04-630
Data Structures and Algorithms for Engineers

David Vernon
Carnegie Mellon University Africa

vernon@cmu.edu
www.vernon.eu

Lecture 27

Hashing

- Dictionaries
- Hashing
- Hash functions
- Collision resolution
- Complexity
- Applications

Recall: Lecture 8

Containers and Dictionaries

- Containers
- Dictionaries

Containers and Dictionaries

- *Containers* are data structures that permit storage and retrieval of data items *independent of content*

- *Dictionaries* are data structures that retrieve data based on *key* values (i.e. content)

Containers and Dictionaries

- *Dictionaries* permits access to data items by content
  - You put an item into a dictionary so that you can find it when you need it

Containers and Dictionaries

- Main dictionary operations are
  - Search(D, k) Given a search key k, return a pointer to the element in dictionary D whose key value is k, if one exists
  - Insert(D, x) Given a data item x, add it to the dictionary D
  - Delete(D, x) Given a pointer to a given data item x in the dictionary D, remove it from D

Containers and Dictionaries

- Some dictionary data structures also **efficiently** support other useful operations
  - $\mathrm{Max}(D)$ Retrieve the item with the largest key from D
  - $\mathrm{Min}(D)$ Retrieve the item with the smallest key from D

  These operations allows the dictionary to serve as a priority queue; more on this later

Containers and Dictionaries

- Some dictionary data structures also **efficiently** support other useful operations
  - Predecessor(D, x) Retrieve the item from D whose key is immediately before x in sorted order
  - Successor(D, x) Retrieve the item from D whose key is immediately after x in sorted order

  These operations enable us to iterate through the elements of the data structure

Containers and Dictionaries

- We have defined these container and dictionary operations in an **abstract** manner,

  without reference to their implementation or the implementation of the structure itself

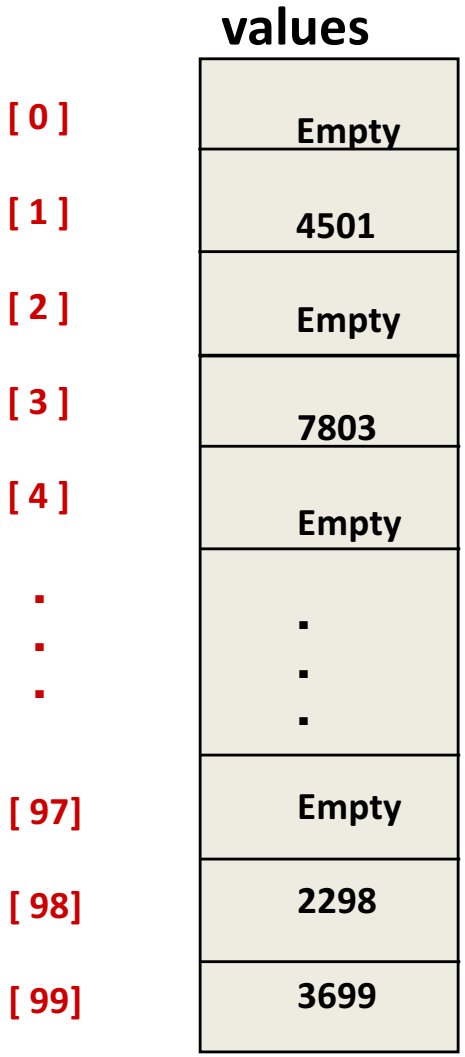
- There are many implementation options

  - Unsorted arrays
  - Sorted arrays
  - Singly-linked lists
  - Doubly-linked lists
  - Binary search trees
  - Balanced binary search trees
  - **Hash tables**
  - Heaps
  - ...

Hashing

- Hash tables are a **very** practical way to maintain a dictionary
- It takes a constant amount of time to look up an item in an array, **if** you have its index … O(1)
- A hash function is a mathematical function to maps keys to integers
  - This integer is used as an index into an array
  - We store our item at that position

Hashing

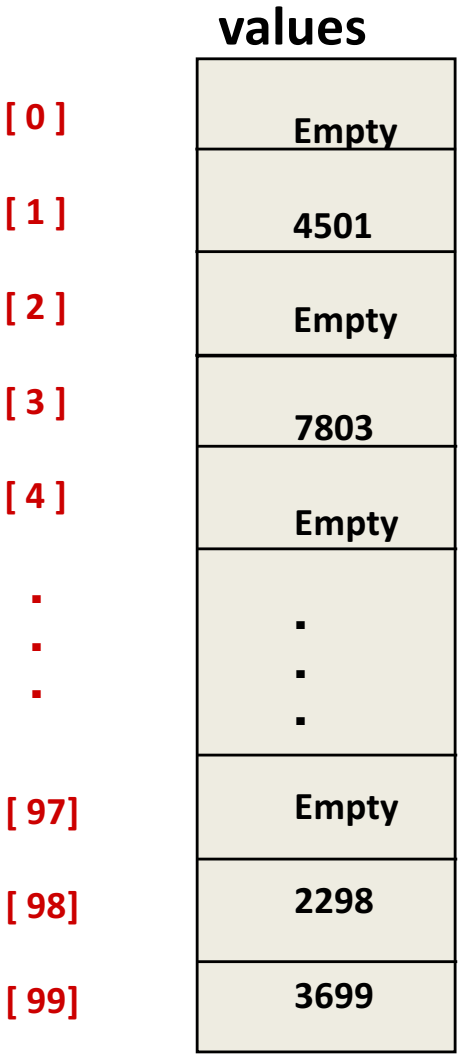


HandyParts company makes no more than 100 different parts. But the parts all have four digit numbers.

This hash function can be used to store and retrieve parts in an array.

Hash(key) = partNum % 100

Placing Elements in the Array



Use the hash function

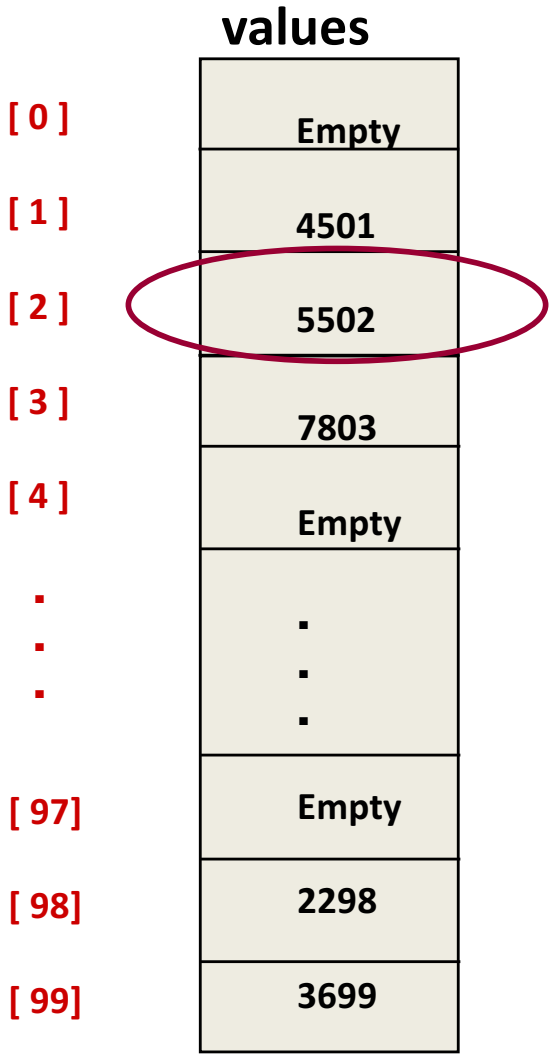
Hash(key) = partNum % 100

to place the element with

part number 5502 in the

array.

Placing Elements in the Array



Next place part number 6702 in the array.

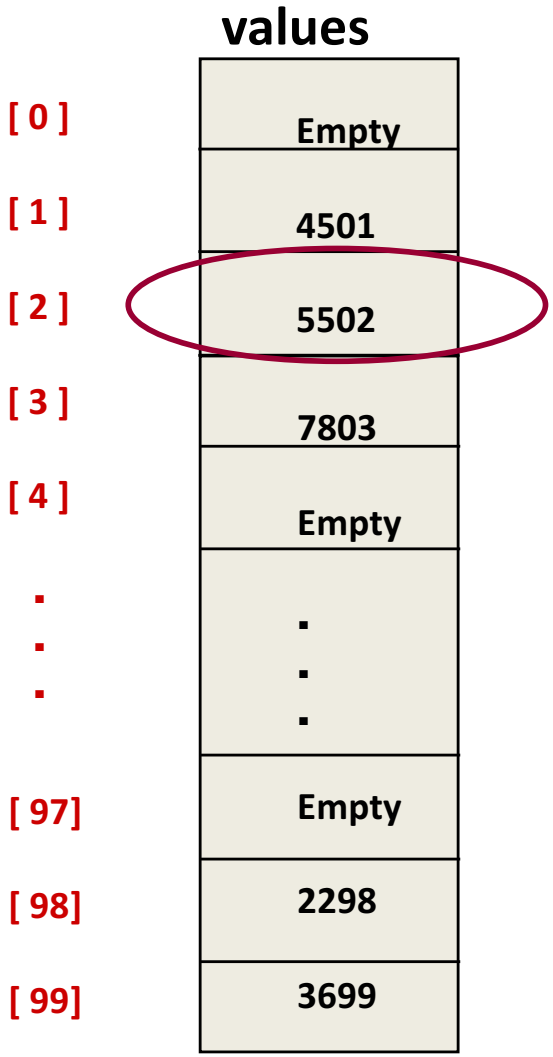
Hash(key) = partNum % 100

6702 % 100 = 2

But values[2] is already occupied.

COLLISION OCCURS

How to Resolve the Collision?

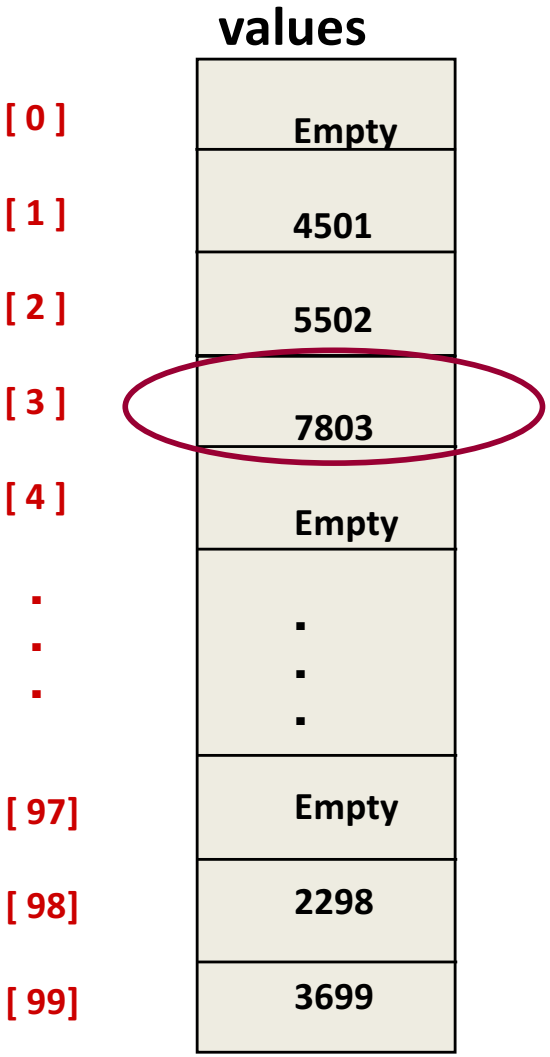


One way is by linear/sequential probing.
This uses the rehash function

(HashValue + 1) % 100

repeatedly until an empty location is found for part number 6702.

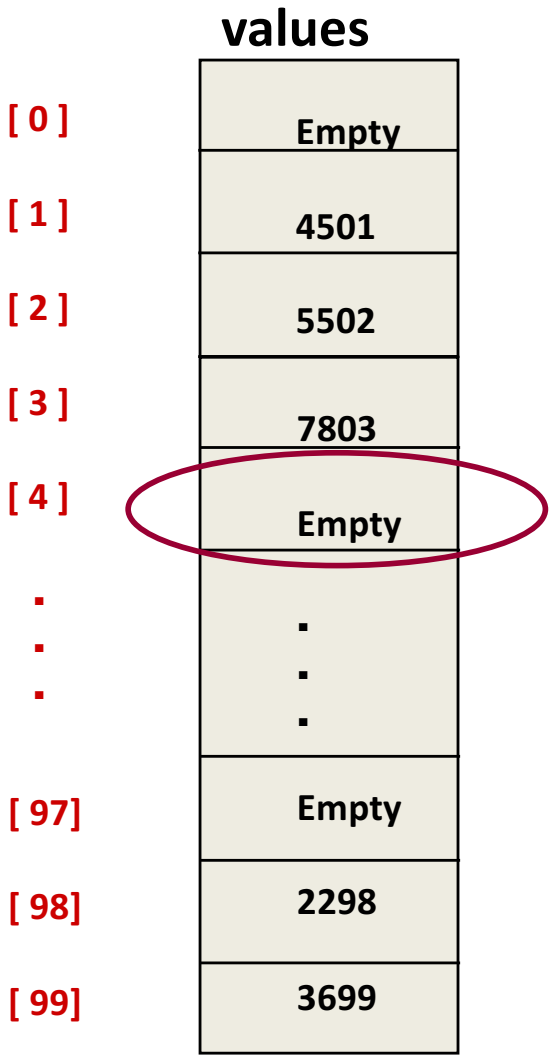
Resolving the Collision



Still looking for a place for 6702 using the function

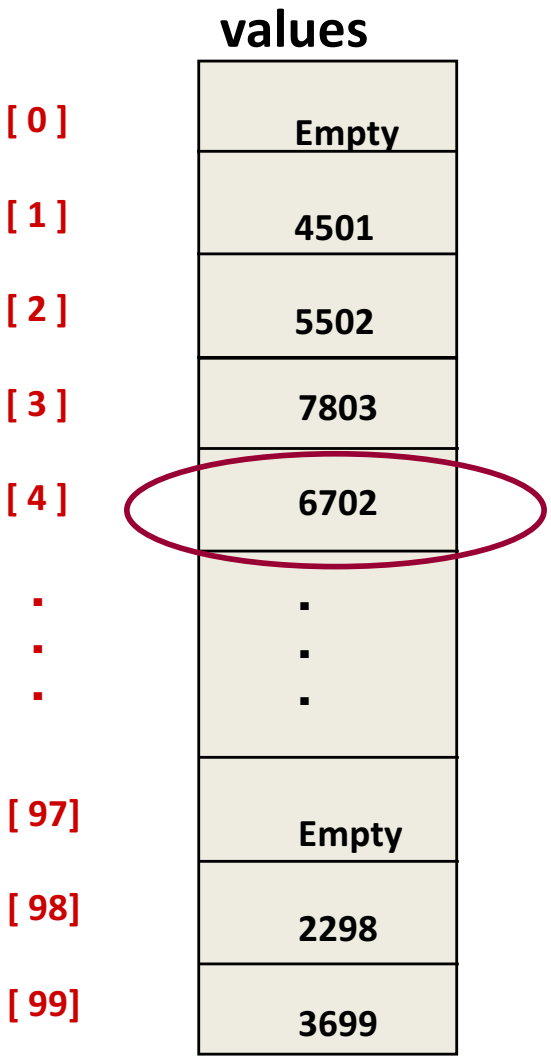
(HashValue + 1) % 100

Collision Resolved



Part 6702 can be placed at the location with index 4.

Collision Resolved



Part 6702 is placed at the location with index 4.

Where would the part with number 4598 be placed using linear probing?

Hashing

- In general, the keys are not so conveniently defined (e.g. part numbers) and they have to be computed
- Typically, they are some alphanumeric string S
- The first step of a hash function is to map each key to a big integer
- Let α be the size of the alphabet on which S is written
- Let $char(c)$ be a function that maps each symbol of the alphabet to a unique integer from 0 to $\alpha - 1$

Hashing

- The hash function

$$H(S) = \sum_{i=0}^{|S|-1} \alpha^{|S|-(i+1)} \times char(s_i)$$

  maps each string to a unique (but large) integer by treating the characters of the string as "digits" in a base-α number system

- The result is unique identified numbers, but they are so large they will quickly exceed the number of slots m in the hash table

- We reduce this number to an integer between 0 and $m - 1$ by taking the remainder of $H(S) \bmod m$

Hashing

- If m, the size of the hash table is selected well, the resulting hash value will be fairly uniformly distributed

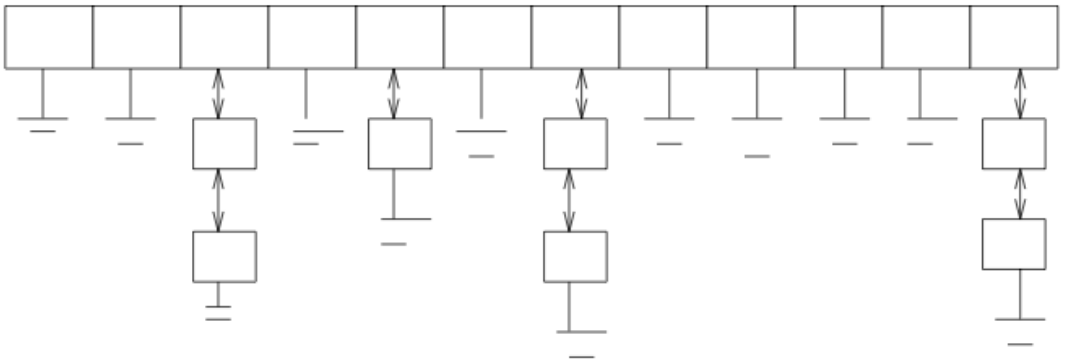
- Ideally, is a large prime not too close to $2^i - 1$

Hashing

- Collisions
  - No matter how good the hash function is, there will sometimes be collisions: two keys mapping to the same number/index
  - One approach to collision resolution: open addressing
    - On insertion, check to see if the desired position is empty
    - If so, insert it
    - If not, find some other place …
    - Simplest approach is sequential probing: look for the next open spot in the table

Hashing

- Collisions
  - No matter how good the hash function is, there will sometimes be collisions: two keys mapping to the same number/index
  - Alternative approach: chaining



Hashing

- Complexity of operations in a hash table
  - Assuming chaining with doubly-linked lists
  - m-element hash table
  - n keys

	Hash table (expected)	Hash table (worst case)
Search(L, k)	$O(n/m)$	$O(n)$
Insert(L, x)	$O(1)$	$O(1)$
Delete(L, x)	$O(1)$	$O(1)$
Successor(L, x)	$O(n+m)$	$O(n+m)$
Predecessor(L, x)	$O(n+m)$	$O(n+m)$
Minimum(L)	$O(n+m)$	$O(n+m)$
Maximum(L)	$O(n+m)$	$O(n+m)$

Hashing

- A hash table is often the best data structure to maintain a dictionary

- Also useful for other applications

  – Efficient string matching via hashing

    Problem: given a text string t and a pattern string p,
    does t contain the pattern p as a substring, and if so where?

Hashing

- A hash table is often the best data structure to maintain a dictionary

- Also useful for other applications

  – String matching

    Simplest algorithm:

    - Overlay p in t at every position in the text
    - Check whether every pattern character matches the text character
    - Complexity $O(nm)$, where $n = |t|$ and $m = |p|$

Hashing

– String matching

Rabin-Karp algorithm:

- Basic idea: if two strings are identical, so are their hash values
- If the two strings are different, the hash values are *almost certainly* different (may need to check, but not often)
- With some clever computation of the hash function (to reduce complexity to constant) the algorithm will usually run in $O(n + m)$

Hashing

- Related applications
  - Is a given document different from all the rest in a large corpus?
  - Is part of this document plagiarized from a document in a large corpus?