Introduction to Cognitive Robotics

Module 3: Mobile Robots

Lecture 4: Closed-loop control and PID control; the go-to-position problem; divide-and-conquer controller

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Terminology

Goal

- Get some process or plant to a desired state
- Maintain that state

Example states:

Water level in a tank
Temperature of water in a tank
Flow rate of a pipeline
Speed of a mobile robot
Position of a mobile robot
Orientation of a mobile robot

These are referred to as "process variables"

"plant" is a term used to refer to the system being controlled
e.g. water tank, pipeline, mobile robot

Terminology

Strictly speaking, "a plant in control theory is the combination of process and actuator"
https://en.wikipedia.org/wiki/Plant_(control_theory)

- Plant: the process or plant to be controlled
- Process variable (PV): the actual state of the process or plant
- Set point (SP): the desired state of the process or plant
- Error: the difference between PV and SP

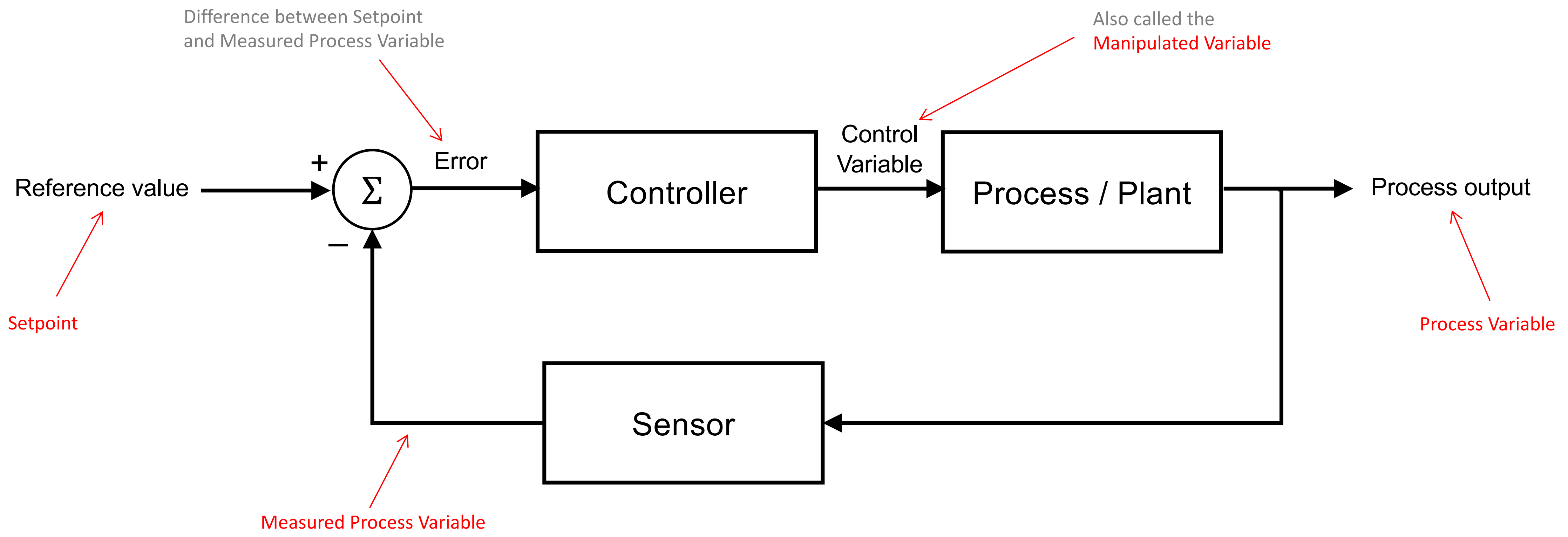
Terminology

- Effector: a mechanism that changes the state of the process or plant (i.e. control action)
- Sensor: a mechanism that measures the state of the process or plant
- Control variable: the input to the effector
- Controller:

Can be a physical device (e.g. a mechanical governor) or software implementing a control algorithm

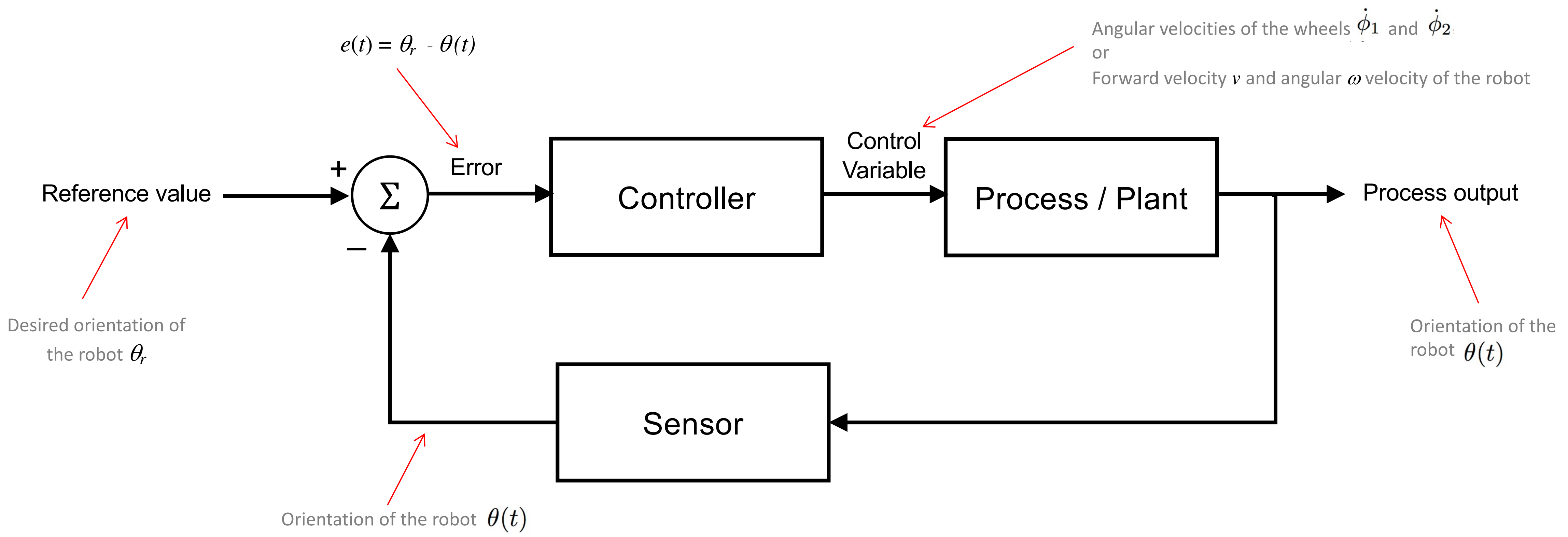
A mechanism to identify the value of the control signal that reduces the error to zero as quickly as possible, without overshoot, in a stable manner

Closed-loop Feedback Control



Closed-loop Feedback Control

For example, controlling the orientation of a mobile robot



$e(t) = \theta_r - \theta(t)$
Angular velocities of the wheels $\dot{\phi}_1$ and $\dot{\phi}_2$
or
Forward velocity v and angular ω velocity of the robot
Reference value
+
Σ
–
Error
Controller
Control Variable
Process / Plant
Process output
Desired orientation of the robot θ_r
Orientation of the robot $\theta(t)$
Sensor
Orientation of the robot $\theta(t)$

Closed-loop Feedback Control

Control variable is a function of the error: $f(e)$

e = error between desired value (i.e. the setpoint) and the actual value (i.e. the measured process value)

PID Controller

Which function?

f = "proportional to e"

f = "proportional to the accumulation of e" ← Integral

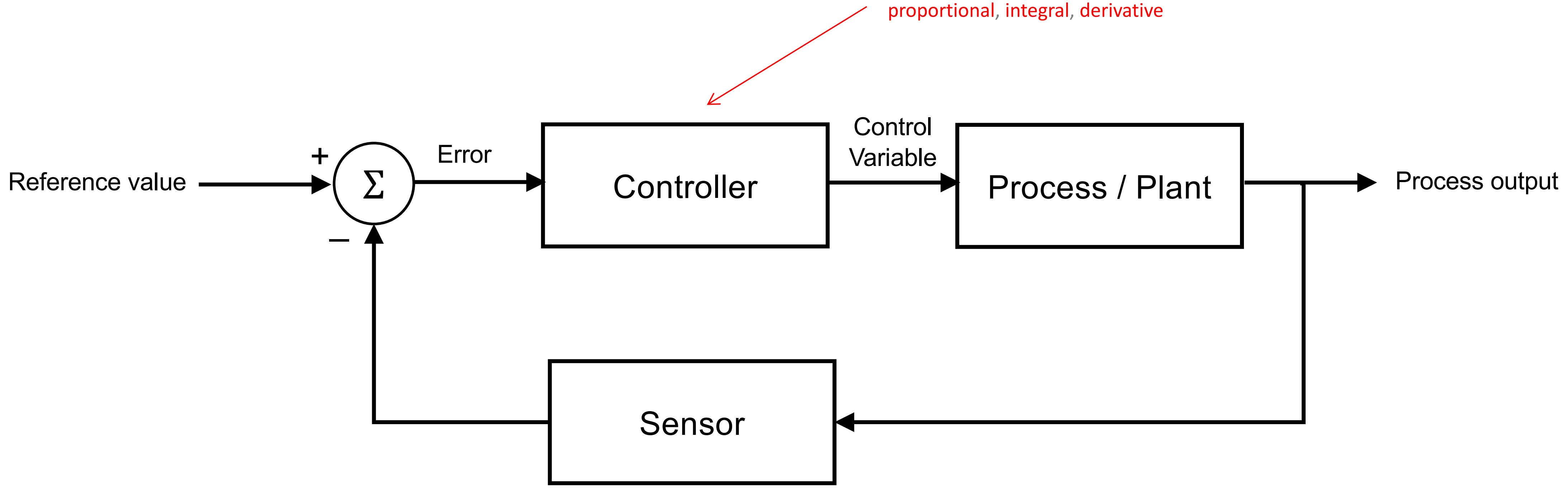
f = "proportional to the rate of change of e" ← Derivative

. . . or a combination of these

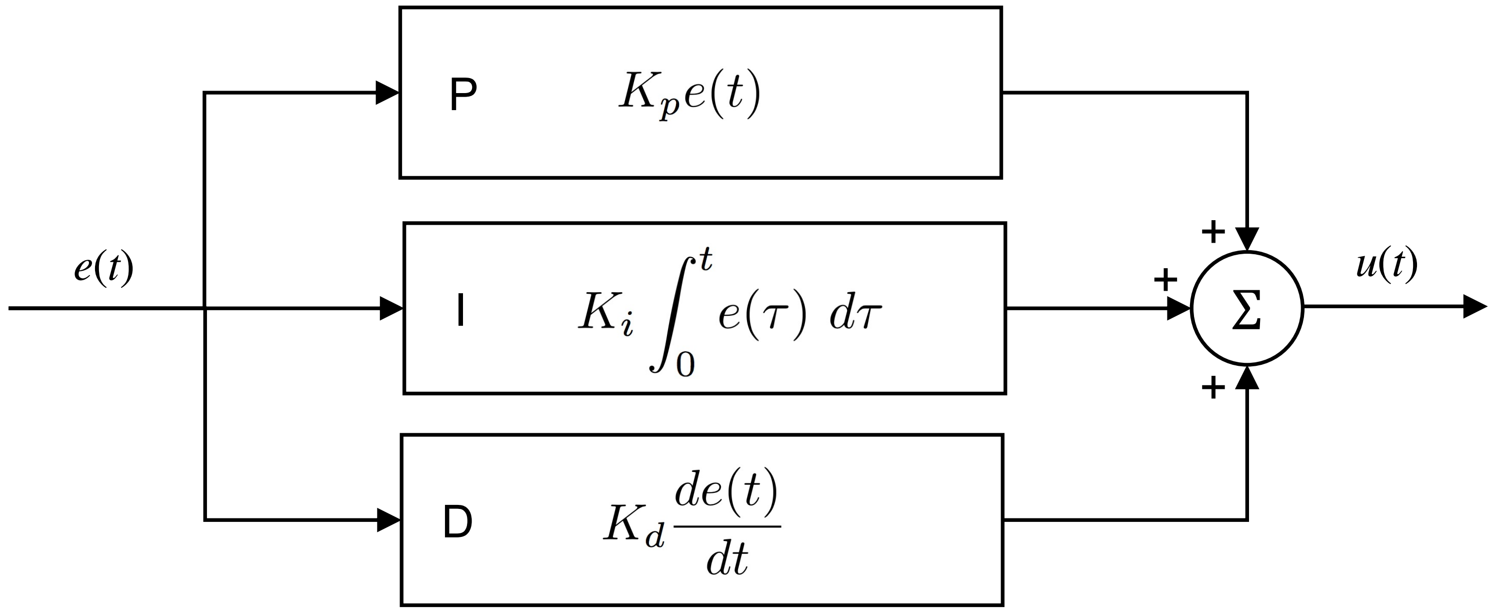
Each component in is modulated by a respective gain: K_p, K_i, K_d

PID Controller

Classical control theory uses a PID controller:
proportional, integral, derivative



PID Controller



$$u(t) = K_p e(t) + K_i \int_0^t e(\tau)\, d\tau + K_d \frac{de(t)}{dt}$$

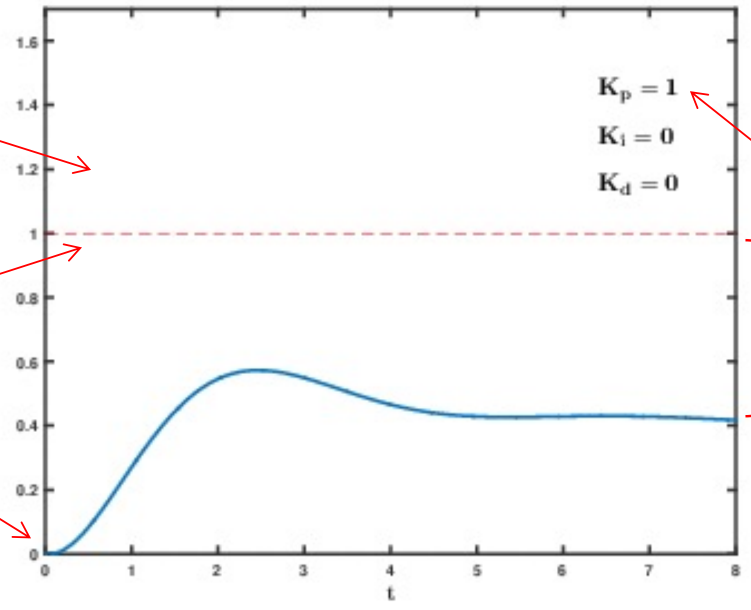
PID Controller

Effect of varying the three gains

The overshoot can be reduced by lowering the K_p but this will increase the time it takes to reduce the error

Setpoint

Initial state of the process variable



https://en.wikipedia.org/wiki/PID_controller

Note that there is a steady-state error for pure proportional control,

(i.e. when the gains of the integral and derivative terms are zero).

The integral term eliminates this.

PID Controller

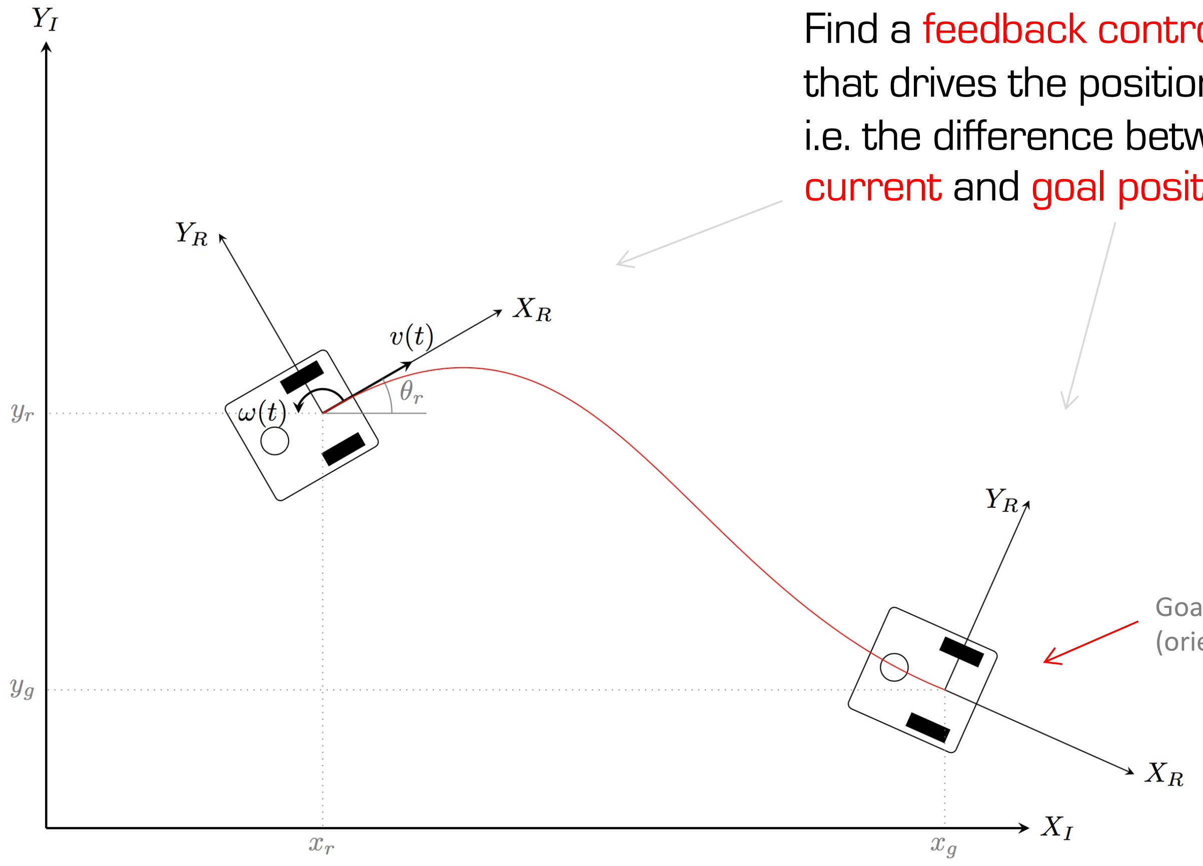
Take-home message

The key to effective PID control is to use the right gain values

but

identifying them is difficult

The Go-to-Position Problem



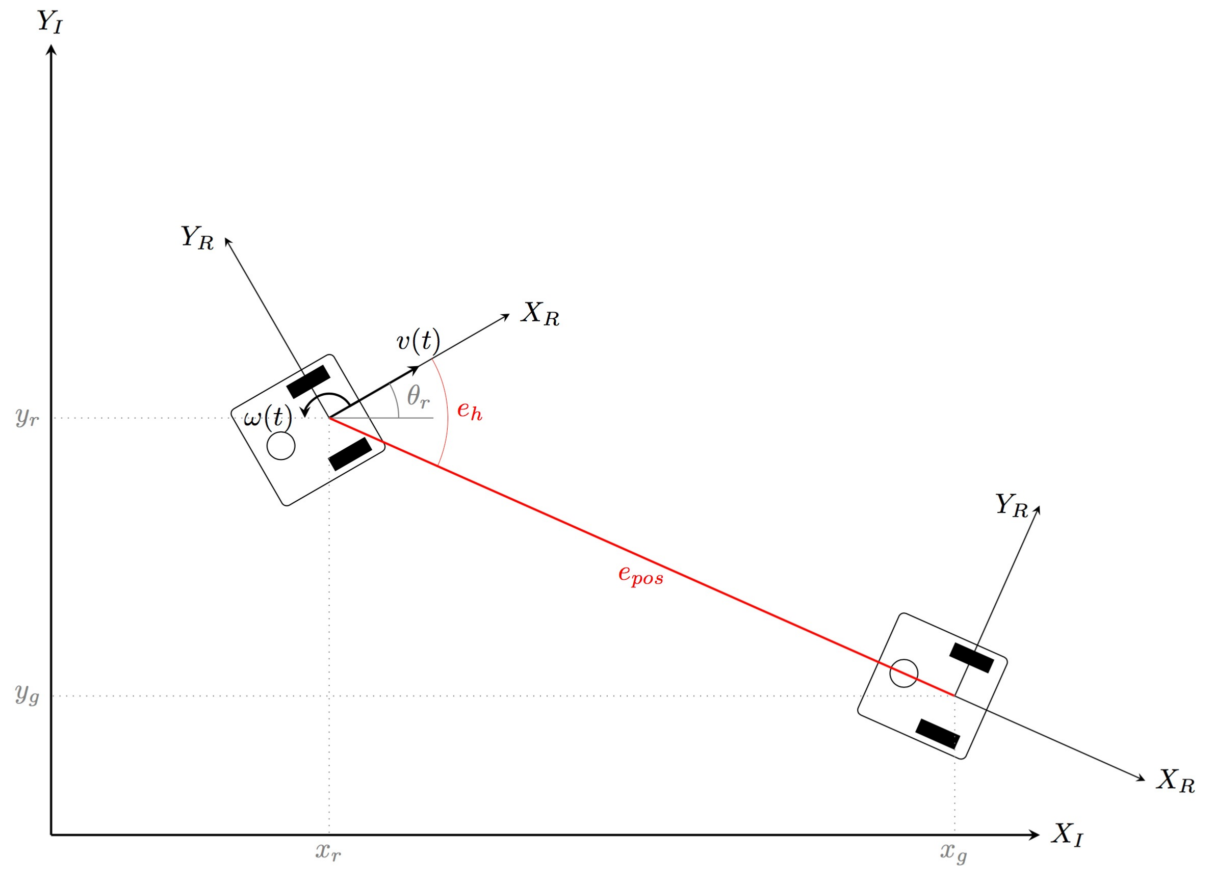
Find a feedback control equation that drives the positional error to zero, i.e. the difference between the current and goal position

Goal position
(orientation is the heading direction on arrival)

The Go-to-Position Problem

- The robot knows its own global current position
  - (x_r, y_r, θ_r)
- It knows the global position of the goal
  - (x_g, y_g)
- Compute error in position and heading
  - (e_{pos}, e_h)

The Go-to-Position Problem



The Go-to-Position Problem

- The robot knows its own global current position
  - (x_c, y_c, θ_c)
- It knows the global position of the goal
  - (x_g, y_g)
- Compute error in position and heading
  - (e_{pos}, e_h)
- Reduce both errors to zero
  - by generating the appropriate forward and angular velocities (v, ω) or, alternatively,
  - by generating the appropriate angular velocities of the wheels, (v_R, v_L) i.e. $(\dot{\phi}_1, \dot{\phi}_2)$

Go-to-Position as a Control Problem
Solution 1: Divide and Conquer

Decompose 2D problem into two 1D problems

- First, correct the heading:

  rotate to reduce the orientation error to zero

- Second, correct the position:

  translate straight ahead to reduce the position error to zero

Go-to-Position as a Control Problem

Solution 1: Divide and Conquer

Algorithm goto1(x_g, y_g) using proportional control

Global variables: the current robot position and orientation (x_r, y_r, θ_r)

Arguments: - the goal position of the robot (x_g, y_g)

- the proportional gains for controlling position and heading

K_p^{pos}

K_p^{h}

- the tolerances on position error Δ_{pos} and orientation error Δ_h

Go-to-Position as a Control Problem

Solution 1: Divide and Conquer

Version A: Control forward and angular velocities of the robot

If not oriented correctly, correct heading

Otherwise, correct position

The difference between the desired heading and the current robot orientation

or some maximum velocity v_{max}

```
Do
    Compute the current position of the robot $(x_r, y_r)$
    Compute the distances from the robot position to the target position $(d_x, dy)$
        $d_x = x_g - x_r$
        $d_y = y_g - y_r$
    Compute the position and heading errors$(e_{pos}, e_h)$
        $e_{pos} = \text{sqrt}(d_x^2 + d_y^2)$
        $e_h = \text{atan2}(d_y, d_x) - \theta_r$
    if $|e_h| > \Delta_h$
        set forward velocity: $v = 0$
        set angular velocity: $\omega = K_p^h e_h$
    else
        set forward velocity: $v = K_p^{pos} e_{pos}$
        set angular velocity: $\omega = 0$
    Send velocities $(v, \omega)$ to the robot
    Pause some time
while $|e_{pos}| > \Delta_{pos}$
Send velocities $(0, 0)$ to the robot
```

Go-to-Position as a Control Problem

Solution 1: Divide and Conquer

Version B: Control angular velocities of the wheels

If not oriented correctly, correct heading

Otherwise, correct position

```
Do
    Compute the current position of the robot $(x_r, y_r)$
    Compute the distances from the robot position to the target position $(d_x, dy)$
        $d_x = x_g - x_r$
        $d_y = y_g - y_r$
    Compute the position and heading errors$(e_{pos}, e_\theta)$
        $e_{pos} = \text{sqrt}\,(d_x^2 + d_y^2)$
        $e_h = \text{atan2}\,(d_y, d_x) - \theta_r$
    if $|e_h| > \Delta_h$
        set right wheel angular velocity: $\dot{\phi}_1 = -K_p^h e_h$
        set left wheel angular velocity: $\dot{\phi}_2 = -\dot{\phi}_1$
    else
        set right wheel angular velocity: $\dot{\phi}_1 = K_p^{pos} e_{pos}$
        set left wheel angular velocity: $\dot{\phi}_2 = \dot{\phi}_1$
    Send velocities $(\dot{\phi}_1, \dot{\phi}_2)$ to the robot
    Pause some time
while $|e_{pos}| > \Delta_{pos}$
Send velocities $(0, 0)$ to the robot
```

Rotate: left and right wheels go in opposite direction

or some maximum velocity

Translate: left and right wheels go in same direction