Robotics: Principles and Practice

Module 3: Mobile Robots

Lecture 2: Absolute position estimation

David Vernon
Carnegie Mellon University Africa

www.vernon.eu

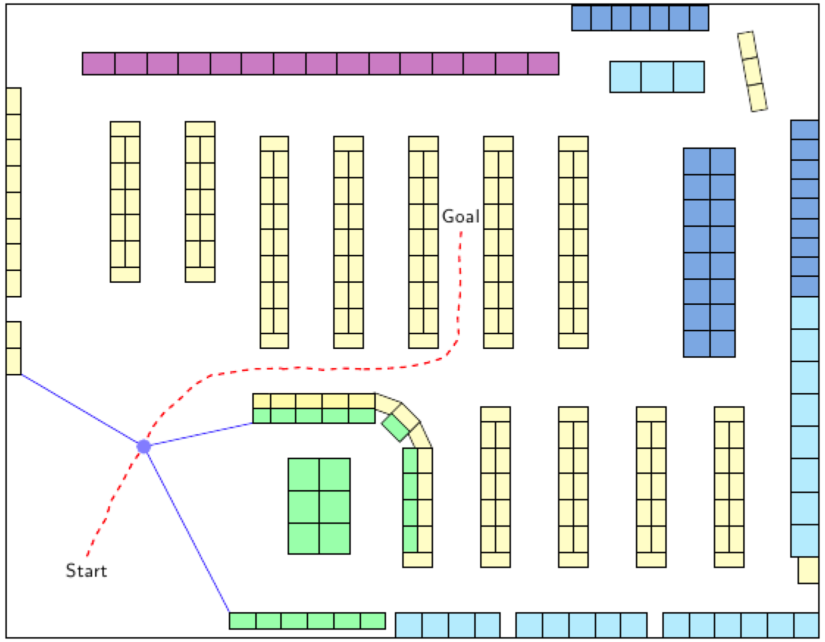
Absolute Position Estimation

Compute the position of the robot (x_r, y_r)

- (x_r, y_r) are the coordinates in a global reference system

Do so by observing several reference points

- The position of the reference points must be known



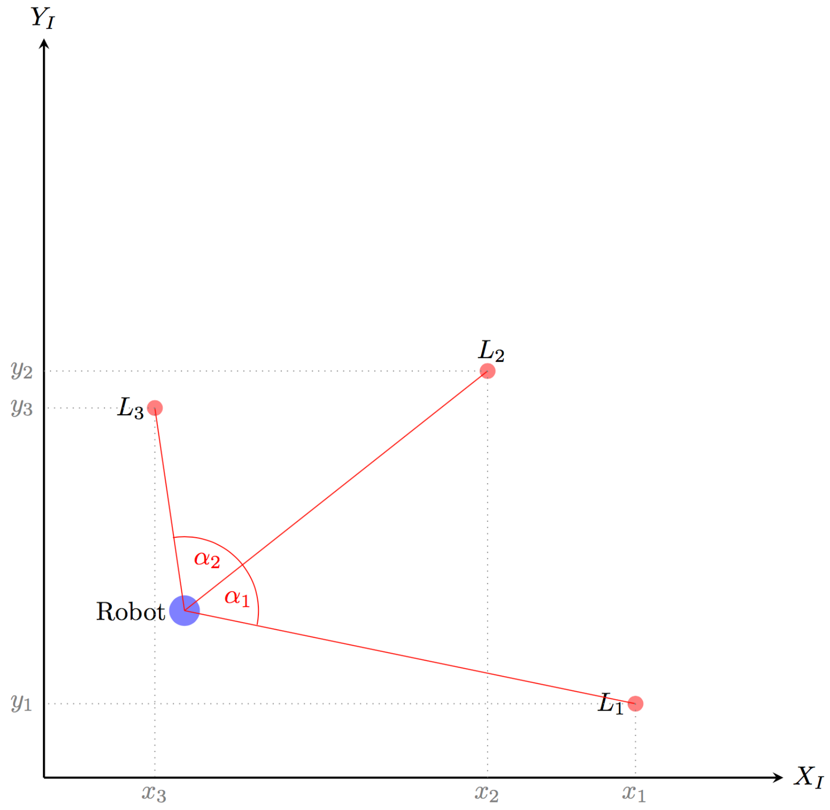
Absolute Position Estimation

Three-point triangulation

- We know the absolute position of three points (objects)

  $L_1 = (x_1, y_1); L_2 = (x_2, y_2); L_3 = (x_3, y_3)$

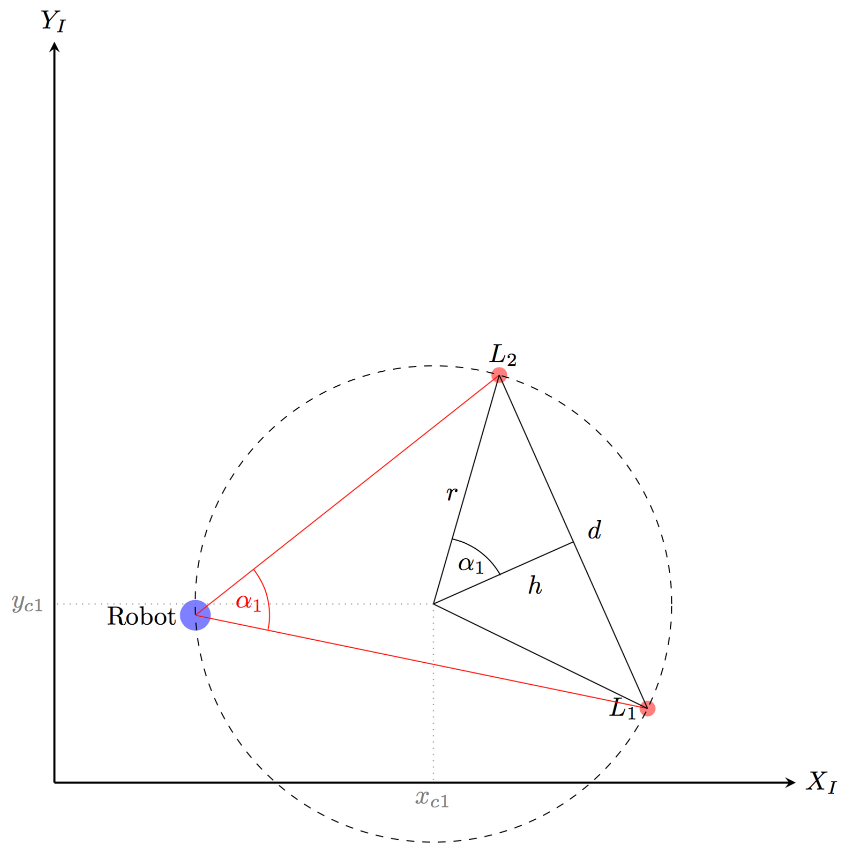
- We observe the relative angles α_1 and α_2 between them
  - We do not observe the distances
- Problem: find the absolute position of the robot



Absolute Position Estimation

– The robot is somewhere on a circle containing L_1 and L_2 with radius r and centre (x_{c1}, y_{c1}) … why?

See explanation on the next slide



Absolute Position Estimation

- The robot is somewhere on a circle containing L_1 and L_2 with radius r and centre (x_{c1}, y_{c1}) … why?

- The locus of points M (the possible positions of the robot)
  - that have a given angle α_1
  - subtended by a given line segment

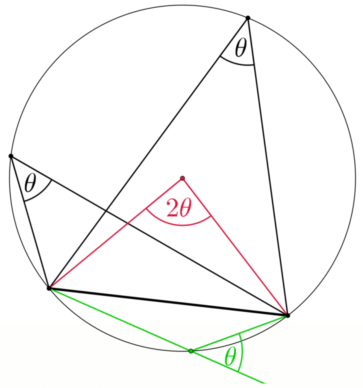
  is a circle

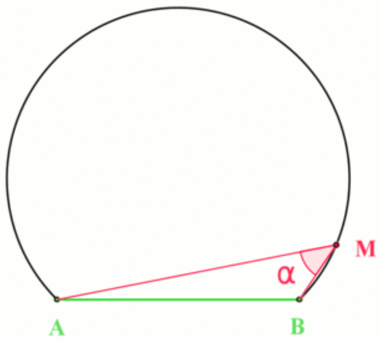
- The line segment is a chord of the circle

- The end points of the line segment define an arc on that circle

"The inscribed angle theorem states that an angle θ inscribed in a circle is half of the central angle 2θ that subtends the same arc on the circle. Therefore, the angle does not change as its vertex is moved to different positions on the circle."

https://en.wikipedia.org/wiki/Inscribed_angle





Absolute Position Estimation

- The robot is somewhere on a circle containing L_1 and L_2 with radius r and centre (x_{c1}, y_{c1}) ... why?

- We know d and we measure α_1

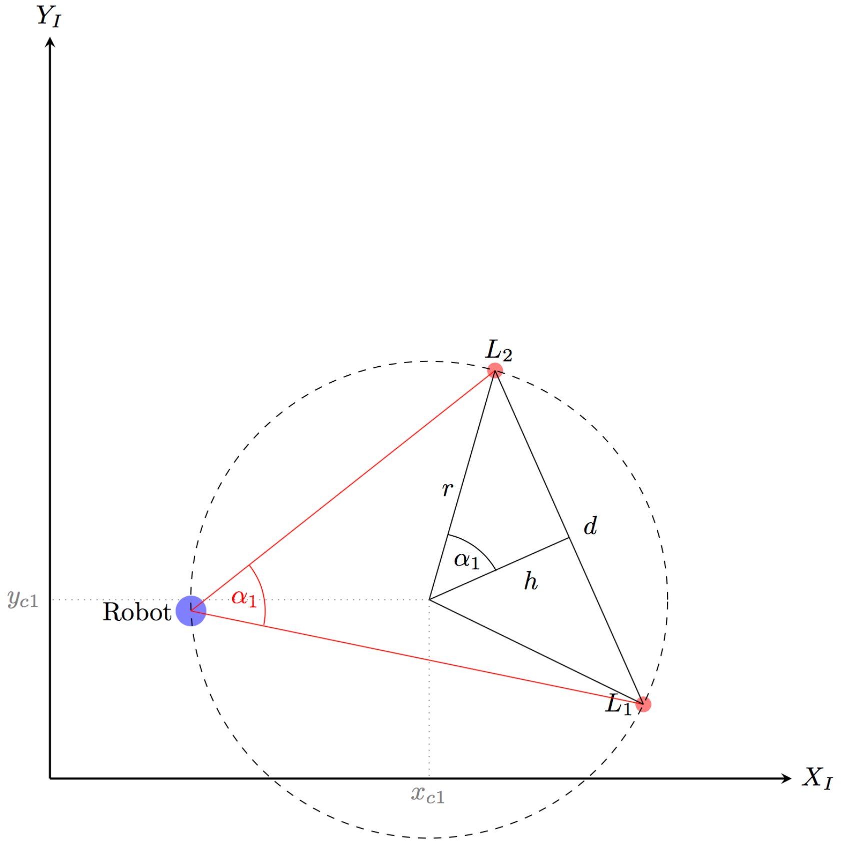
- Compute r and h

$$\sin\alpha_1 = \frac{d/2}{r} \qquad \tan\alpha_1 = \frac{d/2}{h}$$

$$\implies r = \frac{d/2}{\sin\alpha_1} \qquad \implies h = \frac{d/2}{\tan\alpha_1}$$

- Compute the coordinates of the mid-point between L_1 and L_2

- Knowing h, compute the coordinates of the centre of the circle (x_{c1}, y_{c1})



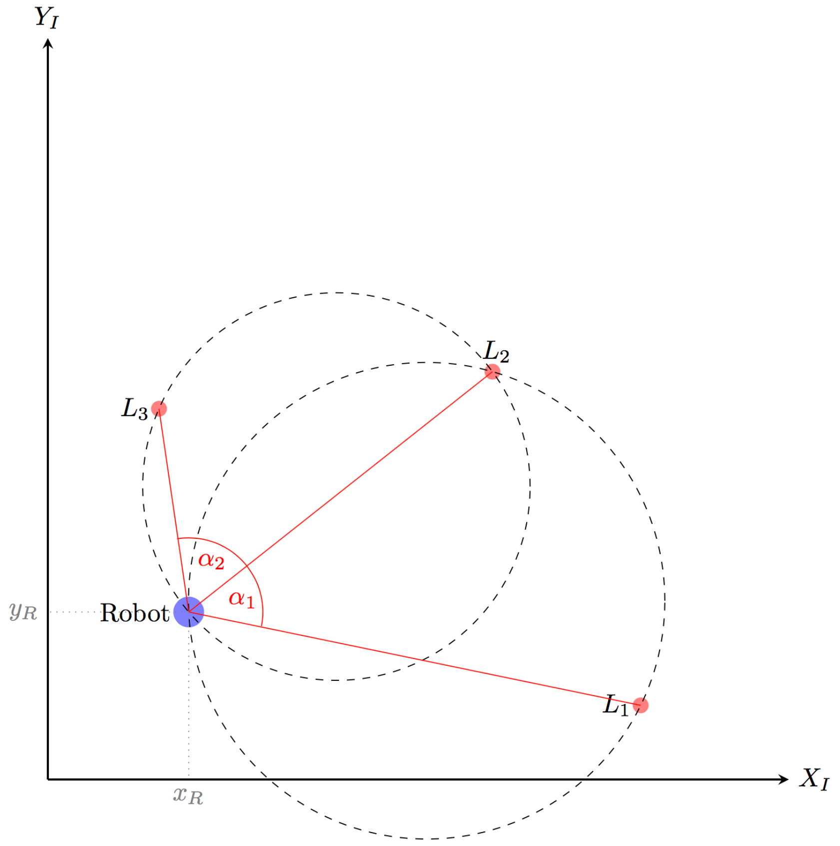
Do this as an exercise

Absolute Position Estimation

- In the same way, we compute the radius and coordinates of the centre of the circle (x_{c2}, y_{c2})

- The robot must be on both circles

  The coordinates of the robot (x_r, y_r)
  is given by the intersection of the two circles



Absolute Position Estimation

Aside

- If the robot can measure the distance to the landmarks, then two landmarks are sufficient to find (x_r, y_r)

- Why?

  These are different circle to the ones in the previous construction

  Because the robot must at the intersection of two circles

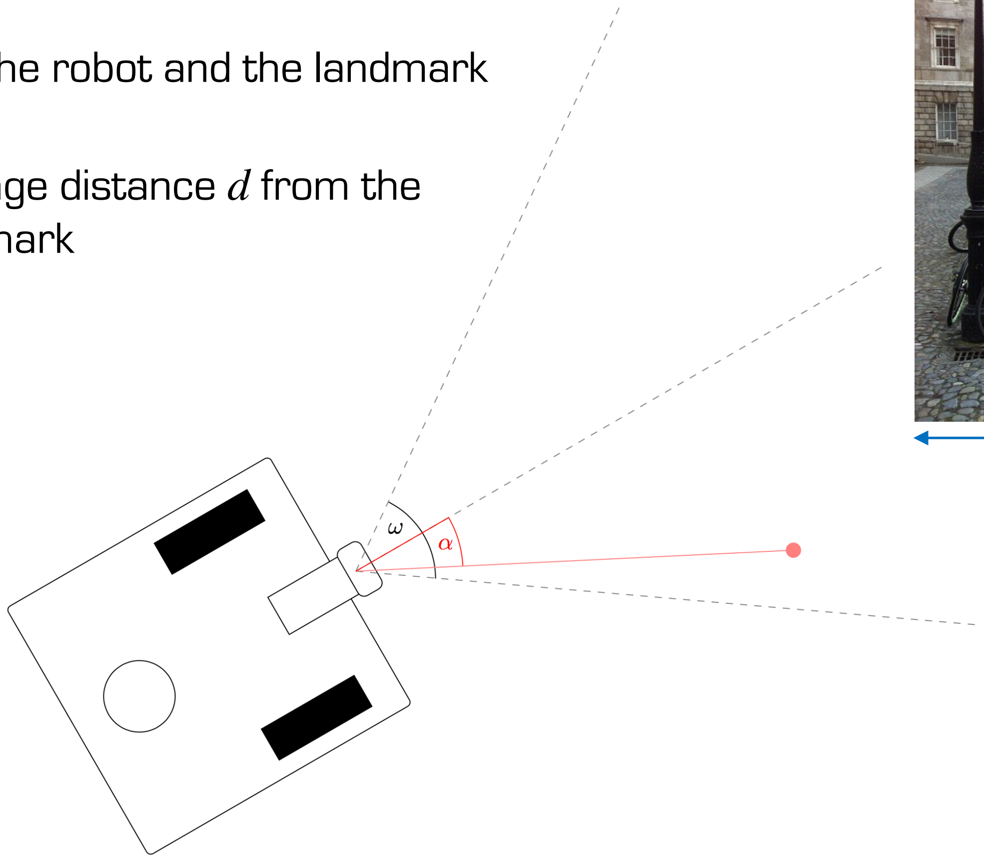
  - One centred at the first landmark and with radius equal to its distance from the robot

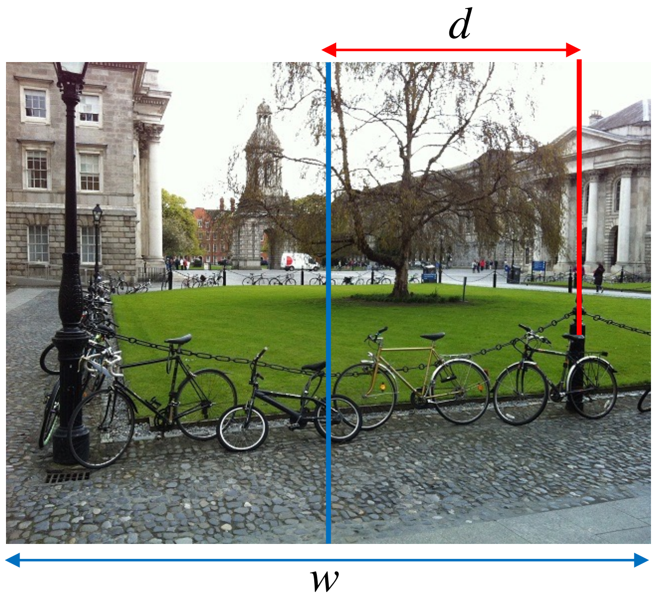
  - One centred at the second landmark and with radius equal to its distance from the robot

Absolute Position Estimation

Measuring angles using visual landmarks

- Compute the angle α between the robot and the landmark
- By measuring the horizontal image distance d from the centre of the image to the landmark





$$\frac{\alpha}{\omega} = \frac{d}{w}$$

$$\implies \alpha = d\frac{\omega}{w}$$

Absolute Position Estimation

Measuring angles using visual landmarks

- The angular field of view angle ω can be computed from the focal length of the lens f and the width of the sensor h

$$\omega = 2 \times \tan^{-1} ({}^{h}/_{2f})$$

- For details, see https://www.edmundoptics.eu/knowledge-center/application-notes/imaging/understanding-focal-length-and-field-of-view/