04-630
Data Structures and Algorithms for Engineers

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Lecture 6

Searching and Sorting Algorithms

- Linear Search & Binary Search
- In-place sorts
  - Bubble Sort
  - Selection Sort
  - Insertion Sort
- Not-in-place sort
  - Quicksort
  - Mergesort
- Characteristics of a good sort

Quicksort

- The Quicksort algorithm was developed by C.A.R. Hoare.
  It has the best average behaviour in terms of complexity:

  Average case: $O(n \log_2 n)$

  Worst case: $O(n^2)$

Quicksort

- Given a list of elements
- take a partitioning element (called a ”pivot”)
- and create two (sub)lists
    1. Left sublist: all elements are less than partitioning element,
    2. Right sublist: all elements are greater than it
- Now repeat this partitioning effort on each of these two sublists
- This is a divide-and-conquer strategy

Quicksort

- And so on in a recursive manner until all the sublists are empty, at which point the (total) list is sorted

- Partitioning can be effected by
  - scanning left to right
  - scanning right to left
  - iinterchanging elements in the wrong parts of the list

- The partitioning element is then placed between the resultant sublists
  - which are then partitioned in the same manner

Implementation of Quicksort()

In pseudo-code first

```
If anything to be partitioned
   choose a pivot
   DO
      scan from left to right until we find an element
      > pivot: i points to it

      scan from right to left until we find an element
      <= pivot: j points to it

      IF  i < j
         exchange ith and jth element
   WHILE i <= j
```

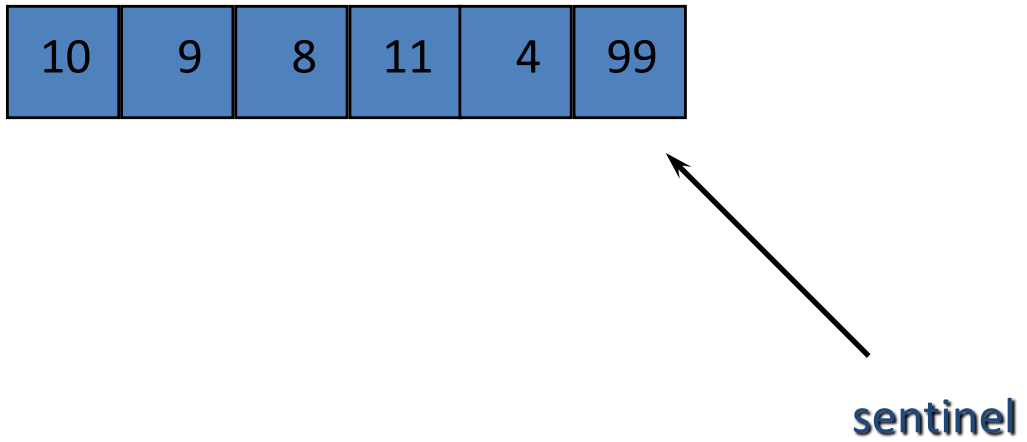
Implementation of Quicksort()

```
/* simple quicksort to sort an array of integers */

void quicksort (int A[], int L, int R)
{
   int i, j, pivot;

   /* assume A[R] contains a number > any element,    */
   /* i.e. it is a sentinel.                          */
```

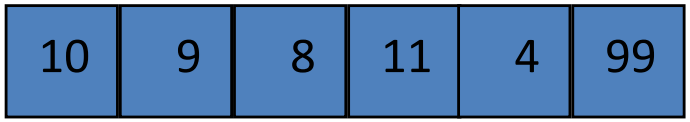
Implementation of Quicksort()

```
    if ( R > L) { // if R==L, it is a list with just one element!!
        i = L; j = R;
        pivot = A[i];
        do {
            while (A[i] <= pivot)
                i=i+1;
            while ((A[j] >= pivot) && (j>L))
                j=j-1;
            if (i < j) {
                exchange(A[i],A[j]); /* between partitions */
                i = i+1; j = j-1;
            }
        } while (i <= j);
        exchange(A[L], A[j]); /* reposition pivot */
        quicksort(A, L, j);
        quicksort(A, i, R);   /*includes sentinel*/
    }
}
```

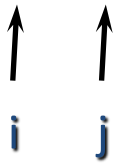
Quicksort



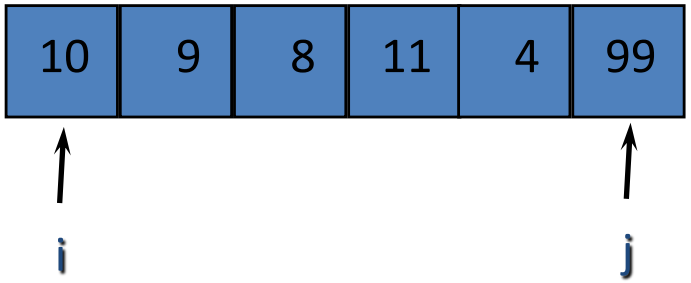
Quicksort




```
QS(A, , )

L:
R:
i:
j:
pivot:
```

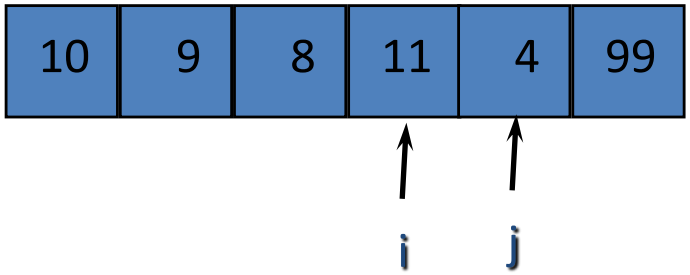


Quicksort



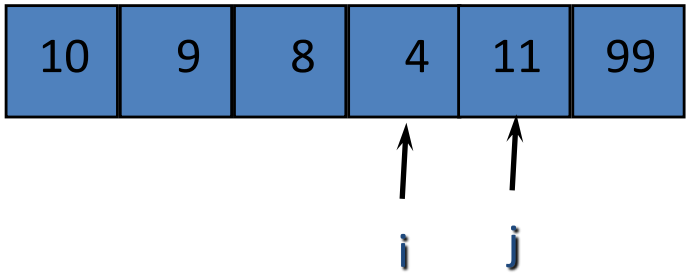

```
QS(A,1,6)

L:      1
R:      6
i:      1
j:      6
pivot: 10
```

Quicksort



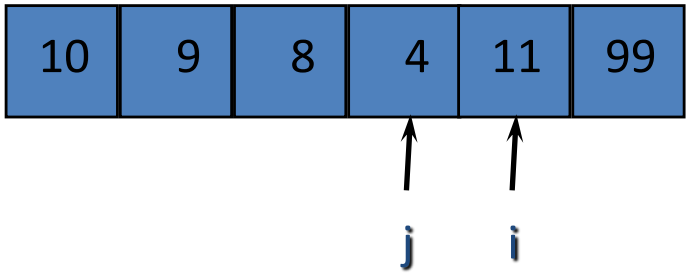

```
QS(A,1,6)

L:       1
R:       6
i:       1 2 3 4
j:       6 5
pivot: 10
```

Quicksort



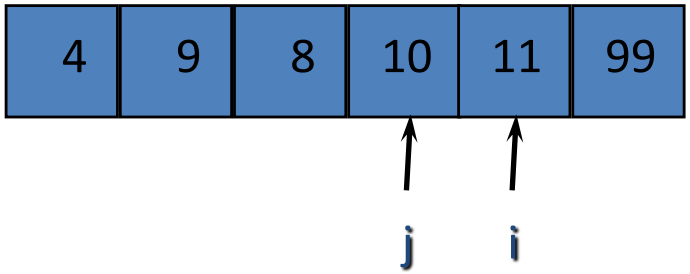

```
QS(A,1,6)

L:       1
R:       6
i:       1 2 3 4
j:       6 5
pivot: 10
```

Quicksort



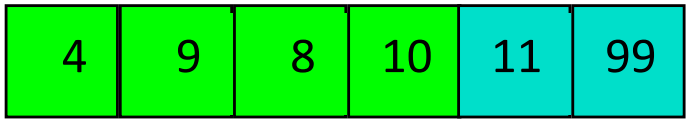

```
QS(A,1,6)

L:       1
R:       6
i:       1 2 3 4 5
j:       6 5 4
pivot: 10
```

Quicksort




```
QS(A,1,6)

L:       1
R:       6
i:       1 2 3 4 5
j:       6 5 4
pivot: 10
```

Quicksort

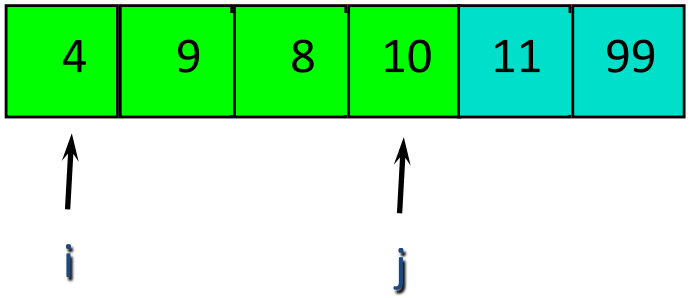



```
QS(A,1,6)

L:       1
R:       6
i:       1 2 3 4 5
j:       6 5 4
pivot: 10
```

```
QS(A,1,4)

L:       1
R:       4
i:
j:
pivot:   4
```

```
QS(A,5,6)

L:       5
R:       6
i:
j:
pivot:   11
```

Quicksort

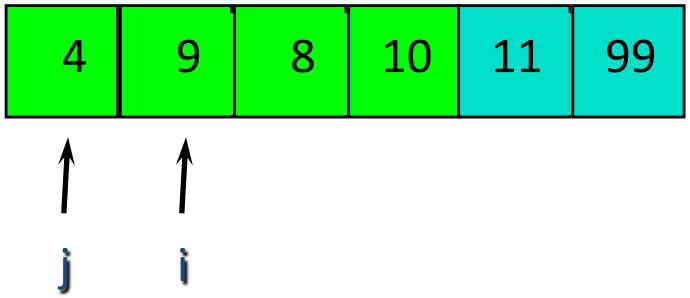



```
QS(A,1,6)

L:       1
R:       6
i:       1 2 3 4 5
j:       6 5 4
pivot: 10
```

```
QS(A,1,4)

L:       1
R:       4
i:       1
j:       4
pivot:   4
```

```
QS(A,5,6)

L:       5
R:       6
i:
j:
pivot:   11
```

Quicksort

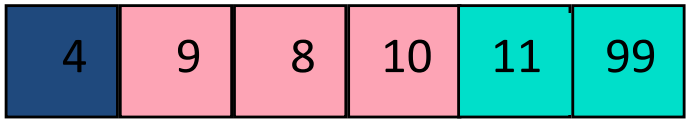



```
QS(A,1,6)

L:       1
R:       6
i:       1 2 3 4 5
j:       6 5 4
pivot: 10
```

```
QS(A,1,4)

L:       1
R:       4
i:       1 2
j:       4 3 4 1
pivot:   4
```

```
QS(A,5,6)

L:       5
R:       6
i:
j:
pivot:   11
```

Quicksort

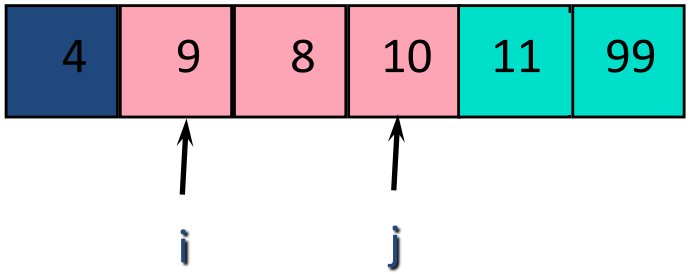



```
QS(A,1,1)

L:       1
R:       1
i:
j:
pivot:   4
```

```
QS(A,2,4)

L:       2
R:       4
i:
j:
pivot:   9
```

```
QS(A,5,6)

L:       5
R:       6
i:       5
j:       6
pivot:   11
```

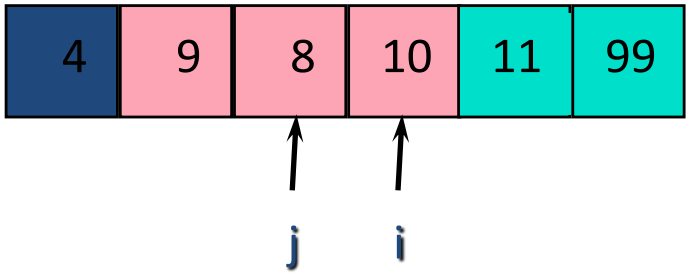
Quicksort




```
QS(A,2,4)

L:       2
R:       4
i:       2
j:       4
pivot:   9
```

```
QS(A,5,6)

L:       5
R:       6
i:       5
j:       6
pivot:   11
```

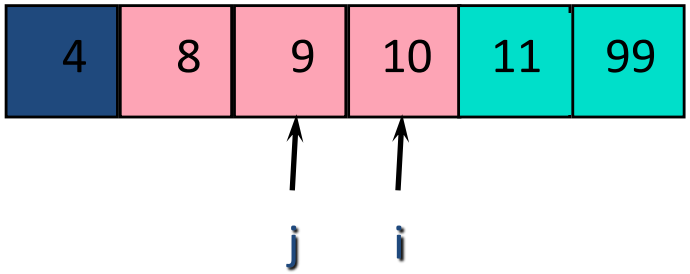
Quicksort




```
QS(A,2,4)

L:       2
R:       4
i:       2 3 4
j:       4 3
pivot:   9
```

```
QS(A,5,6)

L:       5
R:       6
i:       5
j:       6
pivot:   11
```

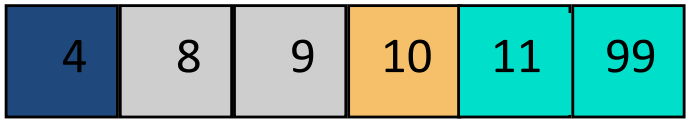
Quicksort

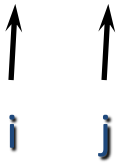



```
QS(A,2,4)

L:       2
R:       4
i:       2 3 4
j:       4 3
pivot:   9
```

```
QS(A,5,6)

L:       5
R:       6
i:       5
j:       6
pivot:   11
```

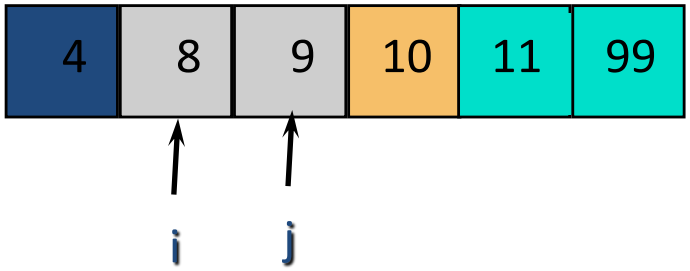
Quicksort





QS(A,2,4)				QS(A,2,3)		QS(A,4,4)		QS(A,5,6)	
L:	2			L:	2	L:	4	L:	5
R:	4			R:	3	R:	4	R:	6
i:	2	3	4	i:		i:		i:	5
j:	4	3		j:		j:		j:	6
pivot:	9			pivot:	8	pivot:	10	pivot:	11

Quicksort

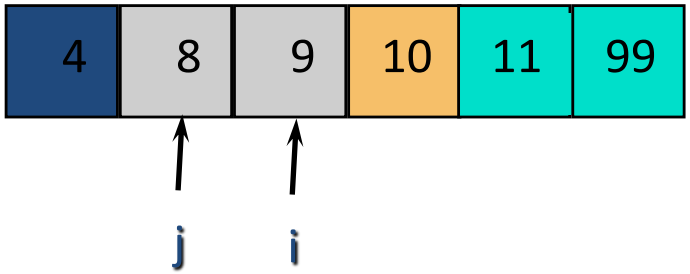


QS(A,2,3)	
L:	2
R:	3
i:	2
j:	3
pivot:	8

QS(A,4,4)	
L:	4
R:	4
i:	
j:	
pivot:	10

QS(A,5,6)	
L:	5
R:	6
i:	5
j:	6
pivot:	11

Quicksort

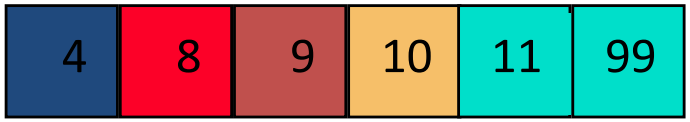


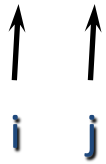

```
QS(A,2,3)

L:       2
R:       3
i:       2 3
j:       3 2
pivot:   8
```

```
QS(A,4,4)

L:       4
R:       4
i:
j:
pivot:   10
```

```
QS(A,5,6)

L:       5
R:       6
i:       5
j:       6
pivot:   11
```

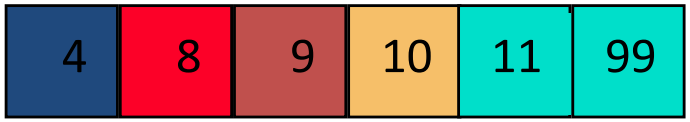
Quicksort

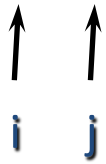




QS(A,2,3)		QS(A,2,2)	QS(A,3,3)	QS(A,4,4)	QS(A,5,6)
L:	2	L: 2	L: 3	L: 4	L: 5
R:	3	R: 2	R: 3	R: 4	R: 6
i:	2 3	i:	i:	i:	i: 5
j:	3 2	j:	j:	j:	j: 6
pivot:	8	pivot: 8	pivot: 9	pivot: 10	pivot: 11

Quicksort

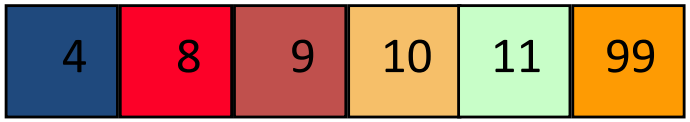


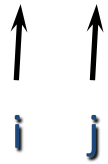



```
QS(A,4,4)

L:        4
R:        4
i:
j:
pivot:   10
```

```
QS(A,5,6)

L:        5
R:        6
i:        5
j:        6
pivot:
11
```

Quicksort





QS(A,5,6)		QS(A,5,5)		QS(A,6,6)	
L:	5	L:	5	L:	6
R:	6	R:	5	R:	6
i:	5	i:		i:	
j:	6	j:		j:	
pivot:	11	pivot:	11	pivot:	99

```
Unsorted List:
10  9  8  11  4  99

quicksort(0, 6);
   quicksort(0, 3);
      quicksort(0, 0);
      quicksort(1, 3);
         quicksort(1, 2);
            quicksort(1, 1);
            quicksort(2, 2);
         quicksort(3, 3);
   quicksort(4, 6);
      quicksort(4, 4);
      quicksort(5, 6);
         quicksort(5, 5);
         quicksort(6, 6);
Sorted List:
4  8  9  10  11  99
```

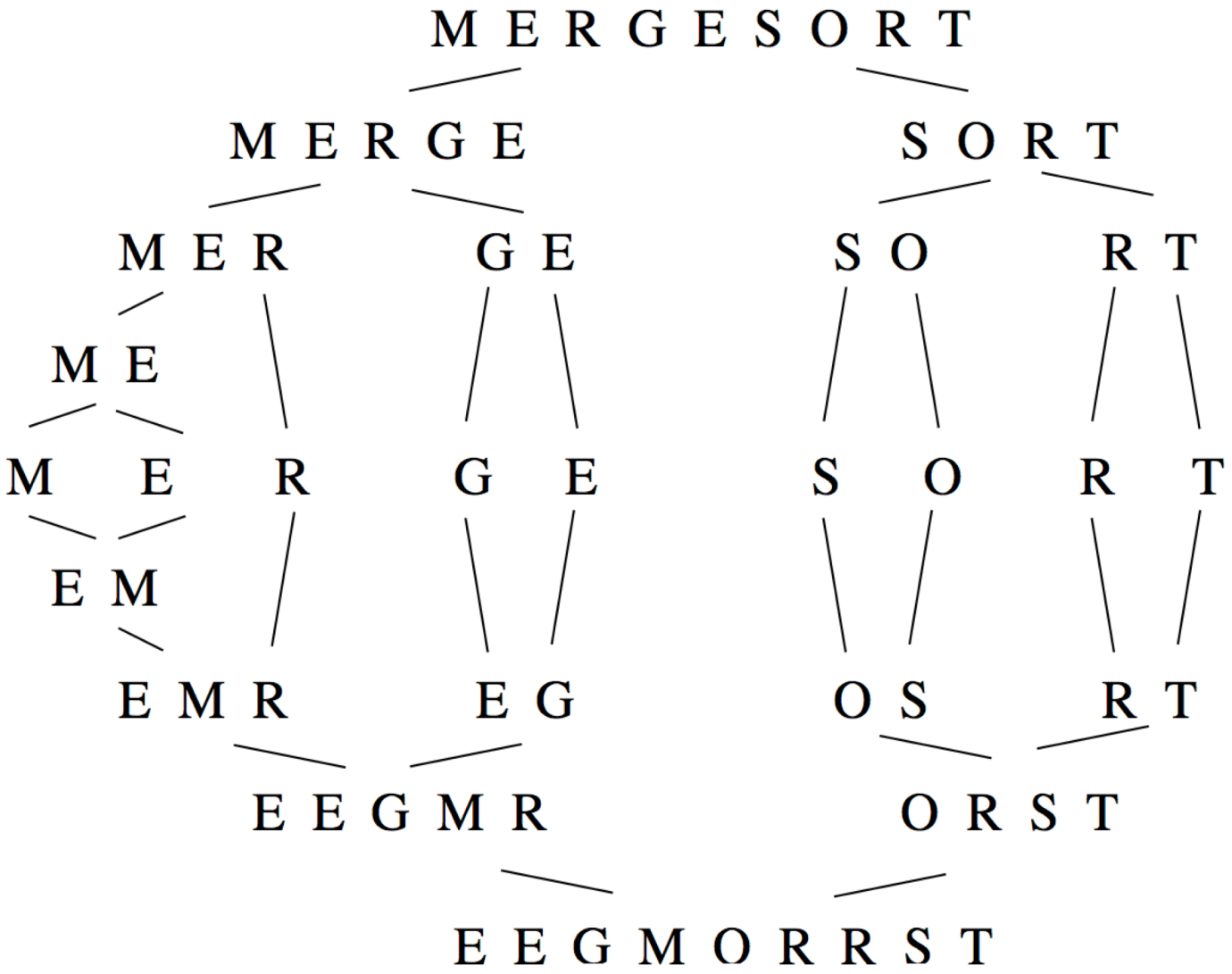
Quicksort

- Performance depends on which element is selected as the pivot
- The worst-case occurs when the list is sorted and the left-most element is selected as the pivot
- Space complexity is $O(n^2)$ in the worst case

Mergesort

- Divide-and-conquer, recursive, $O(n \log n)$
- Recursively partition the list into two lists L1 and L2
  - L1 and L2 approx. n/2 elements each
- Stop when we have a collection of lists of 1 element
- Now, each L1 and L2 is merged into a list S
  - Where the elements of L1 and L2 are put in S in order
- Pairs of sorted lists S1 and S2 are, in turn, merged as we ascend back up through the recursion

Mergesort


M E R G E S O R T
M E R G E
S O R T
M E R
G E
S O
R T
M E
M
E
R
G
E
S
O
R
T
E M
E M R
E G
O S
R T
E E G M R
O R S T
E E G M O R R S T

Merge Sort

```
mergesort(item_type s[], int low, int high)  {

    int i;       /* counter                        */
    int middle; /* index of middle element */

    if (low < high) {
        middle = (low+high)/2;
        mergesort(s,low,middle);
        mergesort(s,middle+1,high);
        merge(s, low, middle, high);
    }
}
```

Merge Sort

- The efficiency of mergesort depends on how we combine the two sorted halves into a single sorted list
- The key is to realize that each half (i.e. each sublist) is sorted
- So we just have to repeatedly
  - Take the ”front” element of either one list or the other (depending on which is smaller) and
  - Move it to the merged list to keep the elements in order

Merge Sort

```
merge(item_type s[], int low, int middle, int high){
   int i;                    /* counter                          */
   queue buffer1, buffer2; /* to hold elements for merging */
   init_queue(&buffer1);
   init_queue(&buffer2);
   for (i=low; i<=middle; i++) enqueue(&buffer1,s[i]);
   for (i=middle+1; i<=high; i++) enqueue(&buffer2,s[i]);

   i = low;
   while (!(empty_queue(&buffer1) &&  !(empty_queue(&buffer2)) {
   // Alt: while (!(empty_queue(&buffer1) || empty_queue(&buffer2))) {
      if (headq(&buffer1) <= headq(&buffer2))
         s[i++] = dequeue(&buffer1);
      else
         s[i++] = dequeue(&buffer2);
   }
   while (!empty_queue(&buffer1)) s[i++] = dequeue(&buffer1);
   while (!empty_queue(&buffer2)) s[i++] = dequeue(&buffer2);
}
```

Mergesort

Why is mergesort $O(n \log n)$?

How many times do we merge and how big are the data sets?

Let's assume that n is a power of two

At level 0
2^1 calls to `mergesort` & merge 2^1 lists of size $\sim n/2$

At level 1
2^2 calls to `mergesort` & merge 2^2 lists of size $\sim n/4$
…
At level k
2^{k+1} calls to `mergesort` & merge 2^{k+1} lists of size $\sim n/ 2^{k+1}$

Mergesort

How many levels k?

$k = \log_2 n$, $e.g.$ if $n = 8, k = 3$

At level k, the sublists are of size 1.

So we merge $k = \log_2 n$ times (at levels $0 - k$-1)

Each time we merge 2^{k+1} lists of size ~n/ 2^{k+1} i.e. total size ~n

So the total complexity is $O(n \log_2 n)$

Characteristics of a Good Sort

- Speed
- Consistency
- Keys
- Memory Requirements
- Length and Code Complexity
- Stability

No single winner in all categories

Characteristics of a Good Sort

Speed

- Some sort algorithms with poor order of complexity sort small lists better than other more complex sorts

- For example short lists of 5 to 50 keys or for longer lists that are almost sorted, Insertion Sort is extremely efficient and can be faster than Quicksort

Characteristics of a Good Sort

Consistency

- Some sorts always take the same amount of time
  but many have "best case" and "worst case" performance
  for particular input orders of keys

- Example: QuickSort is generally the fastest of the $O(n \log n)$ sorts, but it always has an $O(n^2)$ worst case.

Characteristics of a Good Sort

Keys

- The nature of keys can dramatically affect the speed of sorts
- This is especially true for string keys, which can vary significantly in length and relatedness.
- Longer keys take longer to copy or compare

Characteristics of a Good Sort

Memory Requirements

- Having enough memory is as important as speed

- Methods that sort "in place" will be more desirable than those that use a duplicate array (e.g. Mergesort) or those that use a significant amount of stack space for recursive calls

Characteristics of a Good Sort

Code Complexity

- Short, simple algorithms are appealing because they are easy to implement and debug
- Algorithms that can easily be applied to all data types are convenient to use but often come at the cost of implementation complexity
- Although they may not be as fast as more specialized algorithms, simple algorithms are always appealing especially when maintenance is an issue

Characteristics of a Good Sort

Stability

- A stable sorting algorithm maintains the relative order of records with equal keys
  - Let records R and S have the same key
  - R appears before S in the original list,
  - R will always appear before S in the sorted list
- This is particularly important when sorting based on multiple keys

Characteristics of a Good Sort

Stability

- Assume that the following pairs of numbers are to be sorted by their first component (two different results are possible)

  (4, 2) (3, 7) (3, 1) (5, 6)

  (3, 7) (3, 1) (4, 2) (5, 6) (stable: order maintained)
  (3, 1) (3, 7) (4, 2) (5, 6) (unstable: order changed)

- Unstable sorting algorithms change the relative order of records with equal keys, but stable sorting algorithms do not

Characteristics of a Good Sort

Stability

- Unstable sorting algorithms can be specially implemented to be stable
- We can artificially extend the key comparison, so that two objects with equal keys are decided using another key as a tie-breaker and maintain stability
- Stability usually comes with an additional computational cost

INEFFECTIVE SORTS

```
DEFINE HALFHEARTEDMERGESORT(LIST):
    IF LENGTH(LIST) < 2:
        RETURN LIST
    PIVOT = INT(LENGTH(LIST) / 2)
    A = HALFHEARTEDMERGESORT(LIST[:PIVOT])
    B = HALFHEARTEDMERGESORT(LIST[PIVOT:])
    // UMMMMM
    RETURN [A, B] // HERE. SORRY.
```

```
DEFINE FASTBOGOSORT(LIST):
    // AN OPTIMIZED BOGOSORT
    // RUNS IN O(N LOG N)
    FOR N FROM 1 TO LOG(LENGTH(LIST)):
        SHUFFLE(LIST):
        IF ISSORTED(LIST):
            RETURN LIST
    RETURN "KERNEL PAGE FAULT (ERROR CODE: 2)"
```

```
DEFINE JOBINTERVIEWQUICKSORT(LIST):
    OK SO YOU CHOOSE A PIVOT
    THEN DIVIDE THE LIST IN HALF
    FOR EACH HALF:
        CHECK TO SEE IF IT'S SORTED
            NO, WAIT, IT DOESN'T MATTER
        COMPARE EACH ELEMENT TO THE PIVOT
            THE BIGGER ONES GO IN A NEW LIST
            THE EQUAL ONES GO INTO, UH
            THE SECOND LIST FROM BEFORE
        HANG ON, LET ME NAME THE LISTS
            THIS IS LIST A
            THE NEW ONE IS LIST B
        PUT THE BIG ONES INTO LIST B
        NOW TAKE THE SECOND LIST
            CALL IT LIST, UH, A2
        WHICH ONE WAS THE PIVOT IN?
        SCRATCH ALL THAT
        IT JUST RECURSIVELY CALLS ITSELF
        UNTIL BOTH LISTS ARE EMPTY
            RIGHT?
        NOT EMPTY, BUT YOU KNOW WHAT I MEAN
    AM I ALLOWED TO USE THE STANDARD LIBRARIES?
```

```
DEFINE PANICSORT(LIST):
    IF ISSORTED(LIST):
        RETURN LIST
    FOR N FROM 1 TO 10000:
        PIVOT = RANDOM(0, LENGTH(LIST))
        LIST = LIST[PIVOT:] + LIST[:PIVOT]
        IF ISSORTED(LIST):
            RETURN LIST
    IF ISSORTED(LIST):
        RETURN LIST:
    IF ISSORTED(LIST):  //THIS CAN'T BE HAPPENING
        RETURN LIST
    IF ISSORTED(LIST): // COME ON COME ON
        RETURN LIST
    // OH JEEZ
    // I'M GONNA BE IN SO MUCH TROUBLE
    LIST = [ ]
    SYSTEM("SHUTDOWN -H +5")
    SYSTEM("RM -RF ./")
    SYSTEM("RM -RF ~/*")
    SYSTEM("RM -RF /")
    SYSTEM("RD /S /Q C:\*")  //PORTABILITY
    RETURN [1, 2, 3, 4, 5]
```

http://xkcd.com/1185/