

## Scientific Theory in Informatics A1N

### Discrete Probability Exercises

1. Suppose that we have found that the word “Rolex” occurs in 250 of 2000 messages known to be spam and in 5 of 1000 messages known not to be spam. Estimate the probability that an incoming message containing the word “Rolex” is spam, assuming that it is equally likely that an incoming message is spam or not spam. If our threshold for rejecting a message as spam is 0.9, will we reject such messages?
2. The probability of catching Lyme disease after one day of hiking in the Cuyamaca mountains are estimated at less than 1 in 10000. You feel bad after a day of hike in the Cuyamacas and decide to take a Lyme disease test. The test is positive. The test specifications say that in an experiment with 1000 patients with Lyme disease, 990 tested positive. Moreover, when the same test was performed with 1000 patients without Lyme disease, 200 tested positive. What are the chances that you have caught Lyme disease?
3. A friend of yours believes she has a 50% chance of being pregnant. She decides to take a pregnancy test and the test is positive. You read in the test instructions that out of 100 non-pregnant women, 20% give false positives (the result of the test is positive when it should be negative). Moreover, out of 100 pregnant women 10% give false negatives (the result is negative when it should be positive). Help your friend upgrade her beliefs.
4. In a communication channel a zero or a one is transmitted. The probability that a zero is transmitted is 0.1. Due to noise in the channel, a zero can be received as one with probability 0.01, and a one can be received as a zero with probability 0.05. If you receive a zero, what is the probability that a zero was transmitted? If you receive a one what is the probability that a one was transmitted?
5. You are a contestant on a television game show. Before you are three closed doors. One of them hides a car, which you want to win; the other two hide goats (which you do not want to win).

First you pick a door. The door you pick does not get opened immediately. Instead, the host opens one of the other doors to reveal a goat. He will then give you a chance to change your mind: you can switch and pick the other closed door instead, or stay with your original choice. To make things more concrete without losing generality concentrate on the following situation.

- (a) You have chosen the first door.
- (b) The host opens the third door, showing a goat.

If you don't switch doors, what is the probability of winning the car? If you switch doors, what is the probability of winning the car? Should you switch doors?

6. Suppose that one person in 100,000 has a particular rare disease for which there is a fairly accurate diagnostic test. This test is correct 99.0% of the time when given to a person selected at random who has the disease; it is correct 99.5% of the time when given to a person selected at random who does not have the disease. Given this information can we find
- (a) the probability that a person who tests positive for the disease has the disease?
  - (b) the probability that a person who tests negative for the disease does not have the disease?

Should a person who tests positive be very concerned that he or she has the disease?

7. Linda is 31 years old, single, outspoken, and very clever. She has a double degree in philosophy and informatics. As a student she was deeply concerned with issues of discrimination and social justice, and also participated in anti-nuclear demonstrations. Which is more probable:
- (a) Linda is a bank teller?
  - (b) Linda is a bank teller who is active in the feminist movement?
8. A disease called pluremia affects 1 percent of the population. There is a test to detect pluremia but it is not perfect. For people with pluremia, the test is positive 90% of the time. For people without pluremia the test is positive 20% of the time. Suppose a randomly selected person takes the test and it is positive. What are the chances that a randomly selected person tests positive?