Robotics: Principles and Practice

Module 4: Robot Manipulators

Lecture 2: Object pose specification with homogenous transformations and vectors & quaternions

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Recall, we have developed a system where we can

specify the position and orientation of coordinate reference frames anywhere

with respect to each other

w.r.t. station frame of reference

or

with respect to a given base frame

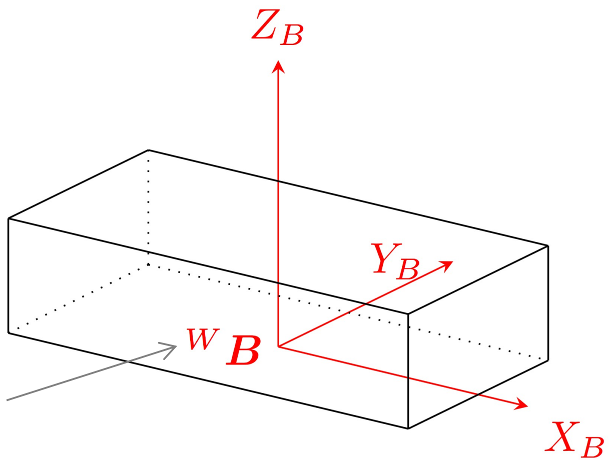
w.r.t. fixed world frame of reference

This, in itself, is not much use since the world you and I know does not have too many coordinate reference frames in it

What we really require is a way of identifying the pose of objects

Position and Orientation:
Six degrees of freedom

The trick, and it is no more than a trick, is to attach a coordinate frame to an object, *i.e.* symbolically glue an XYZ frame into an object simply by defining it to be there

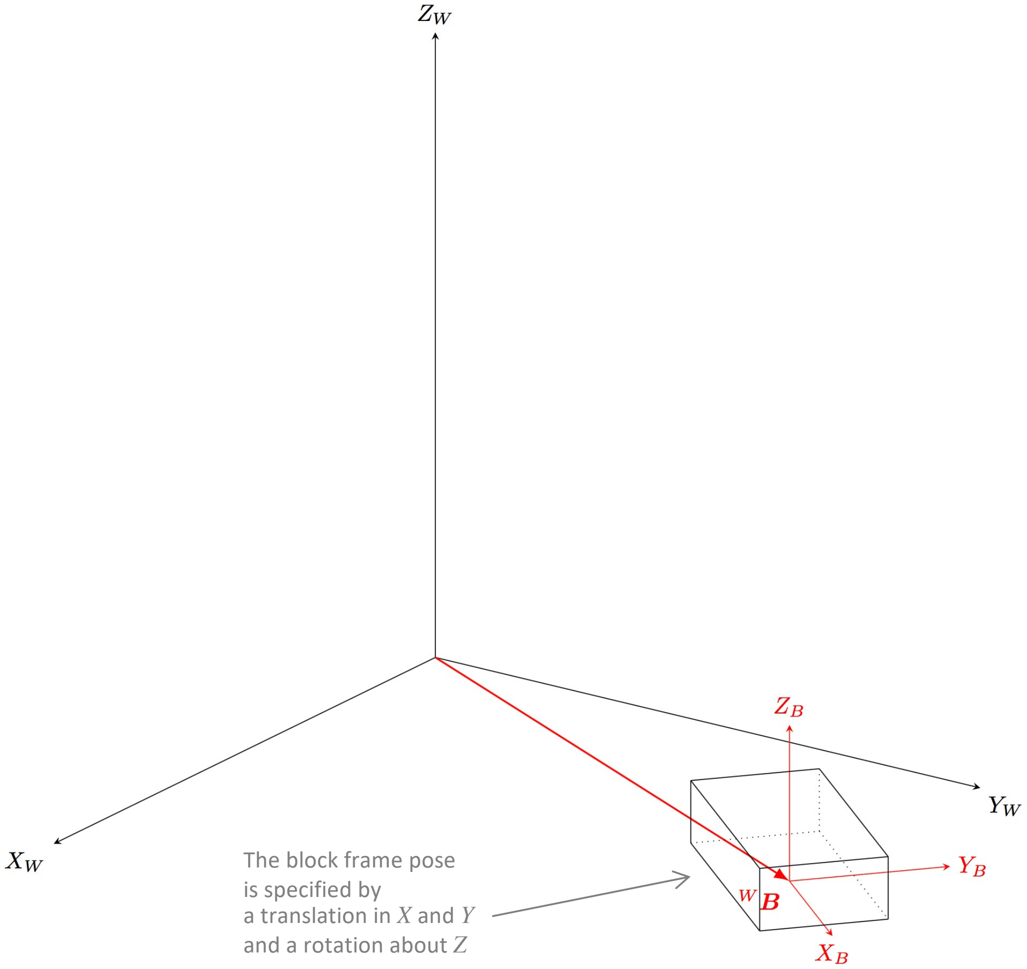


The block frame is defined with respect to the world frame of reference

- As we rotate and translate the coordinate frame, so we rotate and translate objects

- We can arbitrarily position and orient a coordinate frame – and an object – by specifying the required translations and rotations

- Thus, we specify the pose of an object by specifying its associated coordinate frame (homogeneous transformation)



The block frame pose is specified by a translation in X and Y and a rotation about Z

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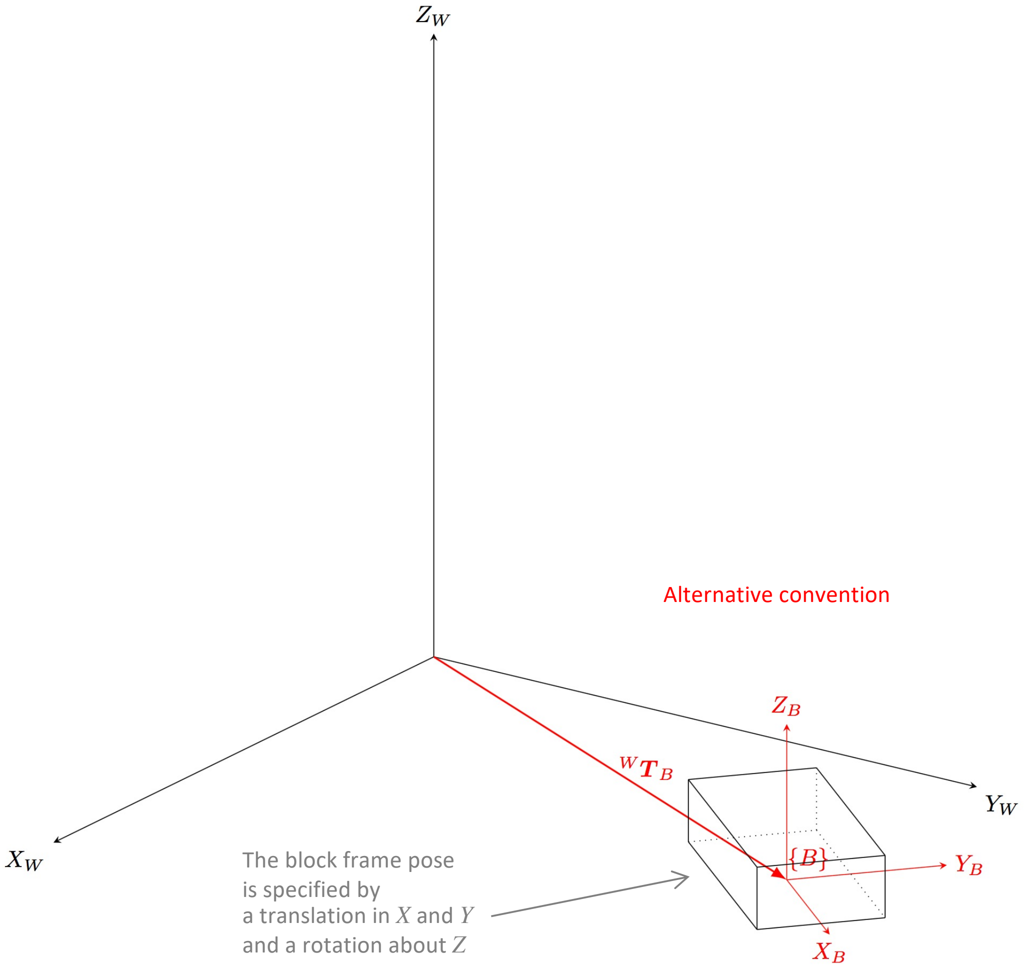


Alternative convention

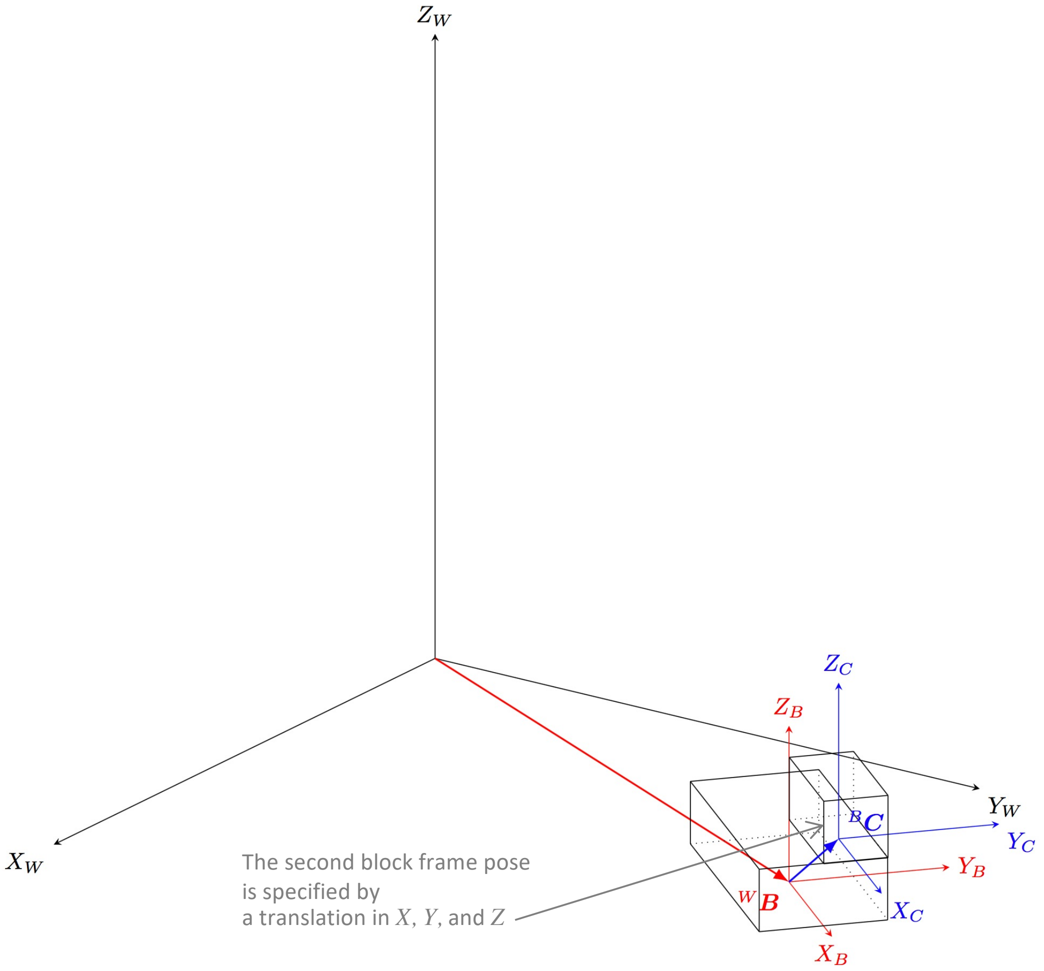
The block frame pose is specified by a translation in X and Y and a rotation about Z

- We can arbitrarily position and orient one object, i.e. its pose, with respect to another object

- How? By specifying the required translations and rotations of its associated coordinate frame (homogeneous transformation)

i.e. ${}^{B}C = \mathrm{Trans}(x, y, z)$

These values represent translations along the X_B, Y_B, Z_B axes;
the values of the translations depend on the dimensions of the objects



The second block frame pose
is specified by
a translation in X, Y, and Z

- We can arbitrarily position and orient one object, i.e. its pose, with respect to another object

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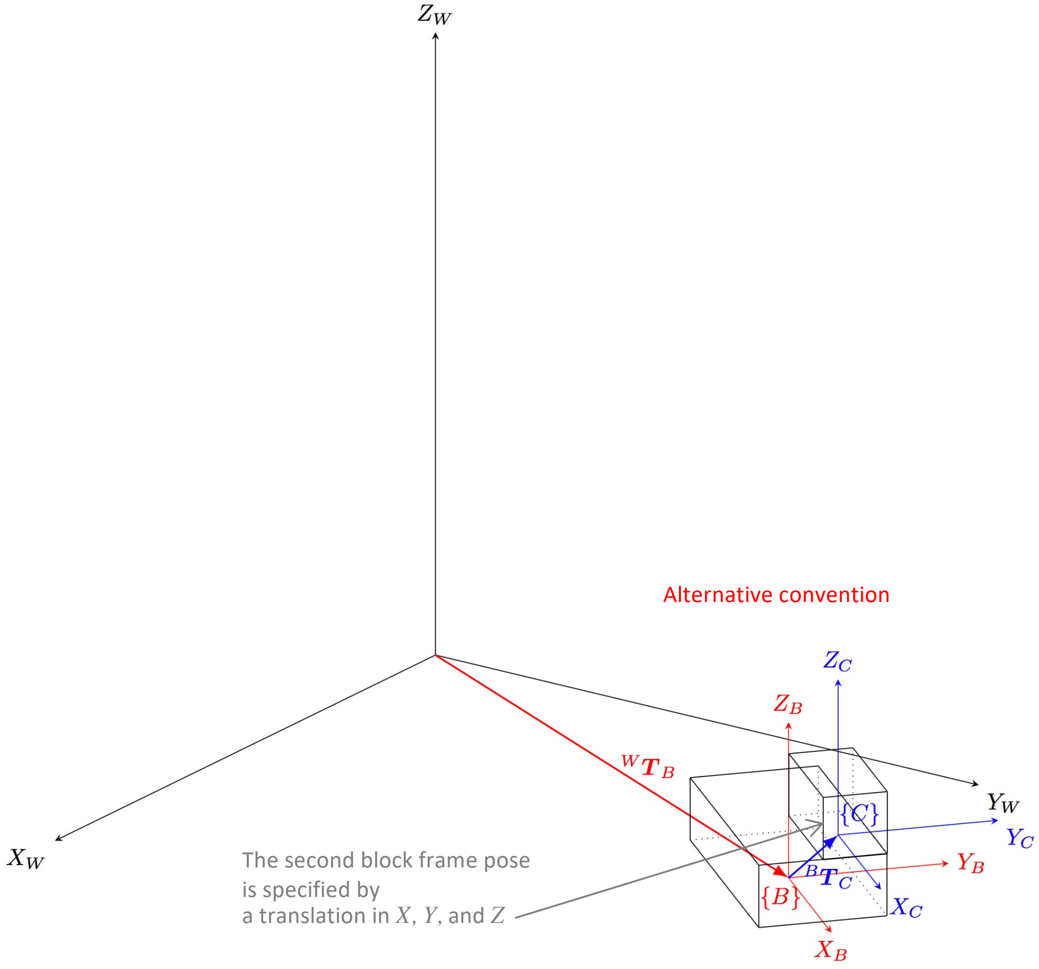


Alternative convention

The second block frame pose
is specified by
a translation in X, Y, and Z

Specifying Pose in ROS

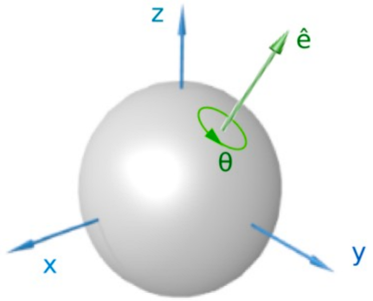
- We will use homogeneous transformations to specify a frame of reference, for end-effector and object pose

- ROS uses a different (but entirely equivalent) approach
  - Specify the origin of the frame as a 3-D vector
  - Specify the orientation of the frame as a quaternion: a single rotation about some (appropriate) axis

Specifying Pose in ROS

- Euler's rotation theorem states that any displacement of a rigid body (in 3D space), such that a point on the rigid body remains fixed,

  is equivalent to a single rotation θ about some axis that runs through the fixed point



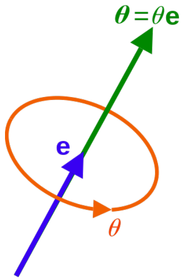
https://en.wikipedia.org/wiki/Euler%27s_rotation_theorem

- The axis of rotation is known as an Euler axis, typically represented by a unit vector **ê**

See: https://en.wikipedia.org/wiki/Rotation_formalisms_in_three_dimensions

Specifying Pose in ROS

- The product by $\theta\,\hat{\mathbf{e}}$ is known as an axis-angle



https://en.wikipedia.org/wiki/Axis–angle_representation

- Quaternions are a simple way to encode this axis–angle representation of a rotation in four numbers

Specifying Pose in ROS

- Quaternions are hypercomplex numbers

$$q = w + \overbrace{x\,\mathbf{i} + y\,\mathbf{j} + z\,\mathbf{k}}^{\text{Vector part (imaginary part)}}$$

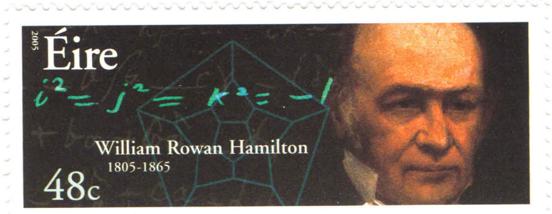
Scalar part (real part)

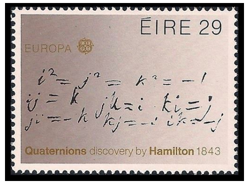
w, x, y, and z are real numbers

quaternion units

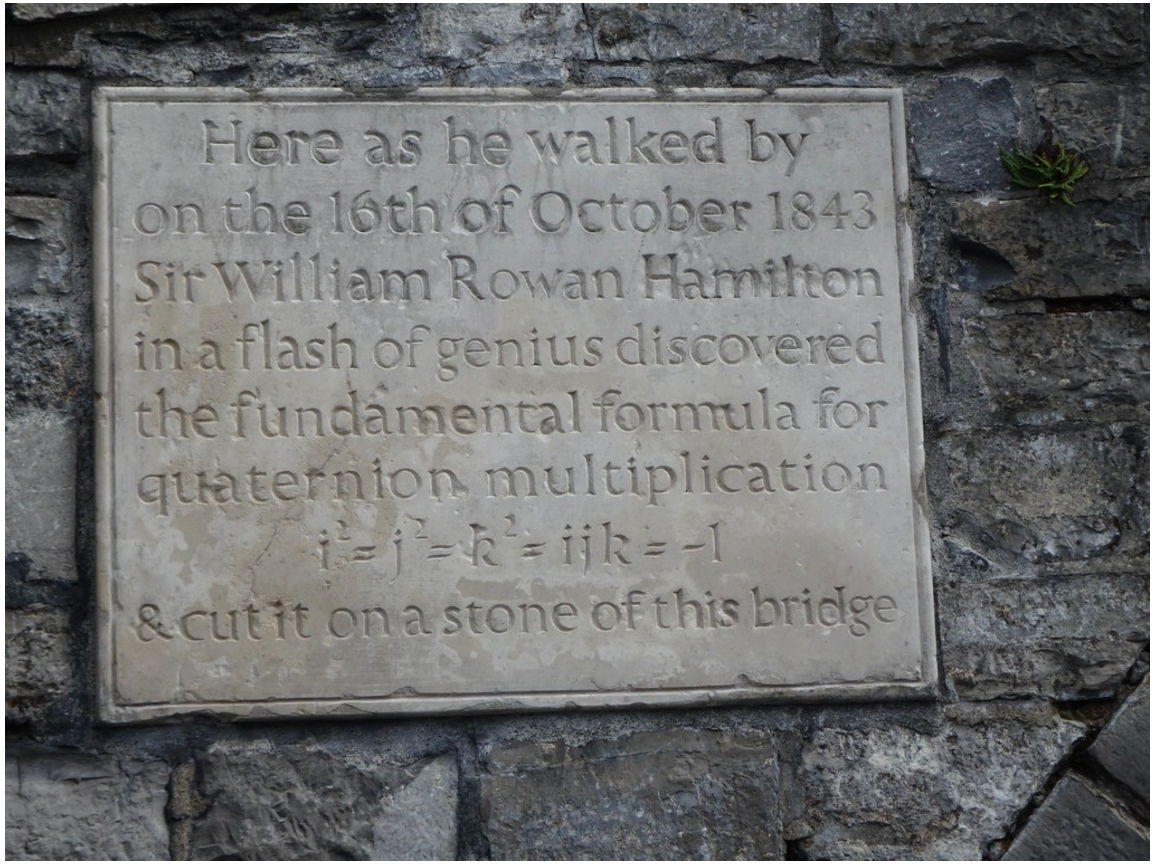
$$\mathbf{i}^2 = \mathbf{j}^2 = \mathbf{k}^2 = \mathbf{i}\,\mathbf{j}\,\mathbf{k} = -1$$

- Discovered by Irish mathematician William Rowan Hamilton in 1843





Specifying Pose in ROS


Here as he walked by
on the 16th of October 1843
Sir William Rowan Hamilton
in a flash of genius discovered
the fundamental formula for
quaternion multiplication
$i^2 = j^2 = k^2 = ijk = -1$
& cut it on a stone of this bridge

Broom Bridge in Dublin, Ireland

https://www.flickr.com/photos/infomatique/44408785822

Specifying Pose in ROS

- A rotation of θ about the Euler axis $\hat{\mathbf{e}} = e_x\,\mathbf{i} + e_y\,\mathbf{j} + e_z\,\mathbf{k}$ is given by

$$\begin{aligned} q &= w + x\,\mathbf{i} + y\,\mathbf{j} + z\,\mathbf{k} \\ &= \cos(\theta/2) + e_x \sin(\theta/2)\,\mathbf{i} + e_y \sin(\theta/2)\,\mathbf{j} + e_z \sin(\theta/2)\,\mathbf{k} \end{aligned}$$

NB: half the angle

- Equivalently

$$q = \begin{pmatrix} w \\ x \\ y \\ z \end{pmatrix}$$

Specifying Pose in ROS

In ROS, we write it slightly differently

$$q = \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} \begin{array}{l} \left.\begin{array}{l} \\ \\ \\ \end{array}\right\} \leftarrow \text{Axis part} \\ \leftarrow \text{Angle part} \end{array}$$

Specifying Pose in ROS

A rotation of -90° about the Z axis would be specified in quaternion notation as

$$q = \begin{bmatrix} 0.7071 \\ 0.0 \\ 0.0 \\ -0.7071 \end{bmatrix}$$

$$q = \begin{bmatrix} 0.0 \\ 0.0 \\ -0.7071 \\ 0.7071 \end{bmatrix} \longleftarrow \text{ROS}$$

Recommended Reading

D. Vernon, Machine Vision – Automated Visual Inspection and Robot Vision, Prentice Hall International, 1991. Chapter 8.

http://vernon.eu/publications/91_Vernon_Machine_Vision.pdf

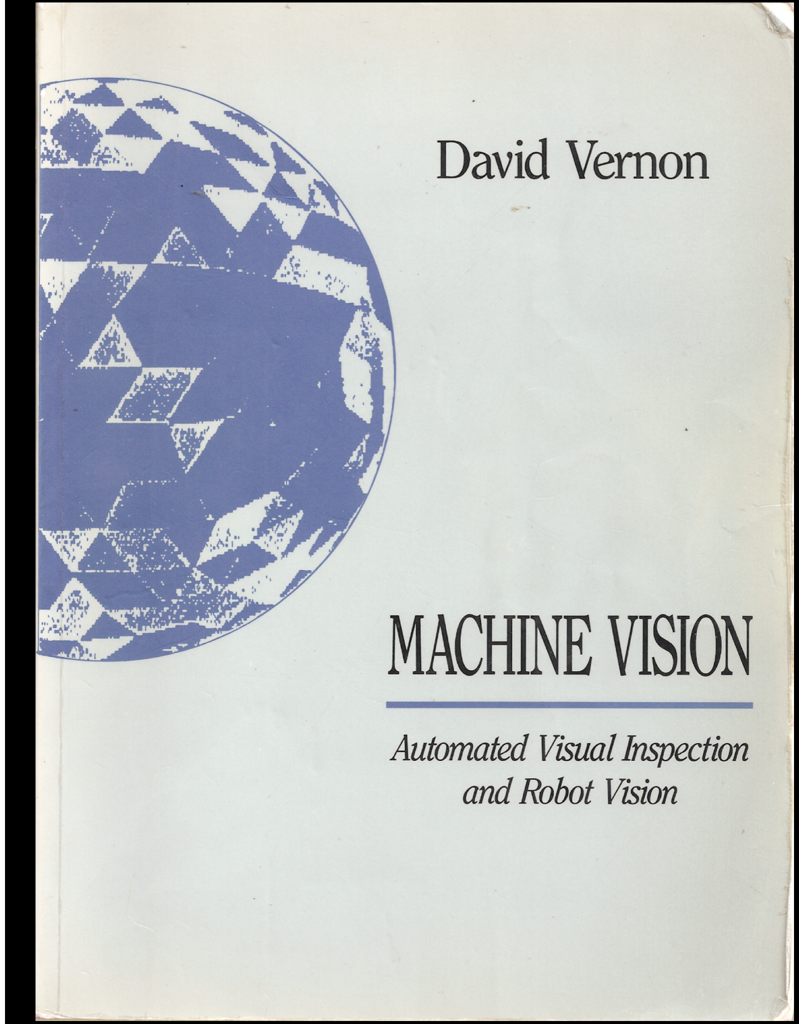
Similar material to that presented in this lecture.

R. P. Paul, Robot Manipulators – Mathematics, Programming, and Control, MIT Press, 1981. Chapter 1.

https://books.google.rw/books?id=UzZ3LAYqvRkC&printsec=frontcover&source=gbs_ViewAPI&redir_esc=y#v=onepage&q&f=false

Similar material to that presented in this lecture but complete comprehensive treatment.

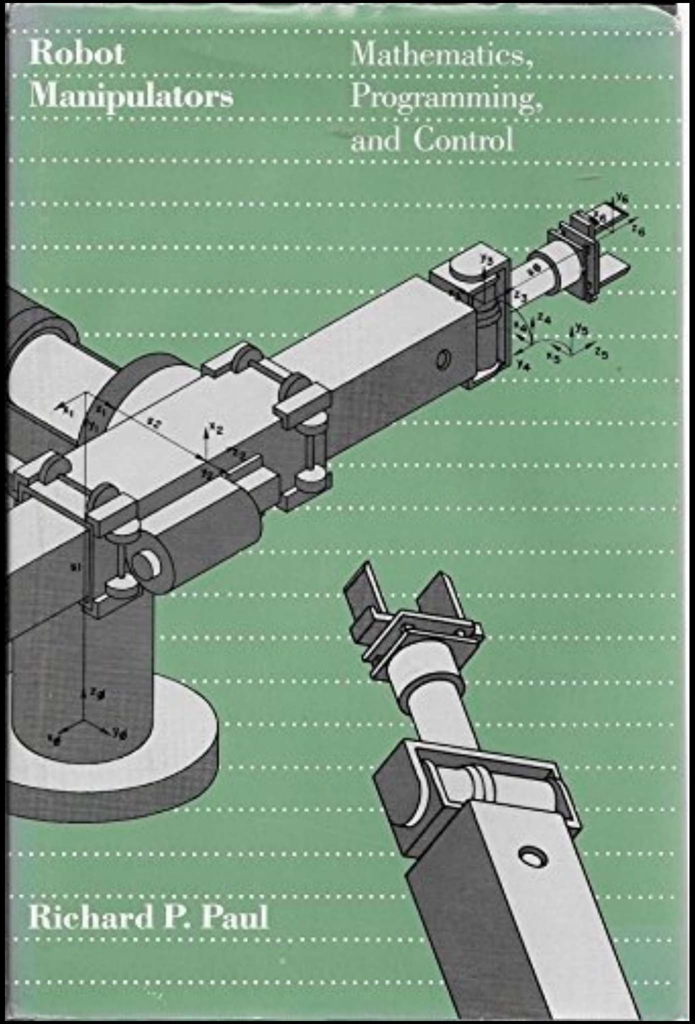
P. Corke, Robotics, Vision and Control, 2nd Edition, Springer, 2017.
Comprehensive contemporary treatment; highly recommended.



David Vernon

MACHINE VISION

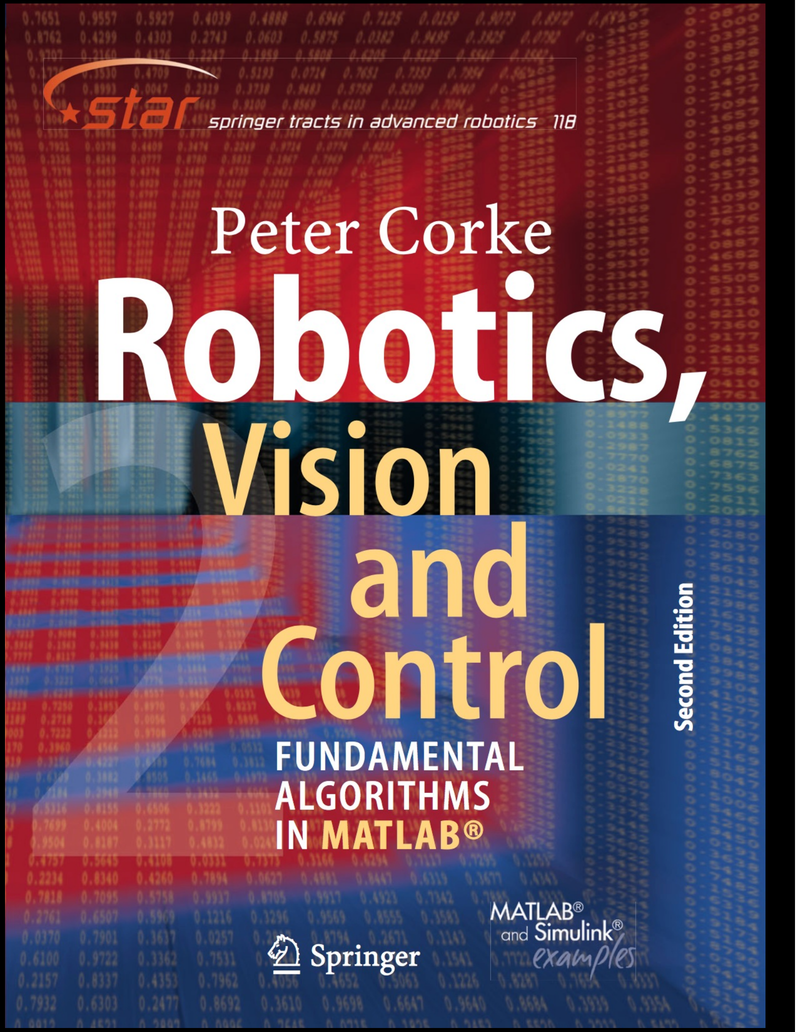
Automated Visual Inspection and Robot Vision



Robot Manipulators

Mathematics, Programming, and Control

Richard P. Paul



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Peter Corke

Robotics, Vision and Control

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