Robotics: Principles and Practice

Module 4: Robot Manipulators

Lecture 6: Denavit-Hartenberg representation; forward kinematics of the LynxMotion AL5D arm

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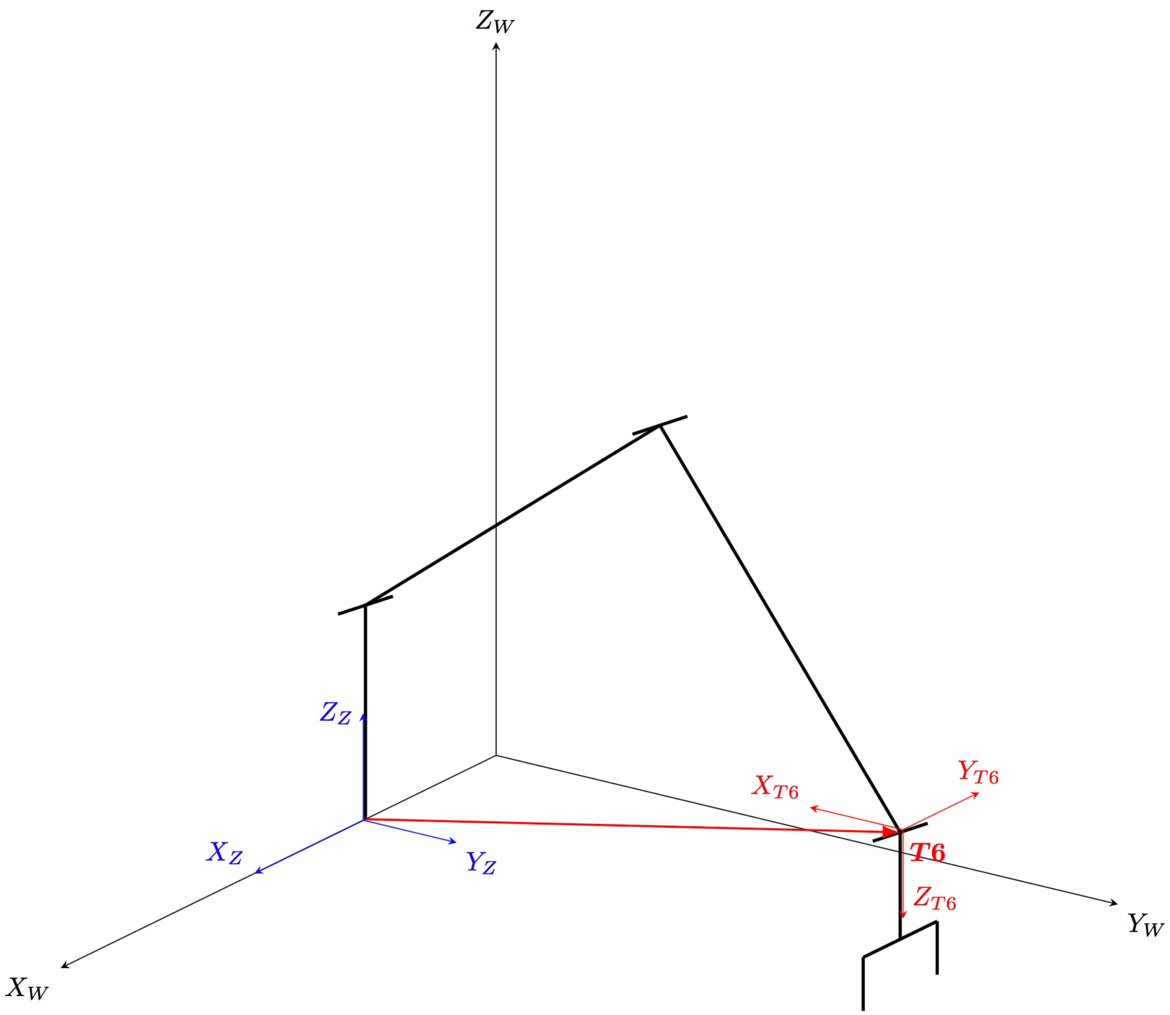


T6 The manipulator wrist

T6 The position of the end of the manipulator with respect to its base at ***Z***

The ***T6*** frame is a computable function of the joint variables

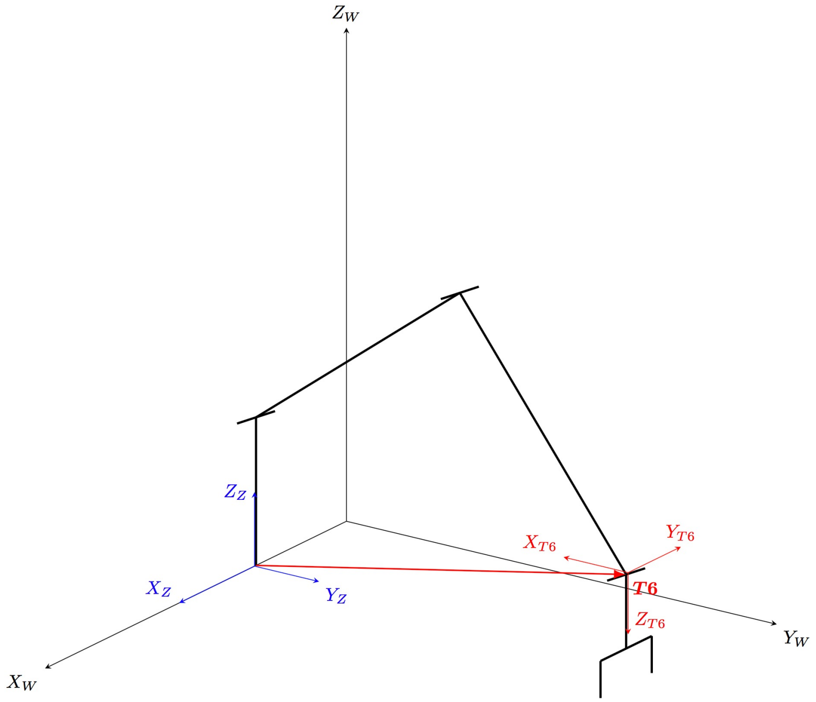
We need to define the relationship between ***Z*** and ***T6*** in terms of the joint angles …
i.e. we need to define the manipulator kinematics

Then we need to find the inverse kinematic solution the allows us to determine the joint values that correspond to a given ***T6***:

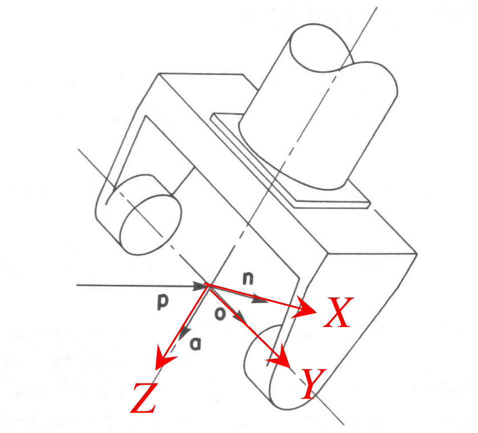
The joint variables are computed for a given ***T6*** frame

Recall that there is a convention that the ***T6*** frame should be embedded in the manipulator

- with the origin at the wrist
- with the Z axis directed outward from the wrist to the gripper
- with the Y axis directed in the plane of movement of the gripper when it is opening and closing
- with the X axis making up a right-hand system



The same convention applies to the $\boldsymbol{E}$ frame that is embedded in a two-finger gripper (end-effector … hence $\boldsymbol{E}$)



(Paul 1981)

Direction of X axis

Direction of Y axis

Direction of Z axis

$$E = \begin{bmatrix} n_x & o_x & a_x & p_x \\ n_y & o_y & a_y & p_y \\ n_z & o_z & a_z & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

n Normal

o Orientation

a Approach

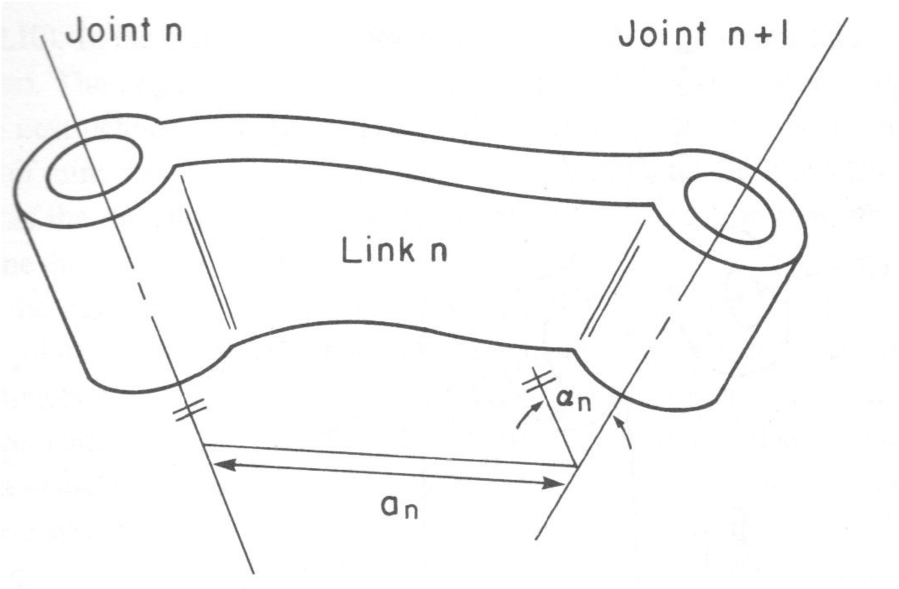
Denavit-Hartenberg Representation

$$T6 = {}^{Z}A_1 \;\; {}^{A1}A_2 \;\; {}^{A2}A_3 \;\; {}^{A3}A_4 \;\; {}^{A4}A_5 \;\; {}^{A5}A_6$$

$$T6 = A_1 \; A_2 \; A_3 \; A_4 \; A_5 \; A_6$$

${}^{Z}A_1$ position and orientation of link 1 w.r.t. the base
${}^{A1}A_2$ position and orientation of link 2 w.r.t. A_1
${}^{A2}A_3$
${}^{A3}A_4$
${}^{A4}A_5$
${}^{A5}A_6$ position and orientation of link 6 w.r.t. A_5

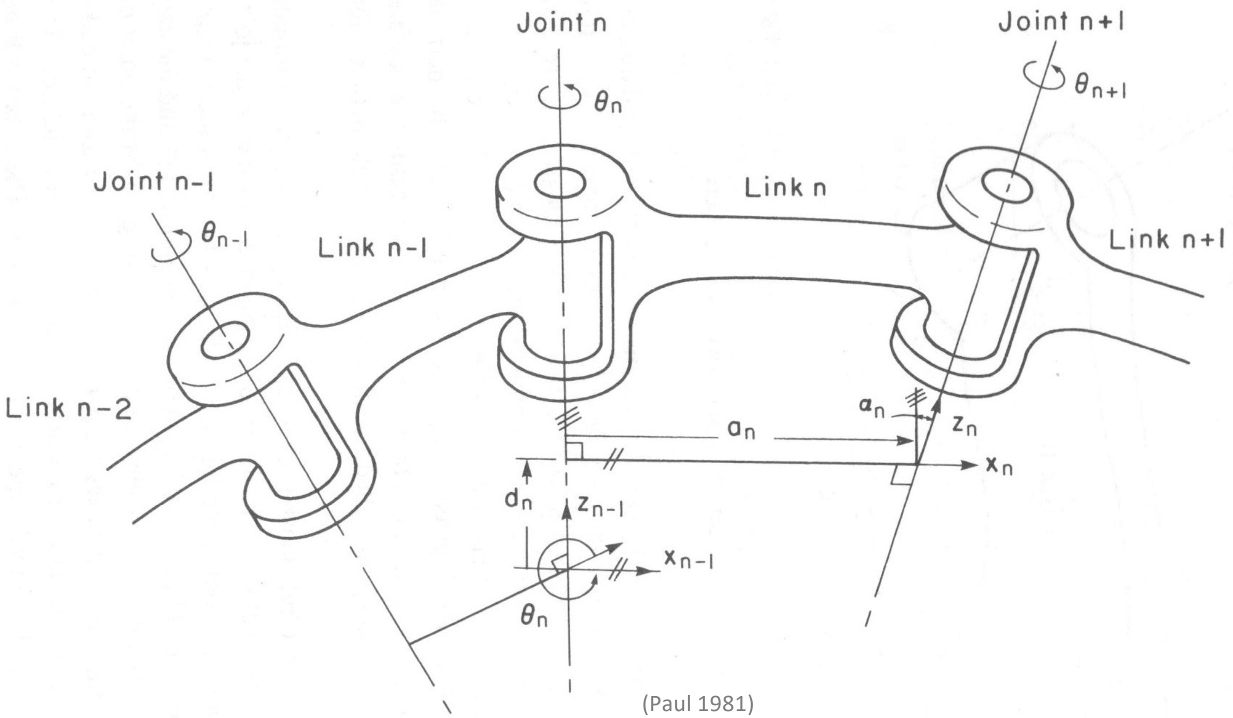
Denavit-Hartenberg Representation



(Paul 1981)

Link **length** (common normal distance) a_n

Link **twist** (angle between the two axes) α_n



(Paul 1981)

A link axis has two normals to it, one for each link

d_n the distance between the normals along the joint n axis

θ_n the angle between the normals measured in a plane normal to the axis

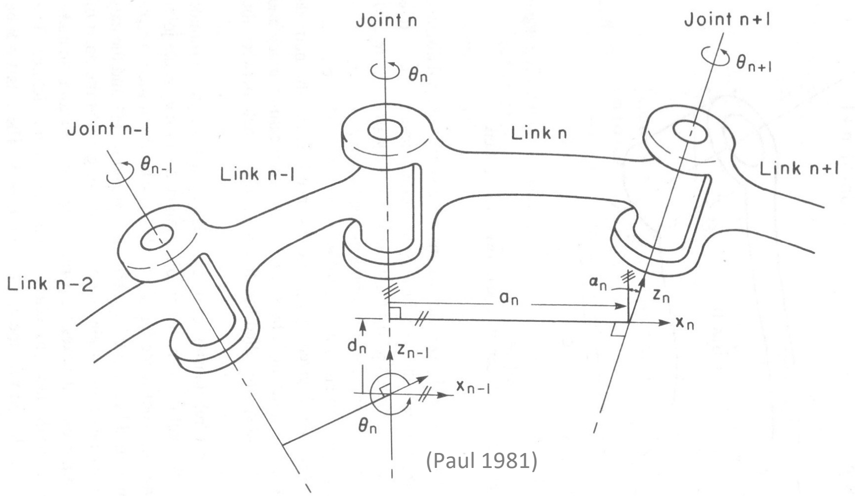
Denavit-Hartenberg Representation

- Assign coordinate frames to each link
- Determine the transformation from link n-1 to link n in terms of the four link parameters a_n α_n d_n θ_n

Denavit-Hartenberg Representation

Assign coordinate frames to each link

- The origin of each coordinate frame of link n is set to be at the intersection of the common normal between the axes of joints n and $n+1$ and the axis of joint $n+1$

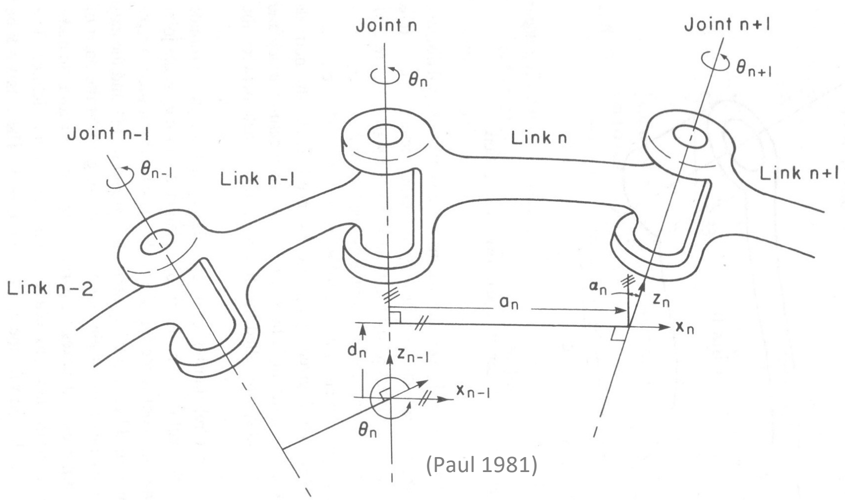


(Paul 1981)

Denavit-Hartenberg Representation

Assign coordinate frames to each link

- In the case of intersecting joint axes, the origin is at the point of intersection of the joint axes

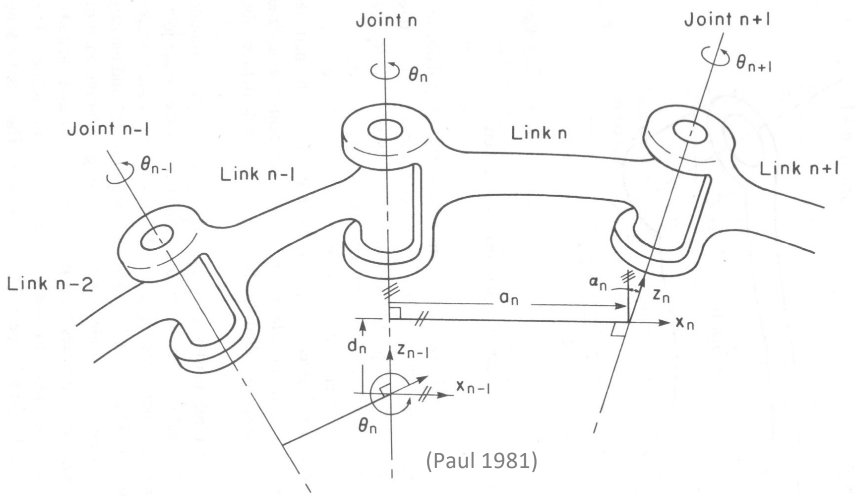


(Paul 1981)

Denavit-Hartenberg Representation

Assign coordinate frames to each link

- In the case of parallel joint axes, the origin is chosen to make the joint distance zero for the next link whose coordinate origin is defined

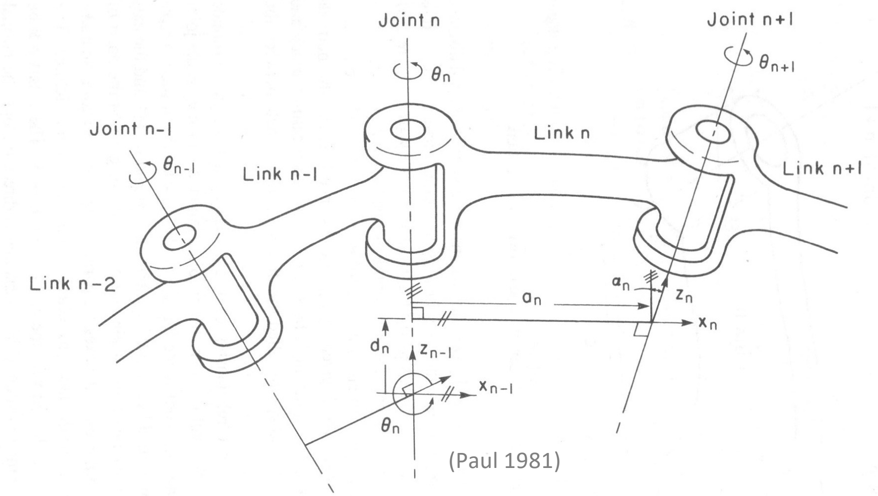


(Paul 1981)

Denavit-Hartenberg Representation

Assign coordinate frames to each link

- The Z axis for link n will be aligned with the axis of joint $n+1$

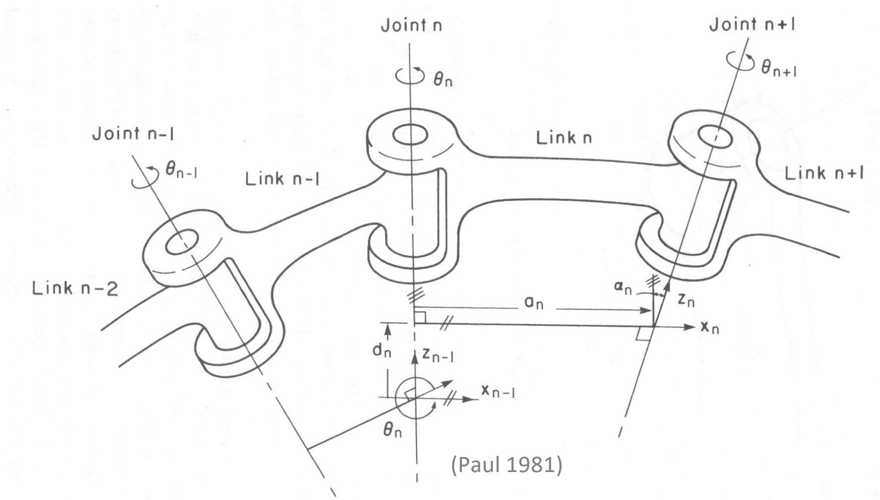


(Paul 1981)

Denavit-Hartenberg Representation

Assign coordinate frames to each link

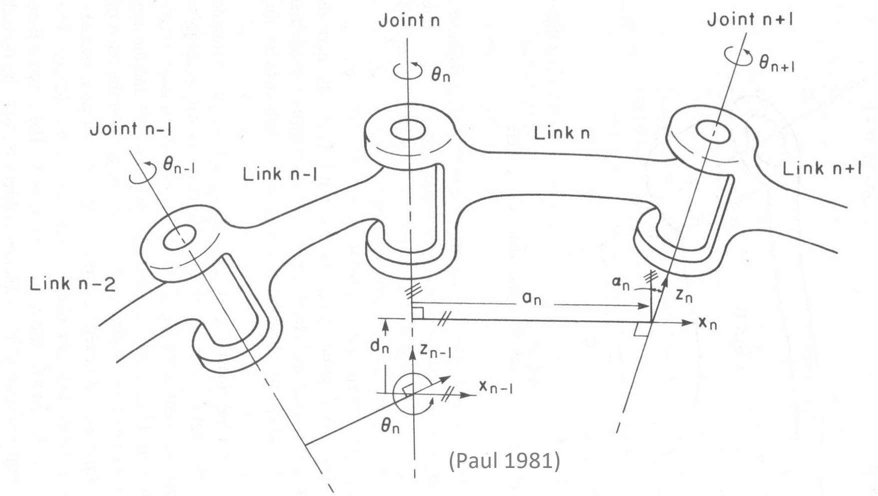
- The X axis will be aligned with any common normal which exists and is directed along the normal from joint to joint $n+1$



(Paul 1981)

Denavit-Hartenberg Representation

- The base is considered to be link 0
- Link 1 is joined to the base (link 0) by joint 1



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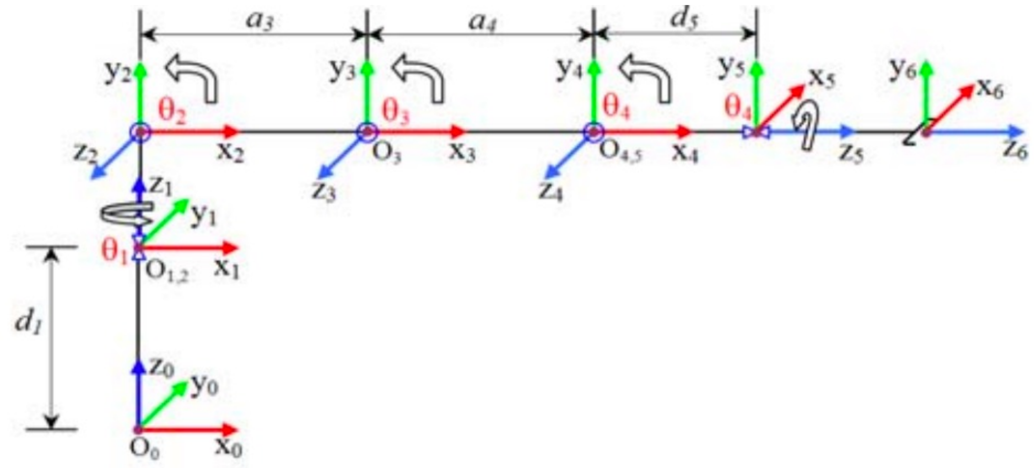
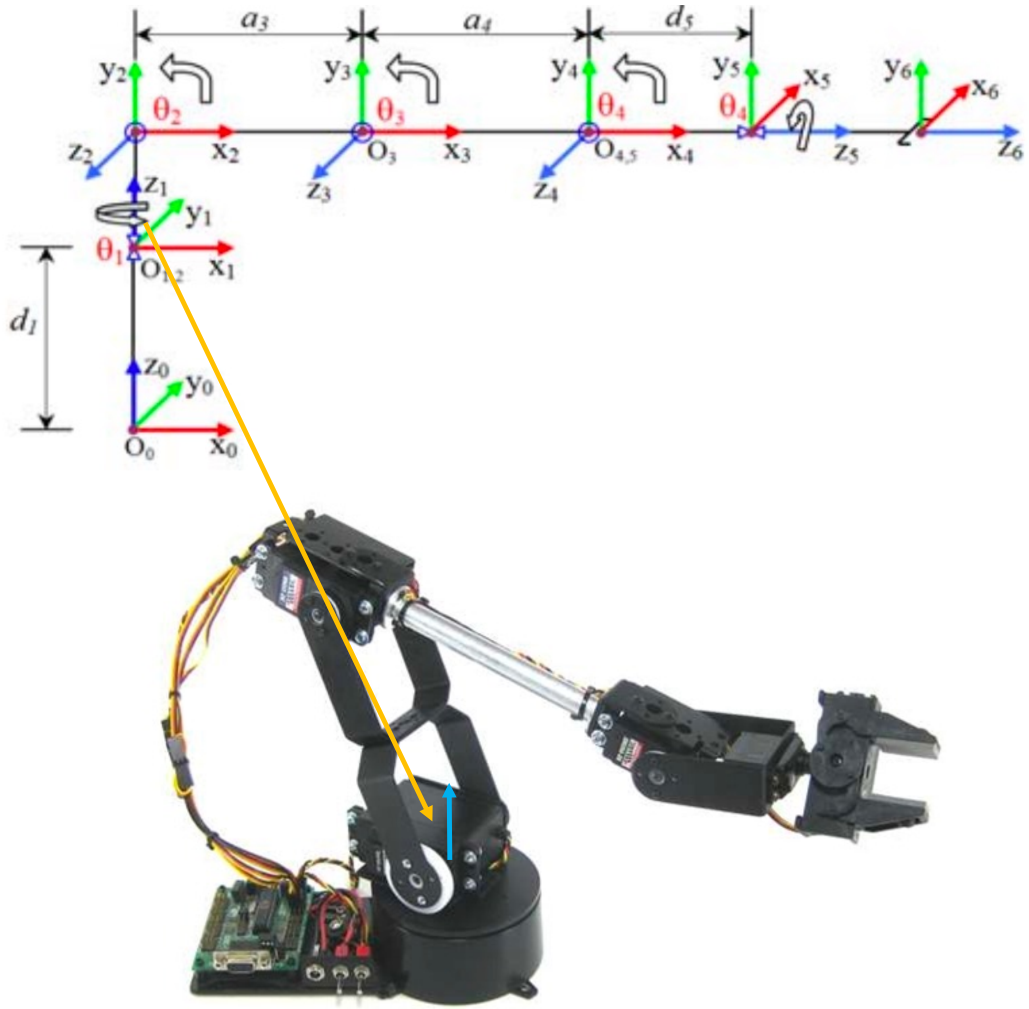


M. A. Qassem, I. Abuhadrous, and H. Elaydi, “Modeling and Simulation of 5 DOF educational robot arm”, 2nd International Conference on Advanced Computer Control (ICACC), 2010.



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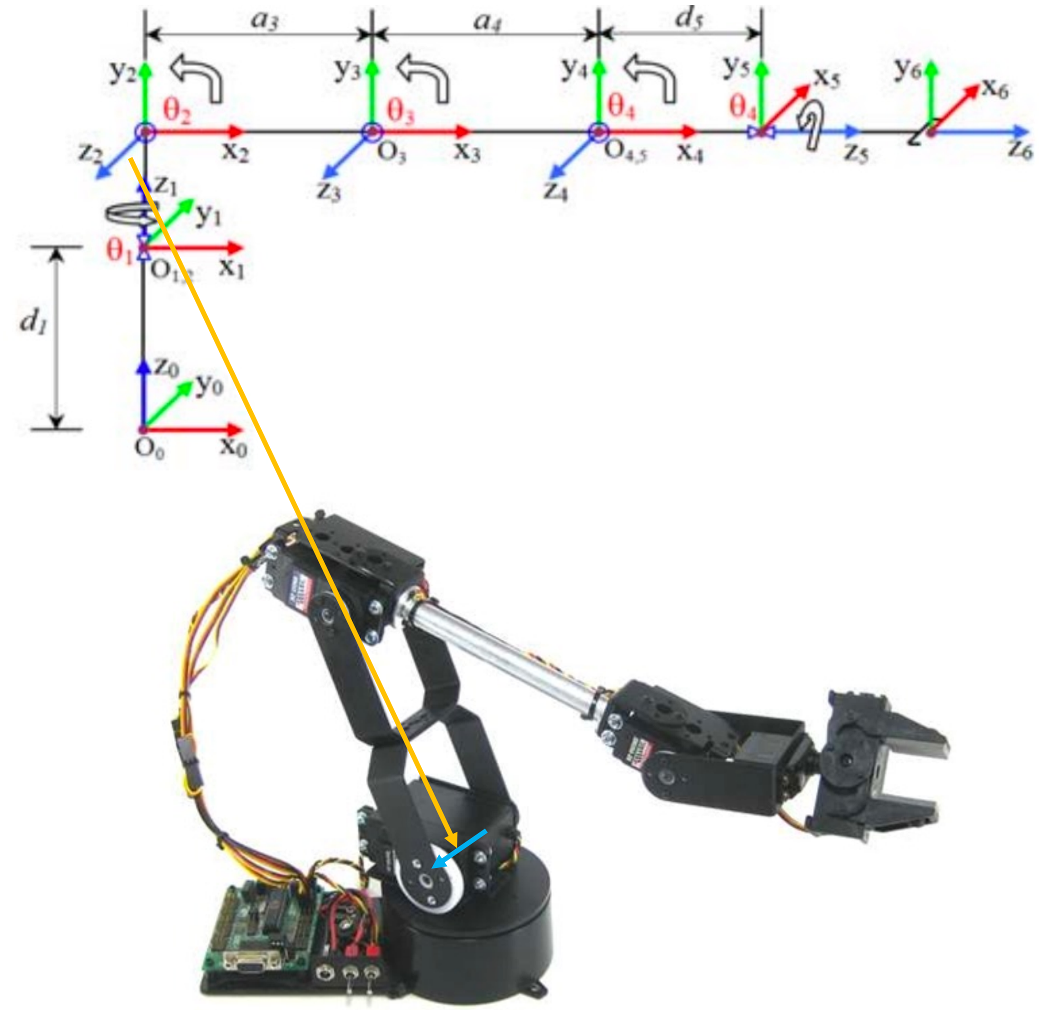

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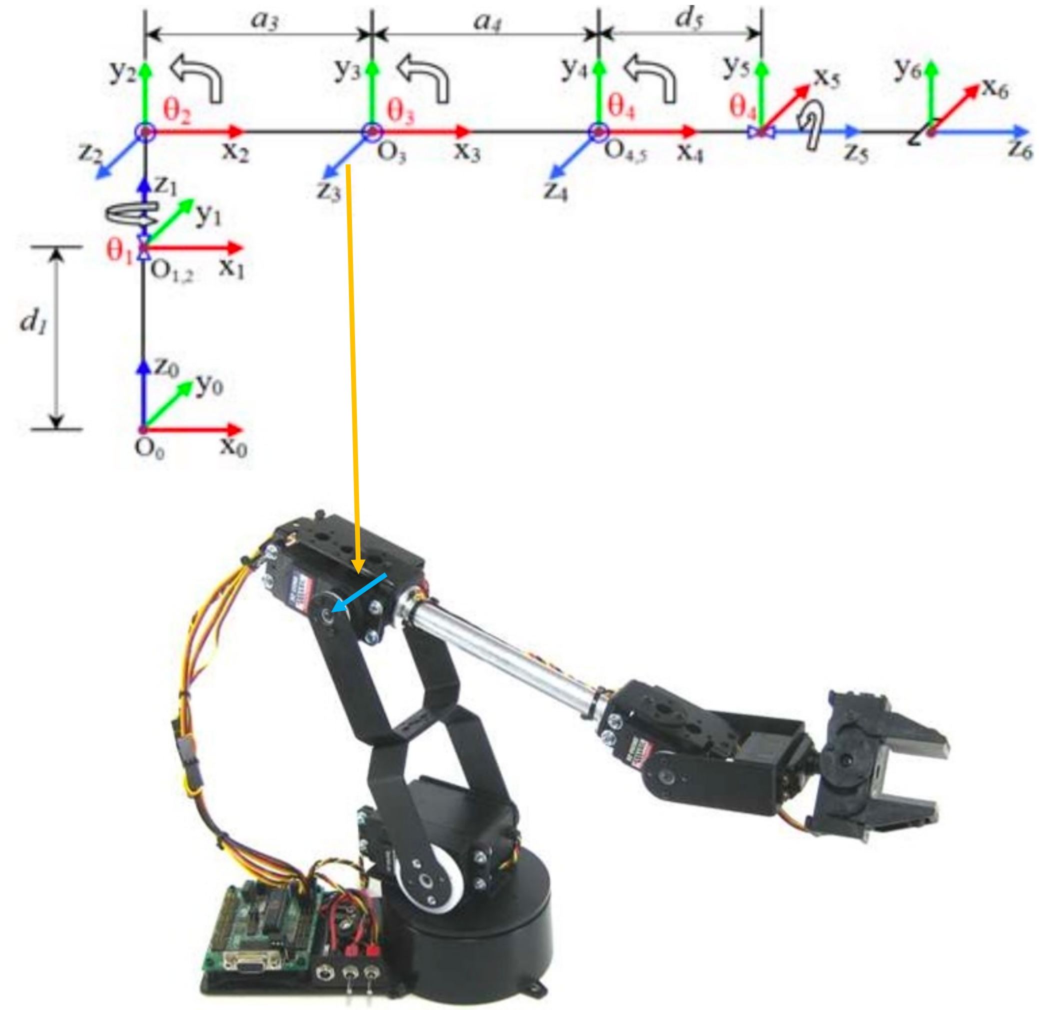

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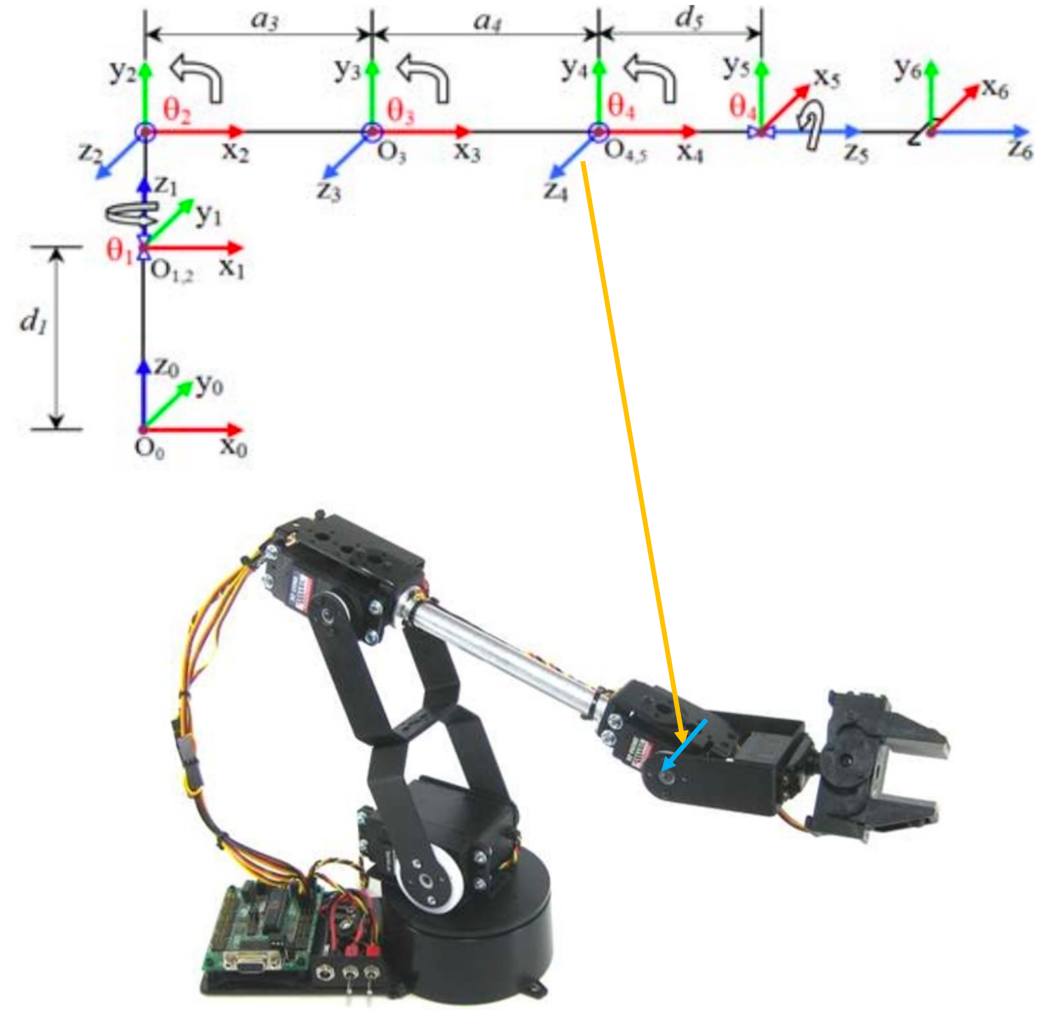

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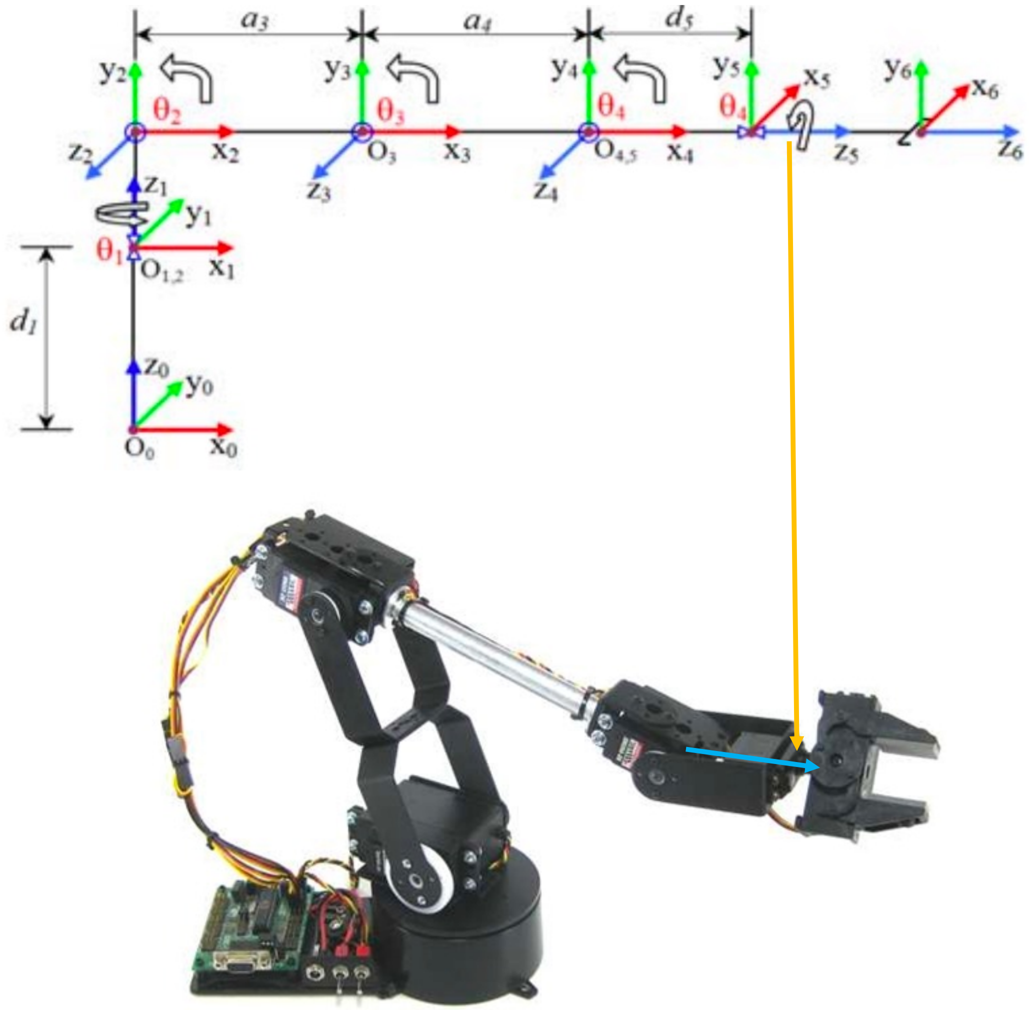

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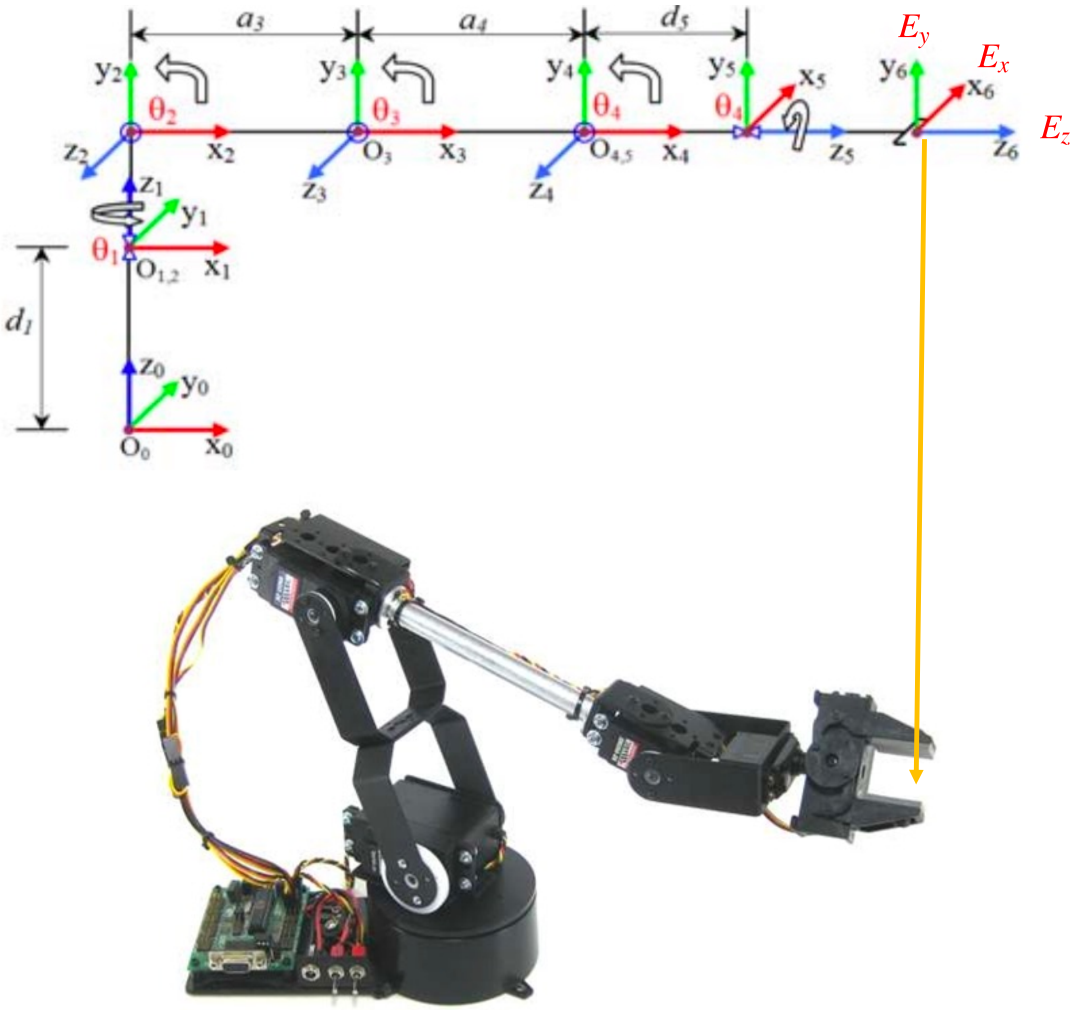

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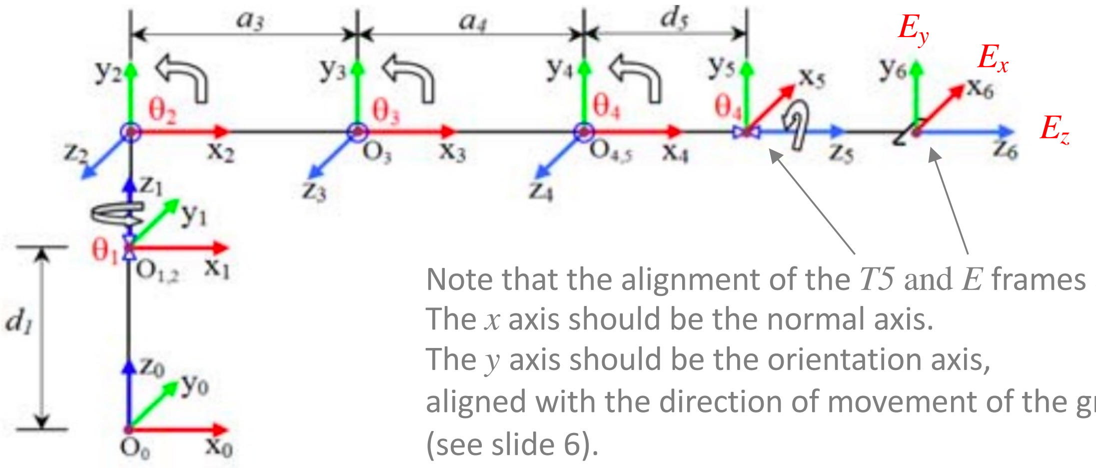

Note that the alignment of the *T5* and *E* frames are incorrect.
The *x* axis should be the normal axis.
The *y* axis should be the orientation axis,
aligned with the direction of movement of the gripper fingers
(see slide 6).

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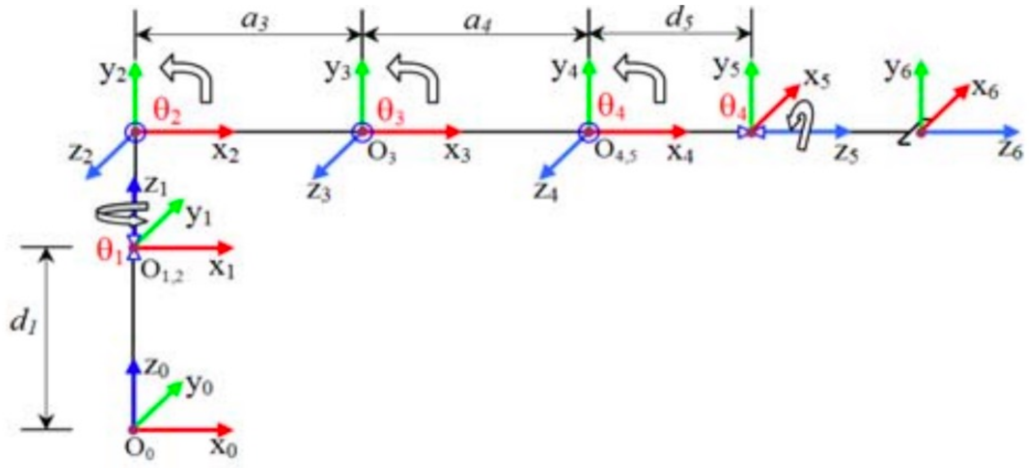

i	α_{i-1}	a_{i-1}	d_i	θ_i
1	0	0	d_1	θ_1^*
2	90	0	0	θ_2^*
3	0	a_3	0	θ_3^*
4	0	a_4	0	$\theta_4 - 90^*$
5	-90	0	d_5	θ_5^*
6	0	0	0	Gripper

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