

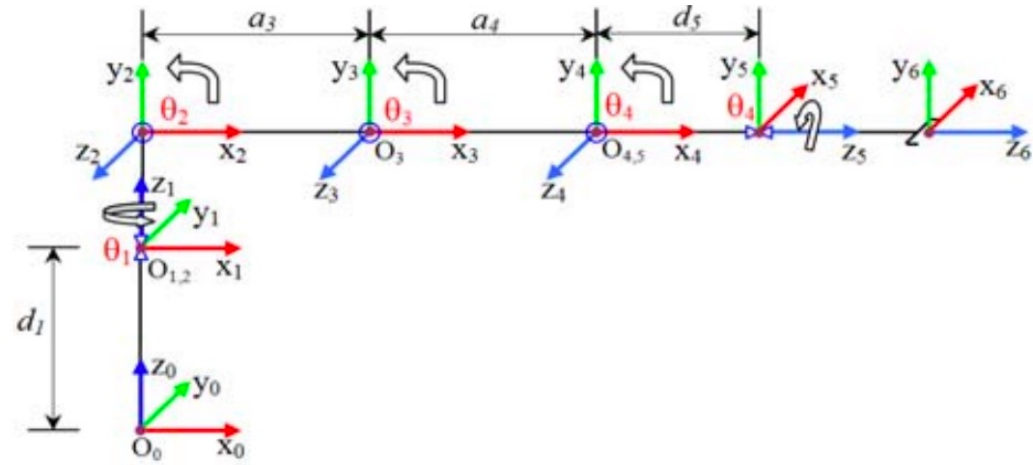
Robotics: Principles and Practice

Module 4: Robot Manipulators

Lecture 6: Inverse kinematics of the LynxMotion AL5D arm

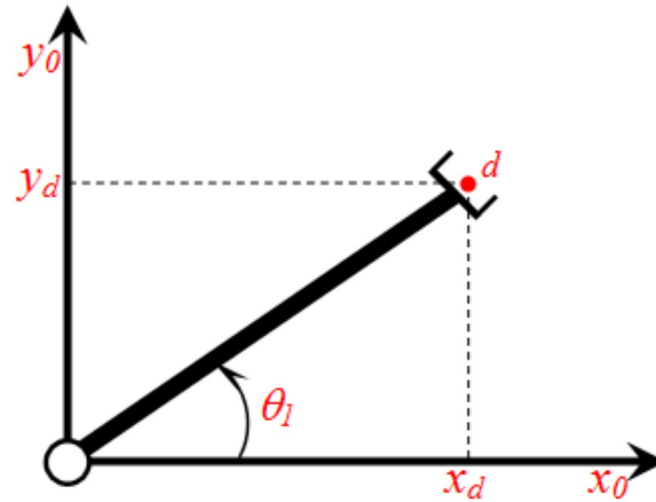
David Vernon
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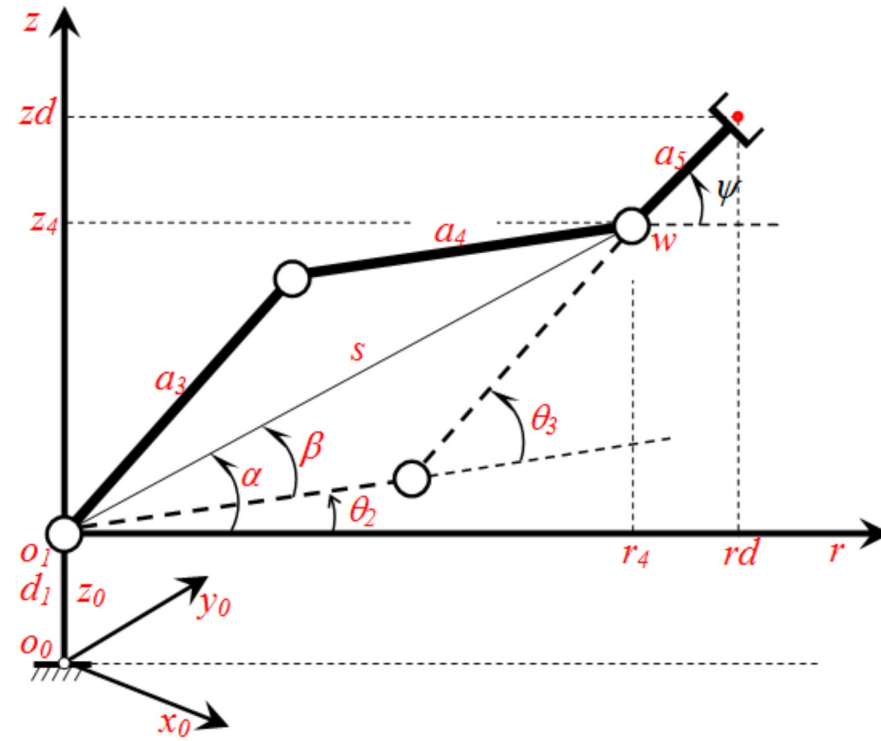
i	α_{i-1}	a_{i-1}	d_i	θ_i
1	0	0	d_1	θ_1^*
2	90	0	0	θ_2^*
3	0	a_3	0	θ_3^*
4	0	a_4	0	$\theta_4 - 90^*$
5	-90	0	d_5	θ_5^*
6	0	0	0	Gripper

M. A. Gasseem, I. Abuhadrous, and H. Elaydi, "Modeling and Simulation of 5 DOF educational robot arm", 2nd International Conference on Advanced Computer Control (ICACC), 2010.



$$\theta_1 = \text{Atan2}(y, x)$$

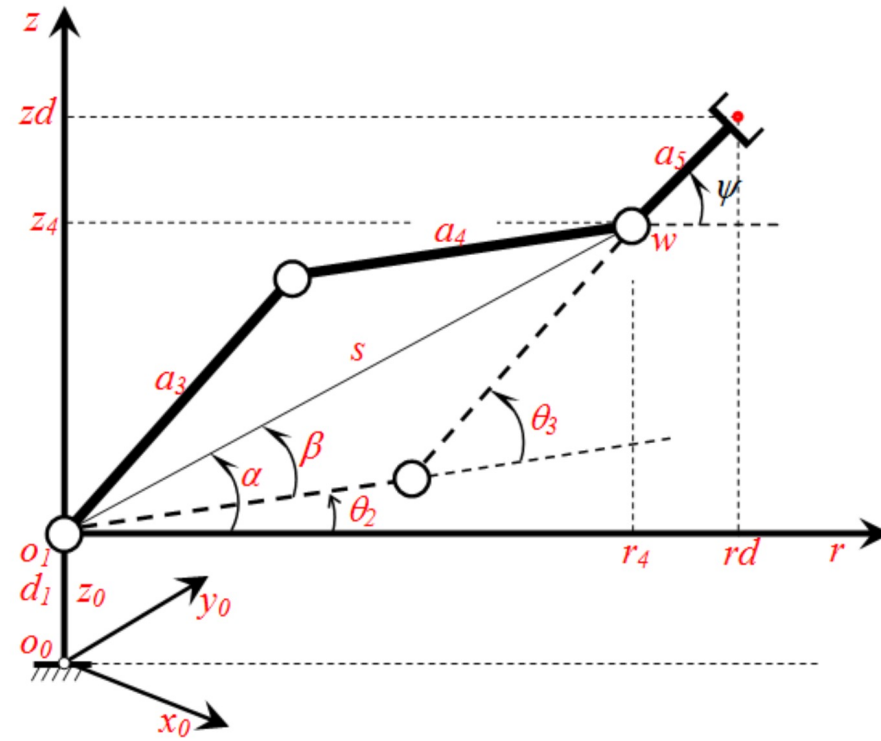
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$$r_4 = a_3 \cos(\theta_2) + a_4 \cos(\theta_2 + \theta_3)$$

$$z_4 = a_3 \sin(\theta_2) + a_4 \sin(\theta_2 + \theta_3) + d_1$$

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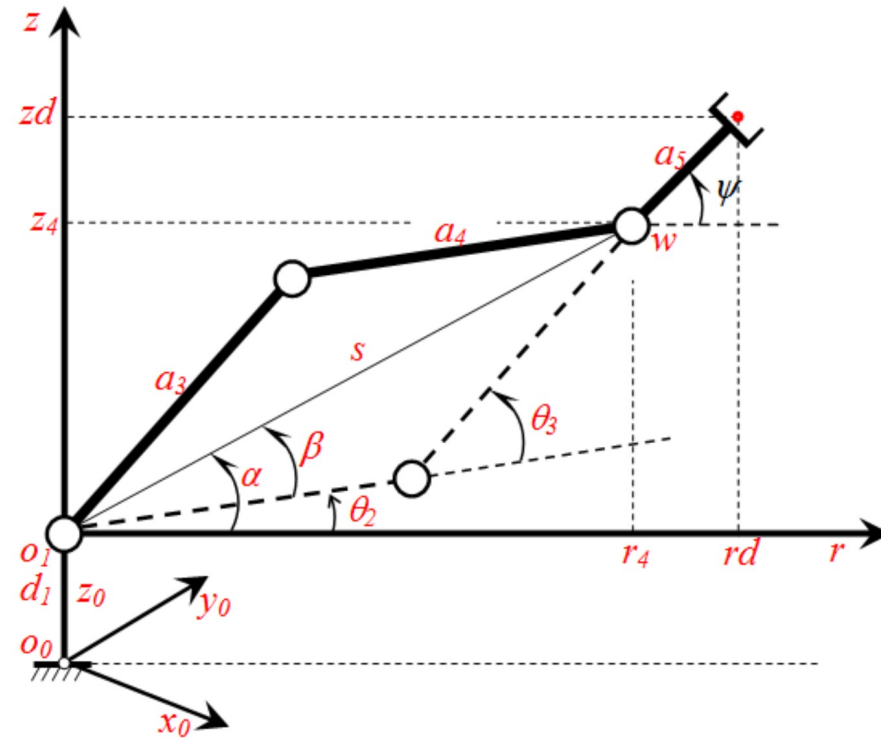


$$\beta = A \tan 2(s^2 + a_3^2 - a_4^2, 2sa_3)$$

$$\alpha = A \tan 2(z_4 - d_1, r_4)$$

$$s = \sqrt{(z_4 - d_1)^2 + r_4^2}$$

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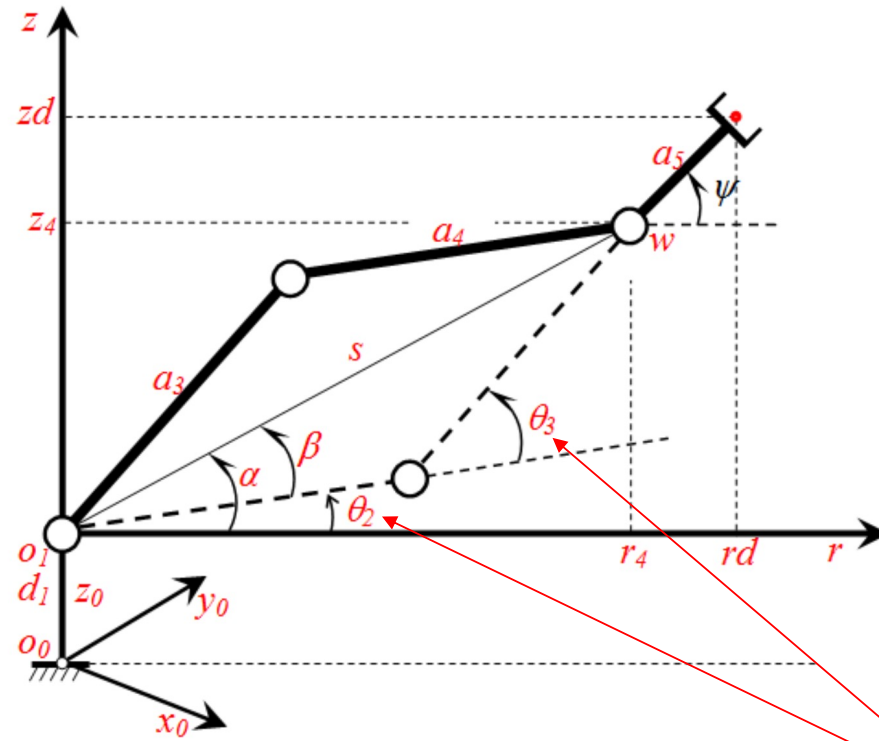


$$\theta_2 = \alpha \pm \beta$$

$$\theta_3 = A \tan 2(s^2 - a_3^2 - a_4^2, 2a_3a_4)$$

$$\theta_4 = \psi - \theta_2 - \theta_3$$

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$$\theta_2 = \alpha \pm \beta$$

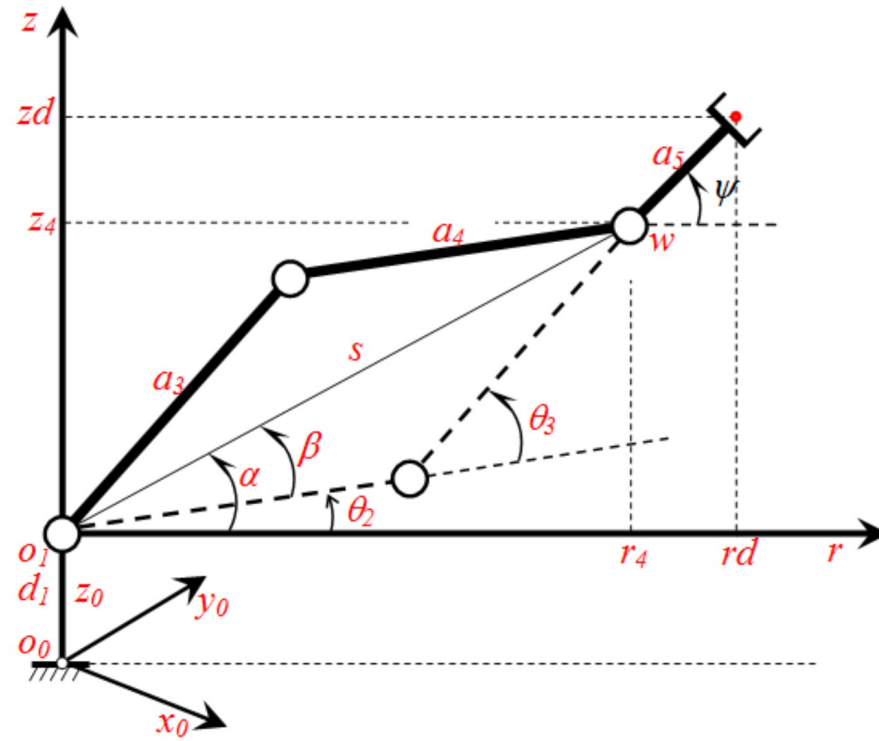
$$\theta_3 = A \tan 2(s^2 - a_3^2 - a_4^2, 2a_3a_4)$$

$$\theta_4 = \psi - \theta_2 - \theta_3$$

This equation for θ_3 is valid for the case where $\theta_2 = \alpha - \beta$, i.e. the case shown in the diagram

It should be negated for the case where $\theta_2 = \alpha + \beta$ i.e. the solution used in the inverse kinematics in the code provided in the accompanying software

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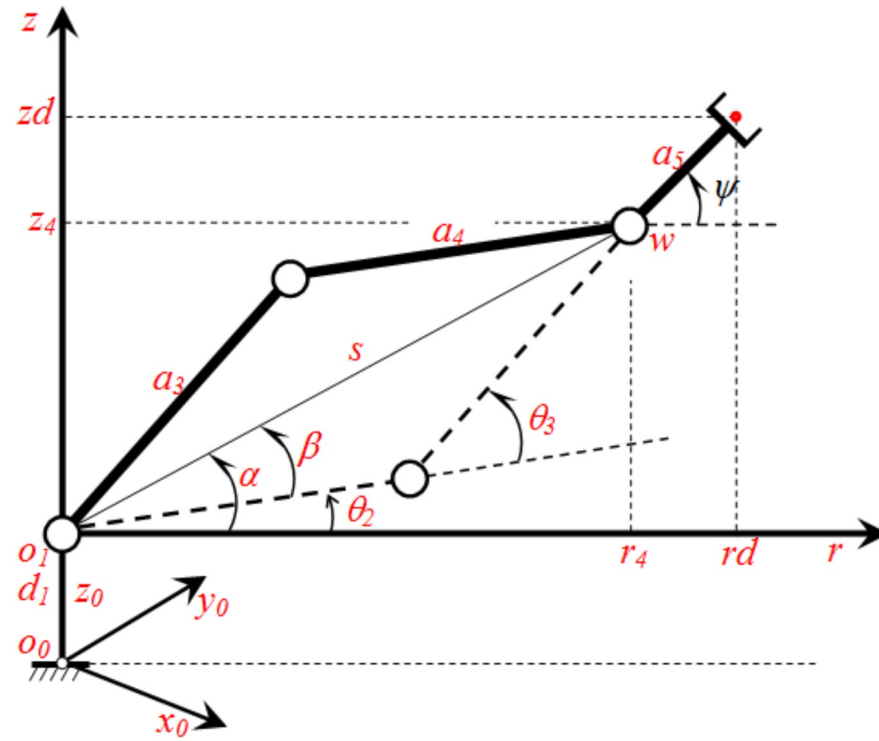
$$\theta_2 = \alpha \pm \beta$$

$$\theta_3 = A \tan 2(s^2 - a_3^2 - a_4^2, 2a_3a_4)$$

$$\theta_4 = \psi - \theta_2 - \theta_3$$

Adjust θ_4 by +90 degrees to if you want to have the T5 z axis aligned with the base frame of reference z axis when T5 is a pure translation

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There is no expression for θ_5 . It is assumed that it is given, and used directly.

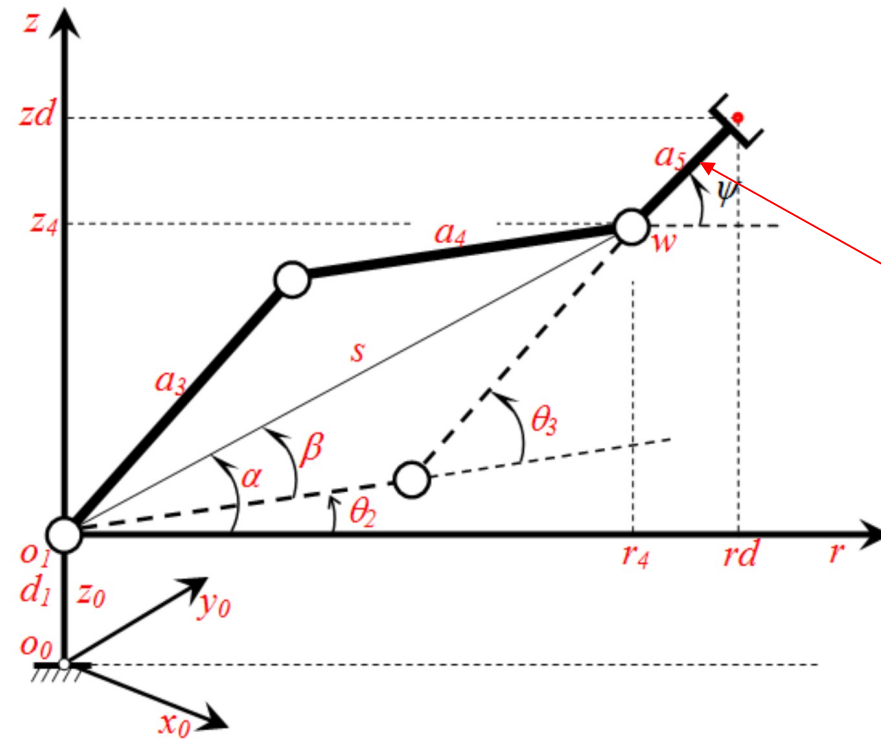
Adjust θ_5 by +90 degrees to if you want to have the T5 z axis aligned with the base frame of reference z axis when T5 is a pure translation

$$\theta_2 = \alpha \pm \beta$$

$$\theta_3 = A \tan 2(s^2 - a_3^2 - a_4^2, 2a_3a_4)$$

$$\theta_4 = \psi - \theta_2 - \theta_3$$

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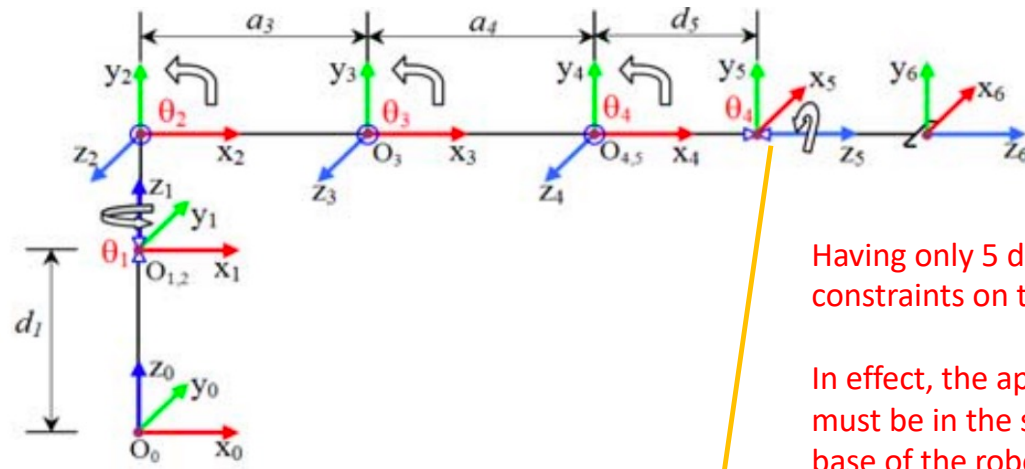
d_5 is not used in this solution (the a_5 is a typo in the figure). It is effectively integrated into the end-effector frame E so that the origin of T_5 is in the wrist, where it should be, and not in the gripper as shown.

$$\theta_2 = \alpha \pm \beta$$

$$\theta_3 = A \tan 2(s^2 - a_3^2 - a_4^2, 2a_3a_4)$$

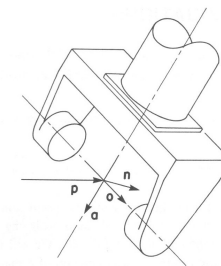
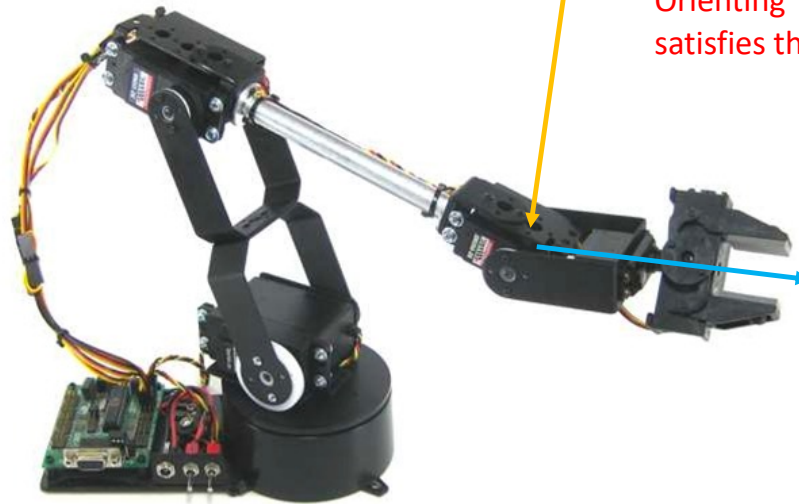
$$\theta_4 = \psi - \theta_2 - \theta_3$$

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Having only 5 degrees of freedom imposes constraints on the achievable pose.

In effect, the approach vector a of the T5 frame must be in the same plane as the vector joining the base of the robot to the origin of T5. Orienting a vertically upwards or downwards satisfies this requirement.



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