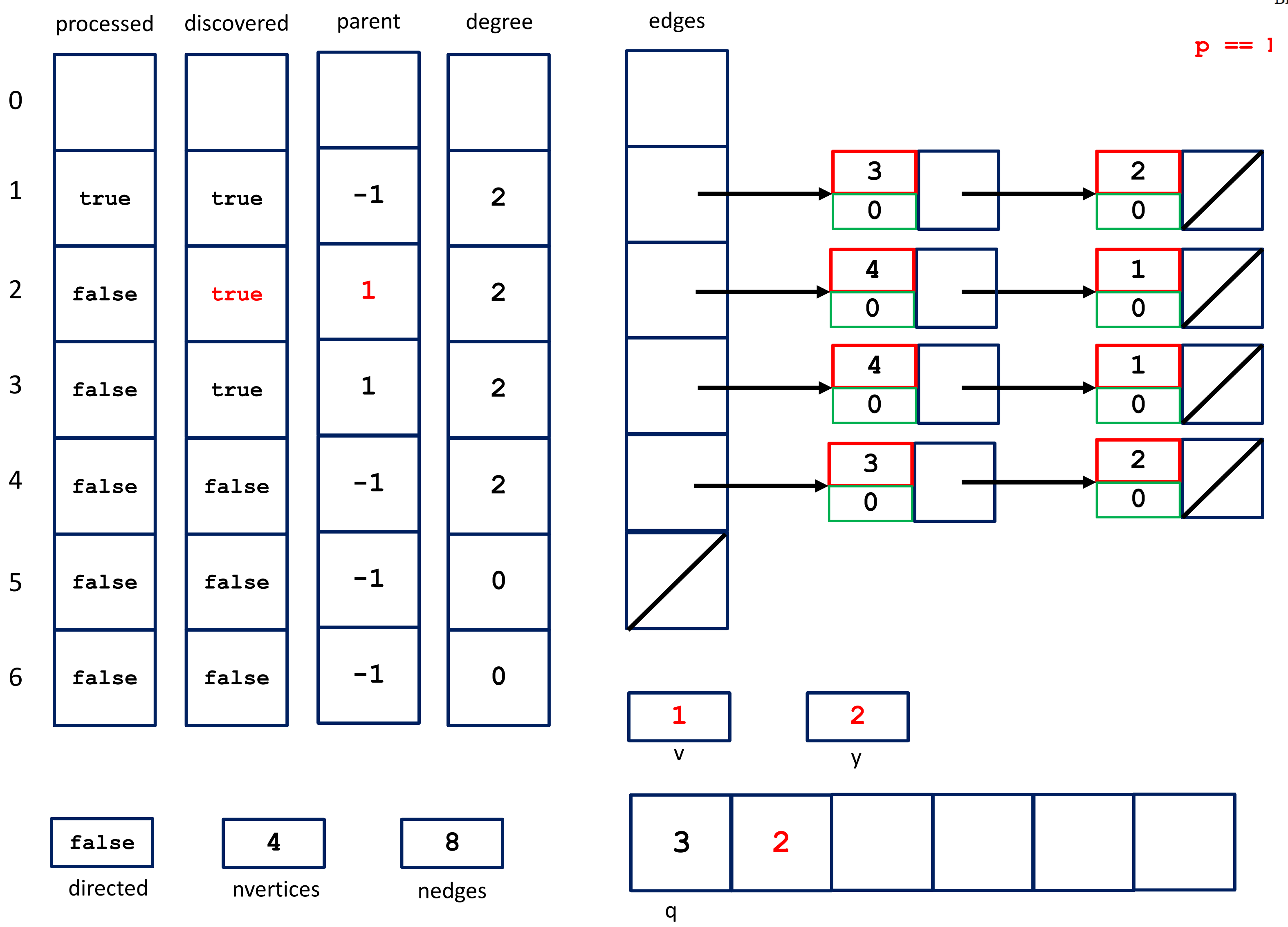
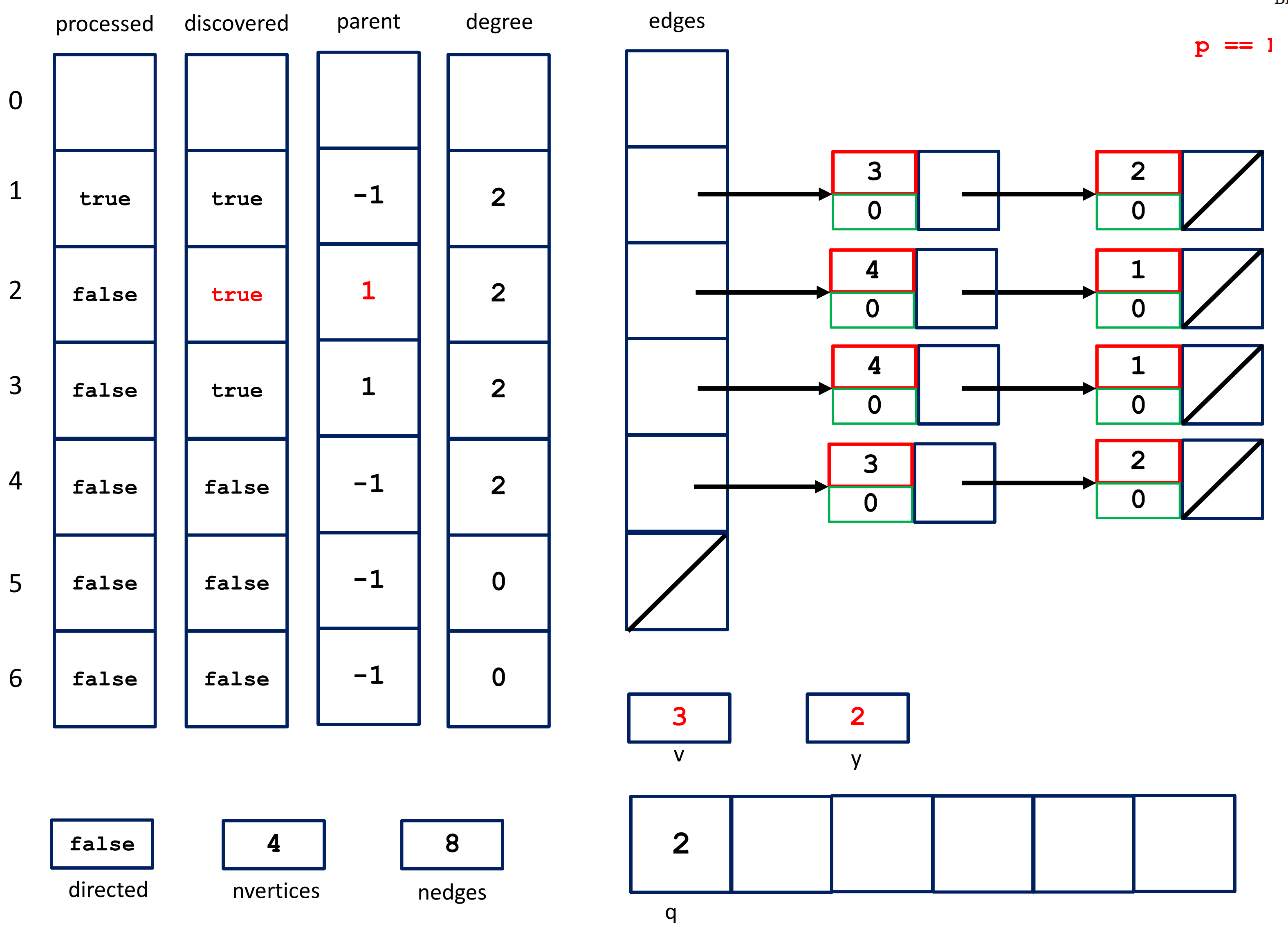

Breadth-First Search

```
    while (empty_queue(&q) == FALSE) {
        v = dequeue(&q);
        process_vertex_early(v);
        processed[v] = true;
        p = g->edges[v];

        while (p != NULL) {
            y = p->y;
            if ((processed[y] == false) || g->directed)
                process_edge(v,y);
            if (discovered[y] == false) {
                enqueue(&q,y);
                discovered[y] = true;
                parent[y] = v;
            }
            p = p->next;
        }
        process_vertex_late(v);
    }
}
```



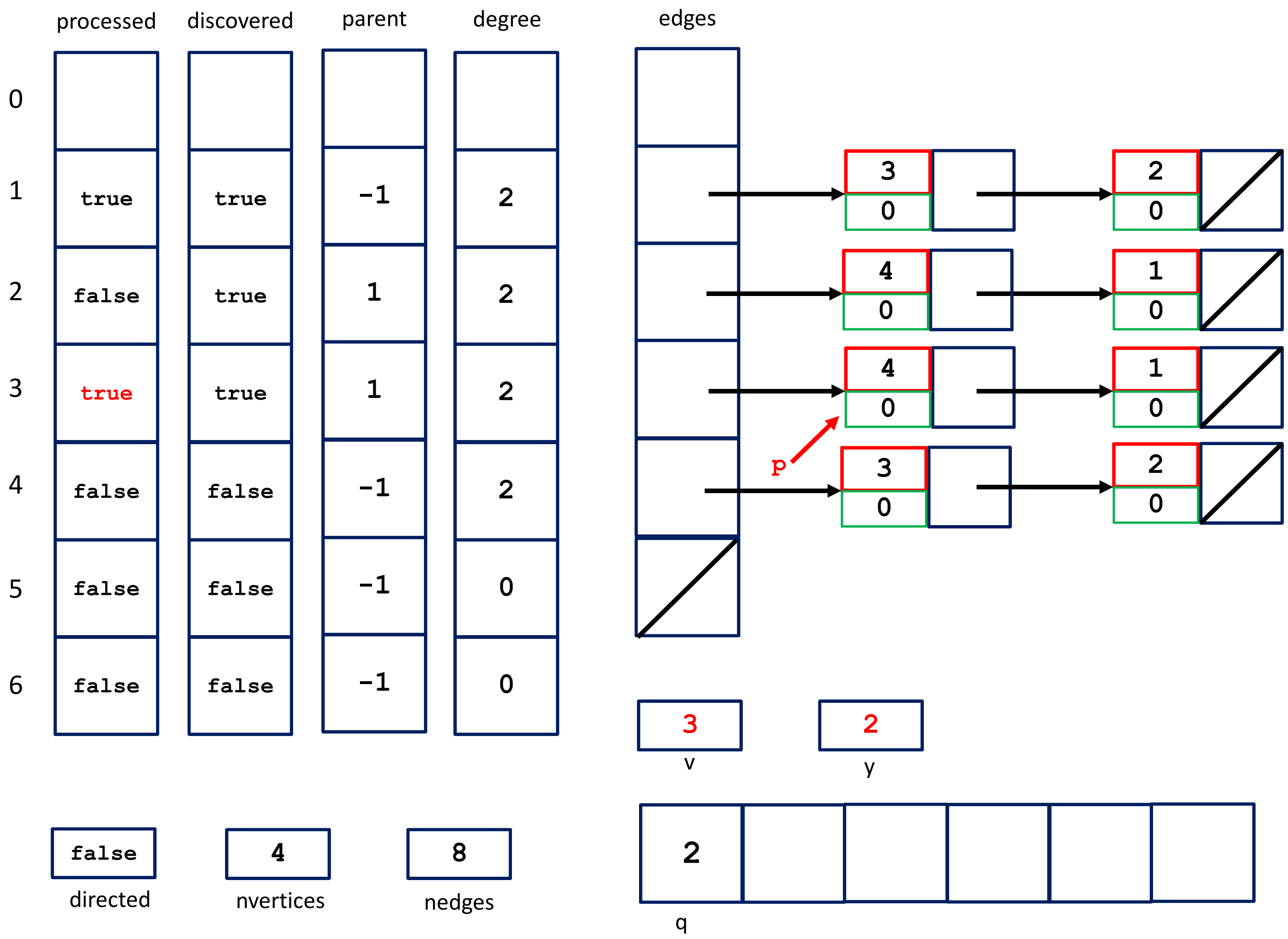

```
BFS(G, s)
    for each vertex u ∈ V[G] − {s} do
        state[u] = "undiscovered"
        p[u] = nil, i.e. no parent is in the BFS tree
    state[s] = "discovered"
    p[s] = nil
    Q = {s}
    while Q ≠ ∅ do
        u = dequeue[Q]
        process vertex u as desired
        for each v ∈ Adj[u] do
            process edge (u, v) as desired
            if state[v] = "undiscovered" then
                state[v] = "discovered"
                p[v] = u
                enqueue[Q, v]
        state[u] = "processed"
```



BFS(G, s)
- for each vertex $u \in V[G] - \{s\}$ do
  - $state[u]$ = "undiscovered"
  - $p[u] = nil$, i.e. no parent is in the BFS tree
- $state[s]$ = "discovered"
- $p[s] = nil$
- $Q = \{s\}$
- while $Q \neq \emptyset$ do
  - u = dequeue[Q]
  - process vertex u as desired
  - for each $v \in Adj[u]$ do
    - process edge (u, v) as desired
    - if $state[v]$ = "undiscovered" then
      - $state[v]$ = "discovered"
      - $p[v] = u$
      - enqueue[Q, v]
  - $state[u]$ = "processed"

Breadth-First Search

```
    while (empty_queue(&q) == FALSE) {
        v = dequeue(&q);
        process_vertex_early(v);
        processed[v] = true;
        p = g->edges[v];

        while (p != NULL) {
            y = p->y;
            if ((processed[y] == false) || g->directed)
                process_edge(v,y);
            if (discovered[y] == false) {
                enqueue(&q,y);
                discovered[y] = true;
                parent[y] = v;
            }
            p = p->next;
        }
        process_vertex_late(v);
    }
}
```




```
BFS(G, s)
    for each vertex u ∈ V[G] − {s} do
        state[u] = "undiscovered"
        p[u] = nil, i.e. no parent is in the BFS tree
    state[s] = "discovered"
    p[s] = nil
    Q = {s}
    while Q ≠ ∅ do
        u = dequeue[Q]
        process vertex u as desired
        for each v ∈ Adj[u] do
            process edge (u, v) as desired
            if state[v] = "undiscovered" then
                state[v] = "discovered"
                p[v] = u
                enqueue[Q, v]
        state[u] = "processed"
```

Breadth-First Search

```
/* The exact behaviour of bfs depends on the functions             */
/*    process vertex early()                                       */
/*    process vertex late()                                        */
/*    process edge()                                               */
/* These functions allow us to customize what the traversal does   */
/* as it makes its official visit to each edge and each vertex.    */
/* Here, e.g., we will do all of vertex processing on entry        */
/* (to print each vertex and edge exactly once)                    */
/* so process vertex late() returns without action                 */

process_vertex_late(int v) {
}

process_vertex_early(int v){
   printf("processed vertex %d\n",v);
}

process_edge(int x, int y) {
   printf("processed edge (%d,%d)\n",x,y);
}
```

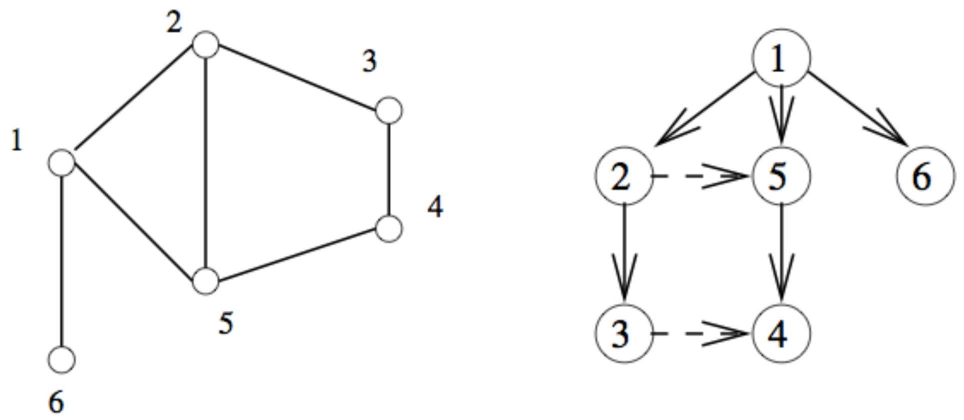
Breadth-First Search

```
/* this version just counts the number of edges                    */

process_edge(int x, int y) {
   nedges = nedges + 1;
}
```

Breadth-First Search

Finding Paths

- The `parent` array in `bfs()` is very useful for finding interesting paths through a graph
- The vertex that discovered vertex `i` is defined as `parent[i]`



vertex	1	2	3	4	5	6
parent	-1	1	2	5	1	1

Breadth-First Search

Finding Paths

- Every vertex is discovered during the course of a traversal, so every node has a parent (except the root)

- The parent relation defines a tree of discovery with the initial search node as the root of the tree

- Because vertices are discovered in order of increasing distance from the root, this tree has a very important property

  - The unique tree path from the root to each node uses the smallest number of edges (and intermediate nodes) possible on any path from the root to that vertex

  - Thus, BFS can be used to find shortest paths in an **unweighted** graph

Breadth-First Search

Finding Paths

- To reconstruct a path, we follow the chain of ancestors from the destination node x to the root
- Note we have to work backwards (we only know the parents)
- We find the path from the target vertex to the root and
  - Either store it and explicitly reverse it using a stack
  - Or construct the path recursively (in which case the stack is implicit)

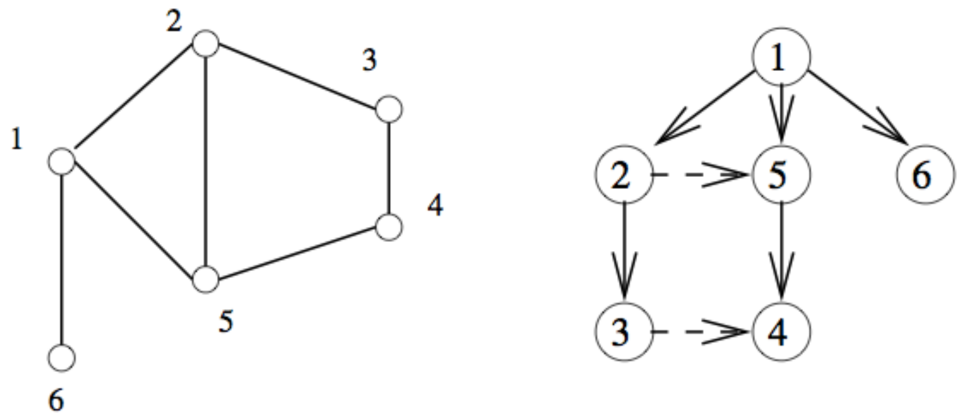
Breadth-First Search

```
bool find_path(int start, int end, int parents[]) {

   bool is_path;

   if (end == -1) {
      is_path = false; // some vertex on the path back from the end
                       // has no parent (not counting start)
   }
   else if ((start == end)) {
       printf("\n%d",start); // or store start in a path DS
       is_path = true;       // we have reached the start vertex
   }
   else {
      is_path = find_path(start,parents[end],parents);
      printf(" %d",end);     // or store end in a path DS
   }
   return(is_path);
}
```

vertex	1	2	3	4	5	6
parent	-1	1	2	5	1	1

Breadth-First Search

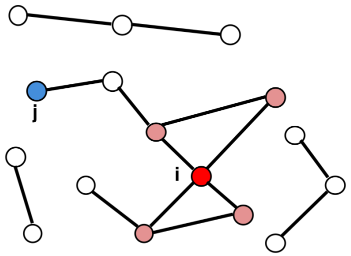
```
find_path(1,4,parent)

-> find_path(1,5,parent) -> find_path(1,1,parent) -> printf(1)
   printf(4)                  printf(5)
```



vertex	1	2	3	4	5	6
parent	-1	1	2	5	1	1

Breadth-First Search

Applications of Breadth-First Search



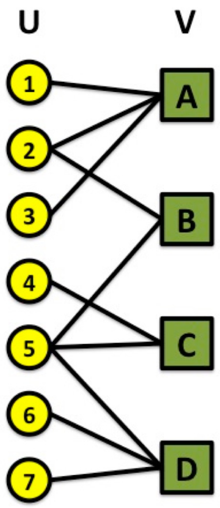
- Identifying connected components
  - A graph is connected if there is a path between any two vertices
  - A connected component of an undirected graph is a maximal set of vertices such that there is a path between every pair of vertices
  - The components are separate “pieces” of the graph such that there is no connection between the pieces
  - Many complicated problems reduce to finding or counting connected components
  - How would you find and label all the components in a graph?

Breadth-First Search

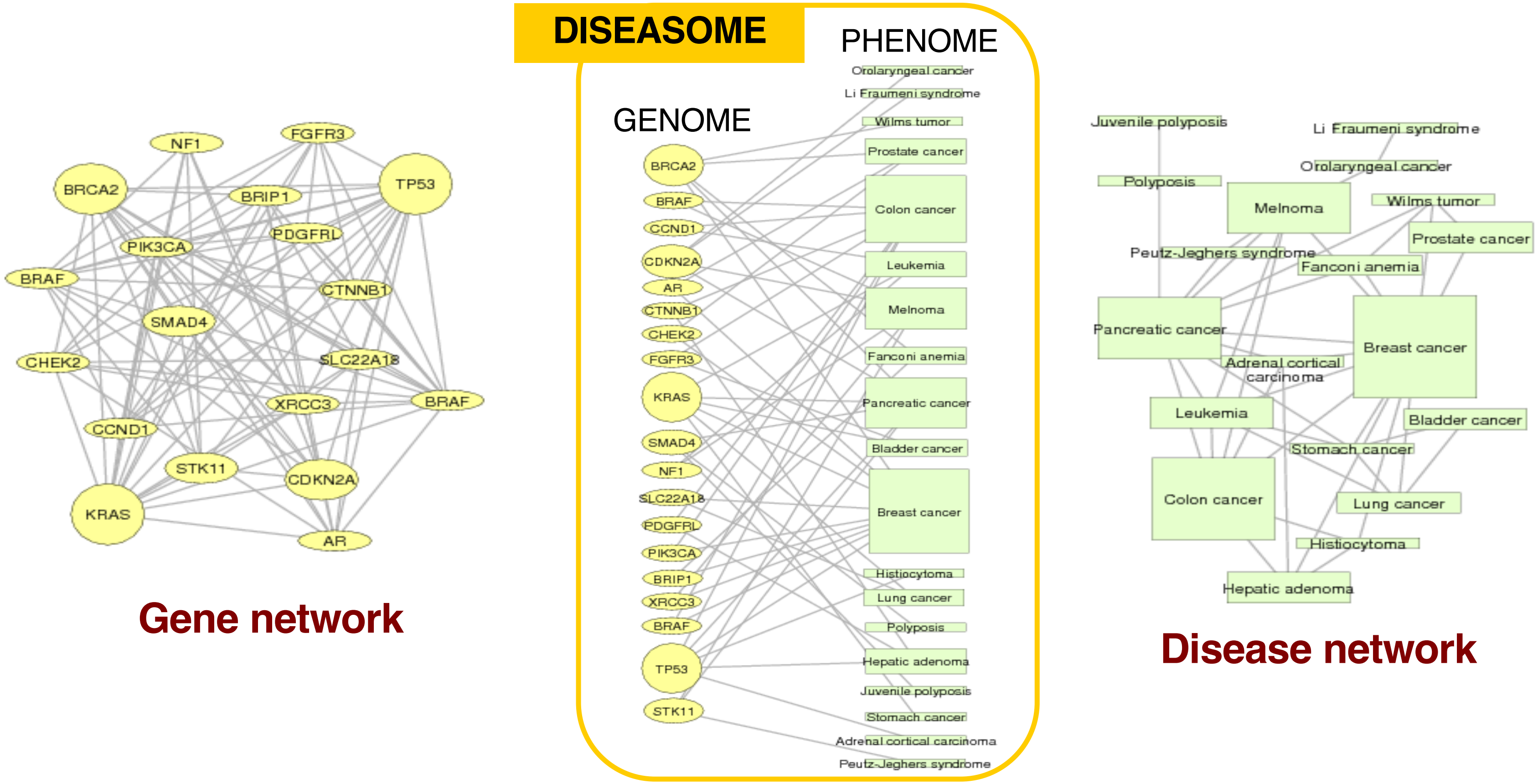
Applications of Breadth-First Search

– Two-Colouring Graphs

- The *vertex-colouring* problem seeks to assign a label (or colour) to each vertex of a graph such that no edge links any two vertices of the same colour
- The goal is use as few colours as possible
- A graph is bipartite if it can be coloured without conflicts using only two colours



Breadth-First Search



Goh, Cusick, Valle, Childs, Vidal & Barabási, PNAS (2007)

Breadth-First Search

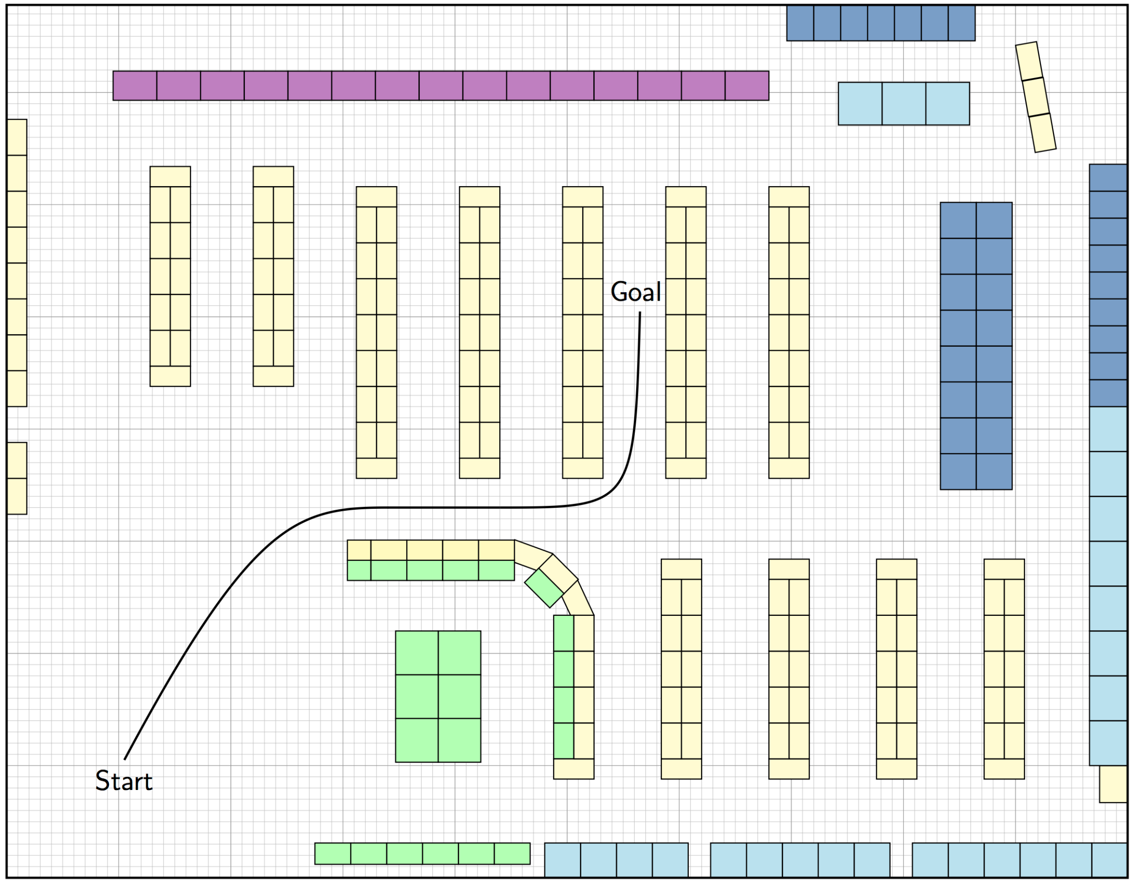
Applications of Breadth-First Search

- Robot path-planning

Breadth-First Search

Applications of Breadth-First Search

Robot path-planning



Map recreated from the following papers:

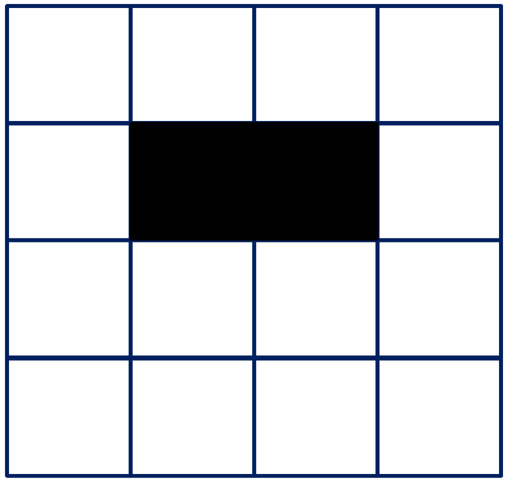
Joho, D., Senk, M., & Burgard, W. (2009). Learning wayfinding heuristics based on local information of object maps. Proceedings of the European Conference on Mobile Robots (ECMR) 2009, 117–122.

Kalff, C., & Strube, G. (2009). Background knowledge in human navigation: a study in a supermarket. Cognitive Processing, 10(2), 225-228.

Breadth-First Search

Applications of Breadth-First Search

- Robot path-planning
  - Represent the map of the environment as an occupancy grid



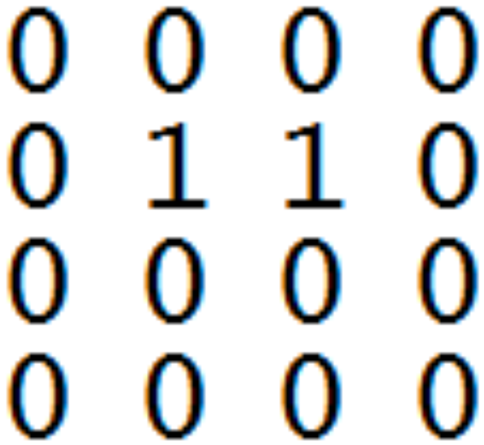
$$\begin{matrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{matrix}$$

Breadth-First Search

Applications of Breadth-First Search

- Robot path-planning
  - Represent the map of the environment as an occupancy grid

0	0	0	0
0	1	1	0
0	0	0	0
0	0	0	0

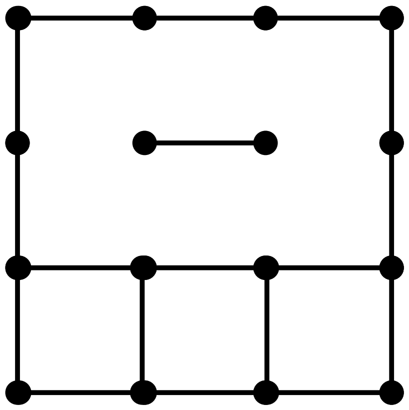


Breadth-First Search

Applications of Breadth-First Search

- Robot path-planning
  - Convert this to a graph

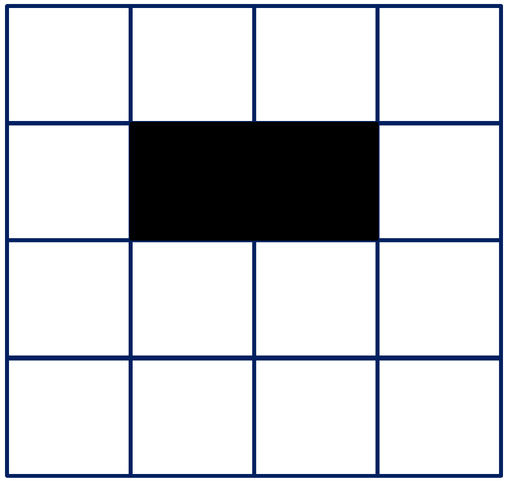
0	0	0	0
0	1	1	0
0	0	0	0
0	0	0	0

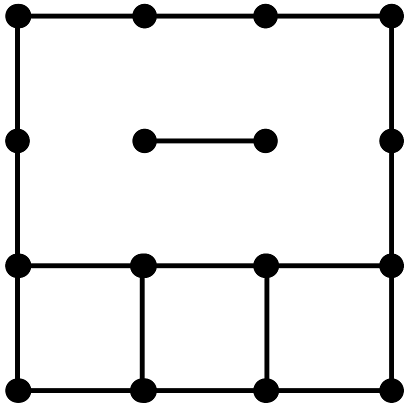


Breadth-First Search

Applications of Breadth-First Search

- Robot path-planning
  - Convert this to a graph



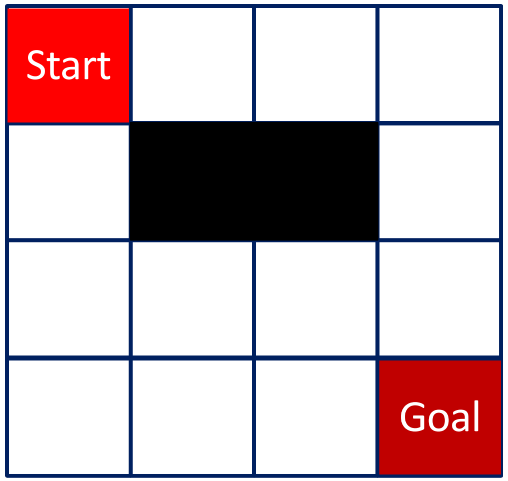


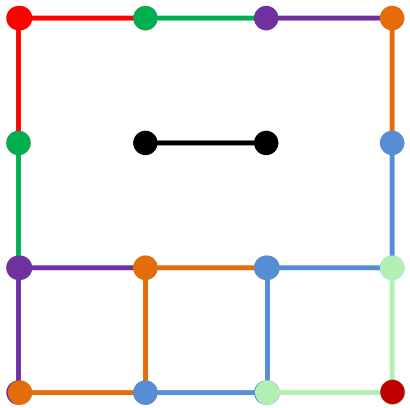
Breadth-First Search

Applications of Breadth-First Search

Robot path-planning

- Do a BFS from the robot start position …
- To find the shortest path to all other vertices





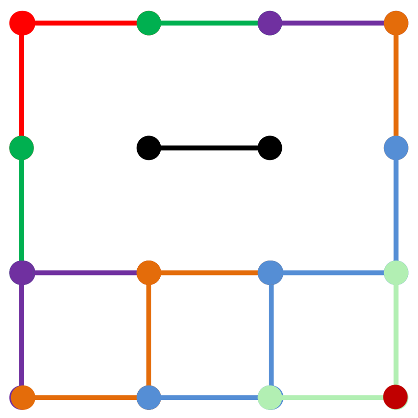
Breadth-First Search

Applications of Breadth-First Search

Robot path-planning

Mark the path from the robot start position to the goal position on the occupancy grid

2	0	0	0
2	1	1	0
2	0	0	0
2	2	2	2



Breadth-First Search

Applications of Breadth-First Search

Robot path-planning

Mark the path from the robot start position to the goal position on the occupancy grid

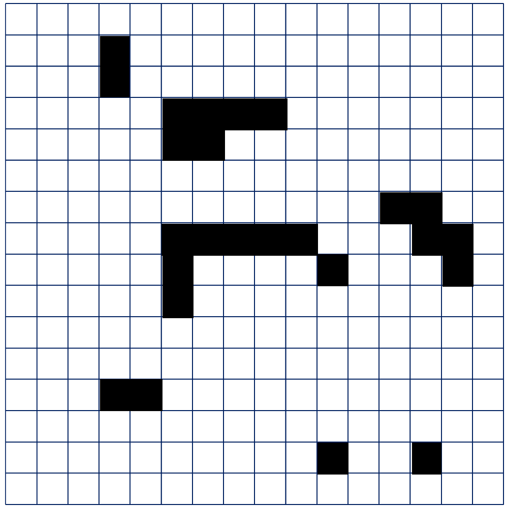
2	0	0	0
2	1	1	0
2	0	0	0
2	2	2	2

2 0 0 0
2 1 1 0
2 0 0 0
2 2 2 2

Breadth-First Search

Applications of Breadth-First Search

Robot path-planning

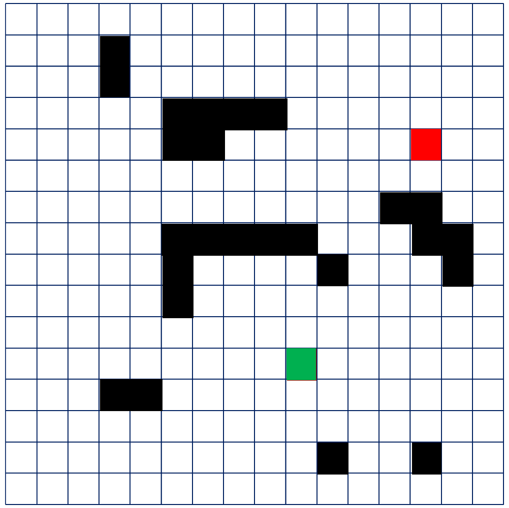


0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	1	1	1	1	0	0	0	0	0	0	0
0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0
0	0	0	0	0	1	1	1	1	1	0	0	0	1	1	0
0	0	0	0	0	1	0	0	0	0	1	0	0	0	1	0
0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	1	0	0	1	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Breadth-First Search

Applications of Breadth-First Search

Robot path-planning

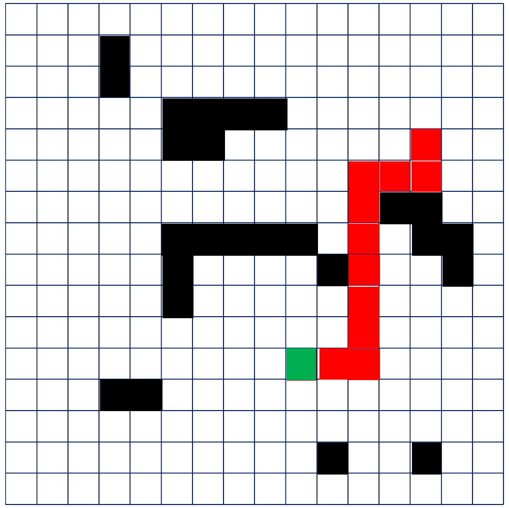


0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	1	1	1	1	0	0	0	0	0	0	0
0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0
0	0	0	0	0	1	1	1	1	1	0	0	0	1	1	0
0	0	0	0	0	1	0	0	0	0	1	0	0	0	1	0
0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	1	0	0	1	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Breadth-First Search

Applications of Breadth-First Search

Robot path-planning



0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	1	1	1	1	0	0	0	0	0	0	0
0	0	0	0	0	1	1	0	0	0	0	0	0	2	0	0
0	0	0	0	0	0	0	0	0	0	0	2	2	2	0	0
0	0	0	0	0	0	0	0	0	0	0	2	1	1	0	0
0	0	0	0	0	1	1	1	1	1	0	2	0	1	1	0
0	0	0	0	0	1	0	0	0	0	1	2	0	0	1	0
0	0	0	0	0	1	0	0	0	0	0	2	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	2	0	0	0	0
0	0	0	0	0	0	0	0	0	2	2	2	0	0	0	0
0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	1	0	0	1	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Breadth-First Search

Applications of Breadth-First Search

Robot path-planning