

# Introduction to Cognitive Robotics

Module 3: Mobile Robots

Lecture 4: Closed-loop control and PID control;  
the go-to-position problem; divide-and-conquer controller

[www.cognitiverobotics.net](http://www.cognitiverobotics.net)

# Terminology

## Goal

- Get some process or plant to a desired state
- Maintain that state

Example states:

Water level in a tank  
Temperature of water in a tank  
Flow rate of a pipeline  
Speed of a mobile robot  
Position of a mobile robot  
Orientation of a mobile robot

These are referred to as "**process variables**"

"plant" is a term used to refer to the system being controlled  
e.g. water tank, pipeline, mobile robot

# Terminology


- **Plant**: the process or plant to be controlled
- **Process variable** (PV): the actual state of the process or plant
- **Set point** (SP): the desired state of the process or plant
- **Error**: the difference between PV and SP

# Terminology

- **Effector**: a mechanism that changes the state of the process or plant (i.e. control action)
- **Sensor**: a mechanism that measures the state of the process or plant
- **Control variable**: the input to the effector

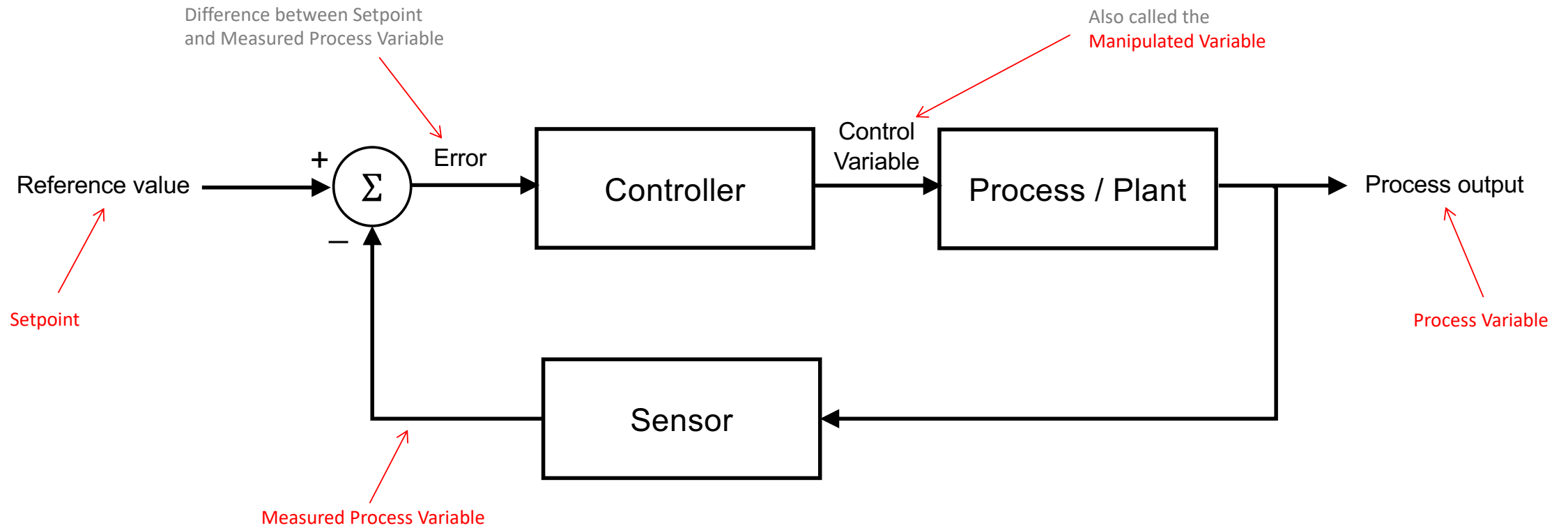
- **Controller**:

Can be a physical device (e.g. a mechanical governor)  
or software implementing a control algorithm



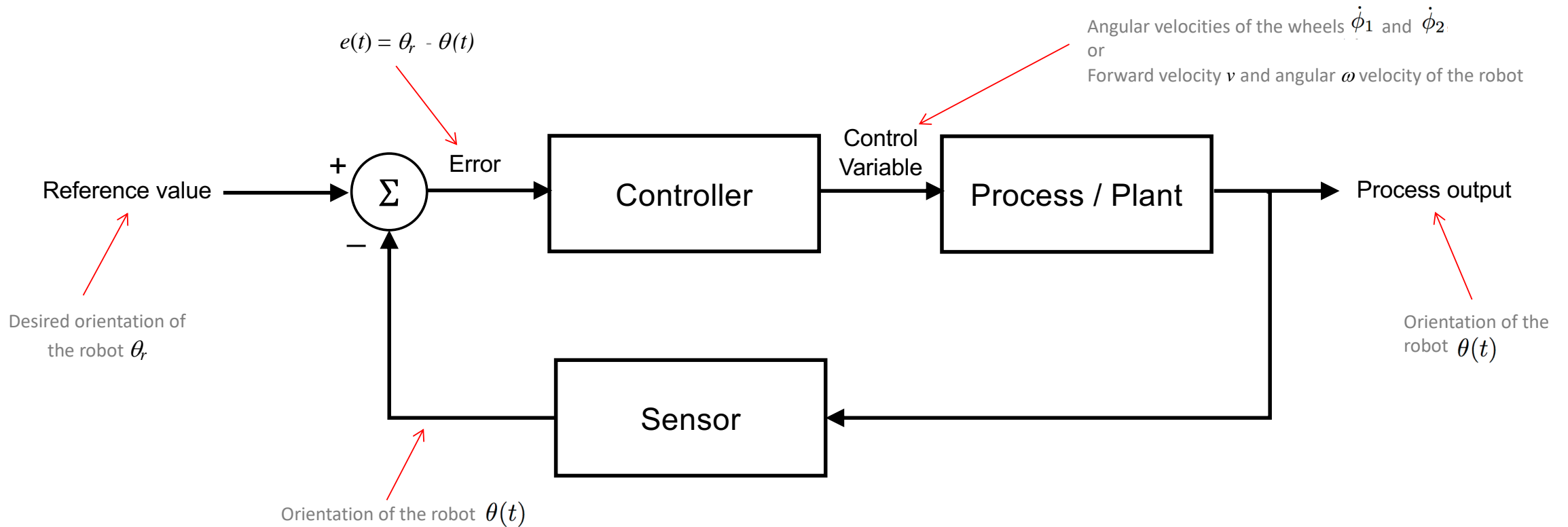
A mechanism to identify the value of the control signal that reduces the error to zero as quickly as possible, without overshoot, in a stable manner

# Closed-loop Feedback Control



# Closed-loop Feedback Control

For example, controlling the orientation of a mobile robot



# Closed-loop Feedback Control

Control variable is a function of the error:  $f(e)$

$e$  = **error** between desired value (i.e. the setpoint) and the actual value (i.e. the measured process value)

# PID Controller

Which function?

$f =$  “proportional to  $e$ ”

$f =$  “proportional to the accumulation of  $e$ ”

$f =$  “proportional to the rate of change of  $e$ ”

Integral

Derivative

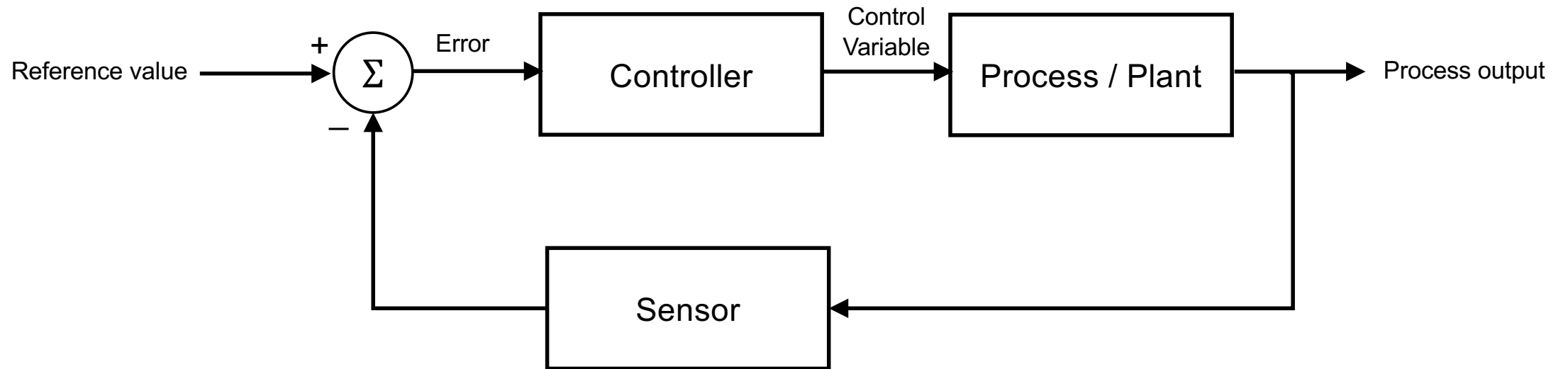
... or a combination of these

Each component in is modulated by a respective gain:  $K_p$ ,  $K_i$ ,  $K_d$

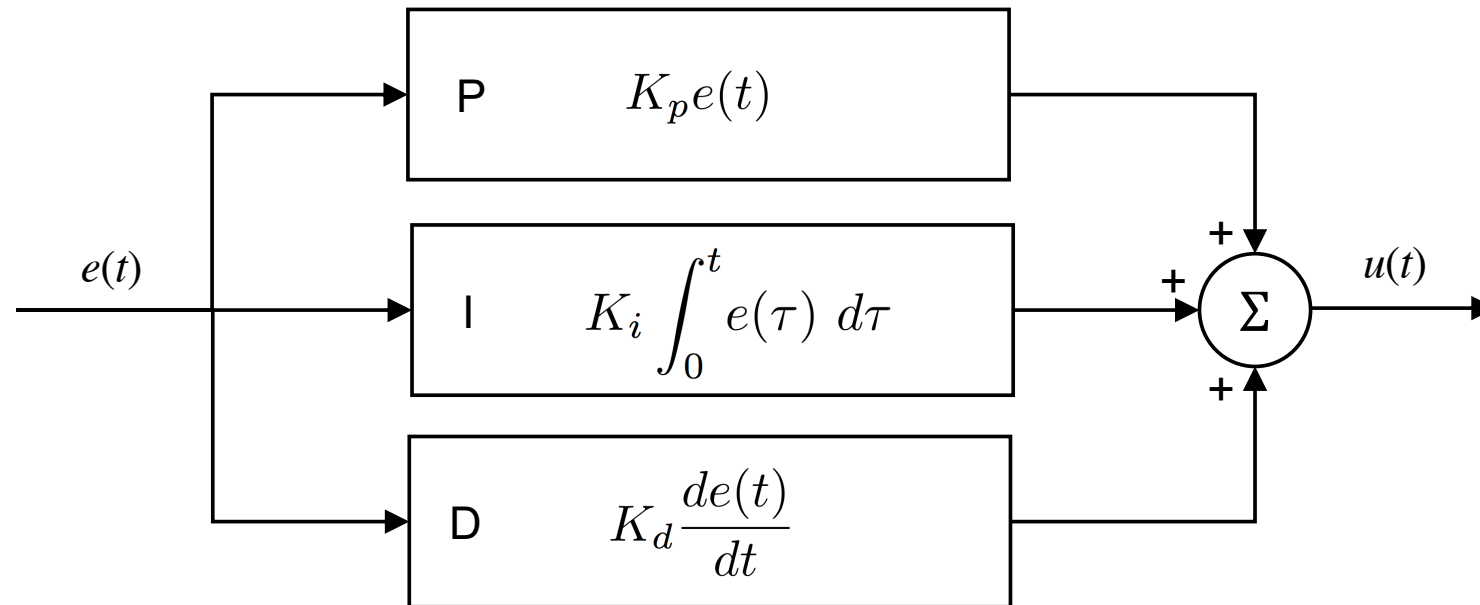


# PID Controller

Classical control theory uses a **PID** controller:  
proportional, integral, derivative



# PID Controller

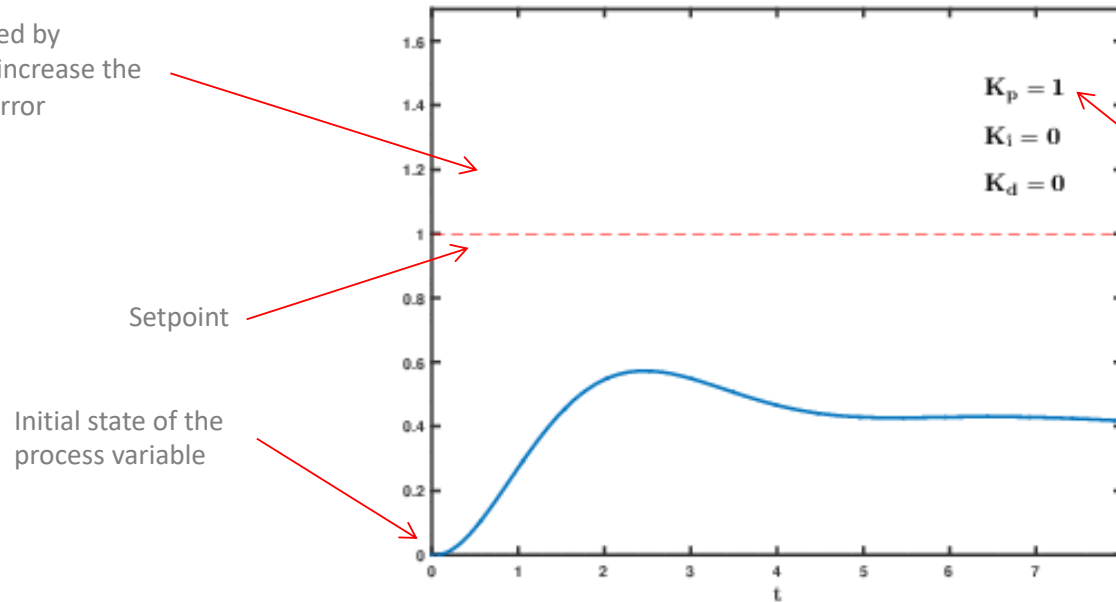


$$u(t) = K_p e(t) + K_i \int_0^t e(\tau) d\tau + K_d \frac{de(t)}{dt}$$

# PID Controller

## Effect of varying the three gains

The overshoot can be reduced by lowering the  $K_p$  but this will increase the time it takes to reduce the error



Note that there is a steady-state error for pure proportional control, (i.e. when the gains of the integral and derivative terms are zero).

The integral term eliminates this.

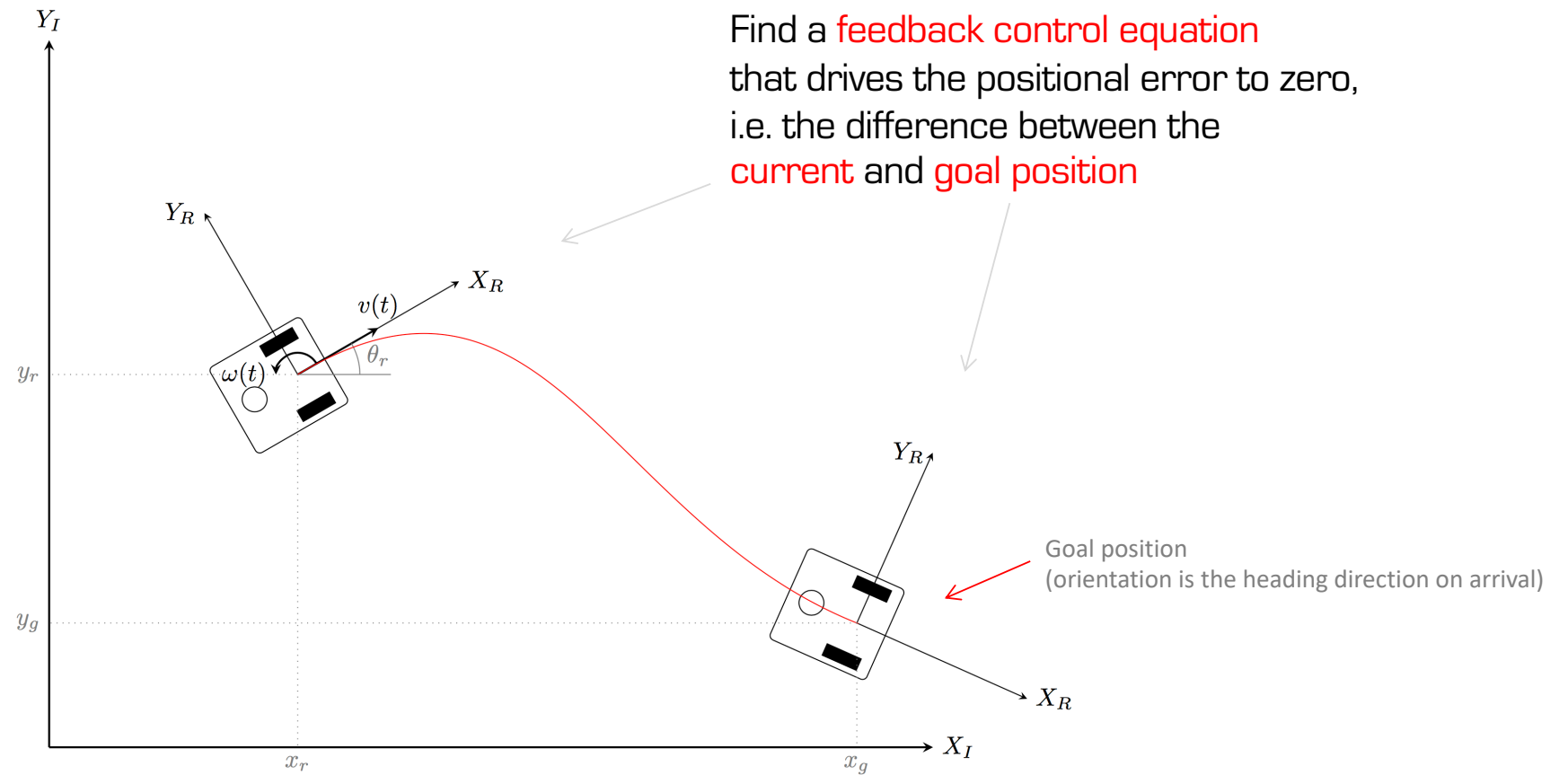
[https://en.wikipedia.org/wiki/PID\\_controller](https://en.wikipedia.org/wiki/PID_controller)

# PID Controller

## Take-home message

The key to effective PID control is to use the right gain values  
but  
identifying them is difficult

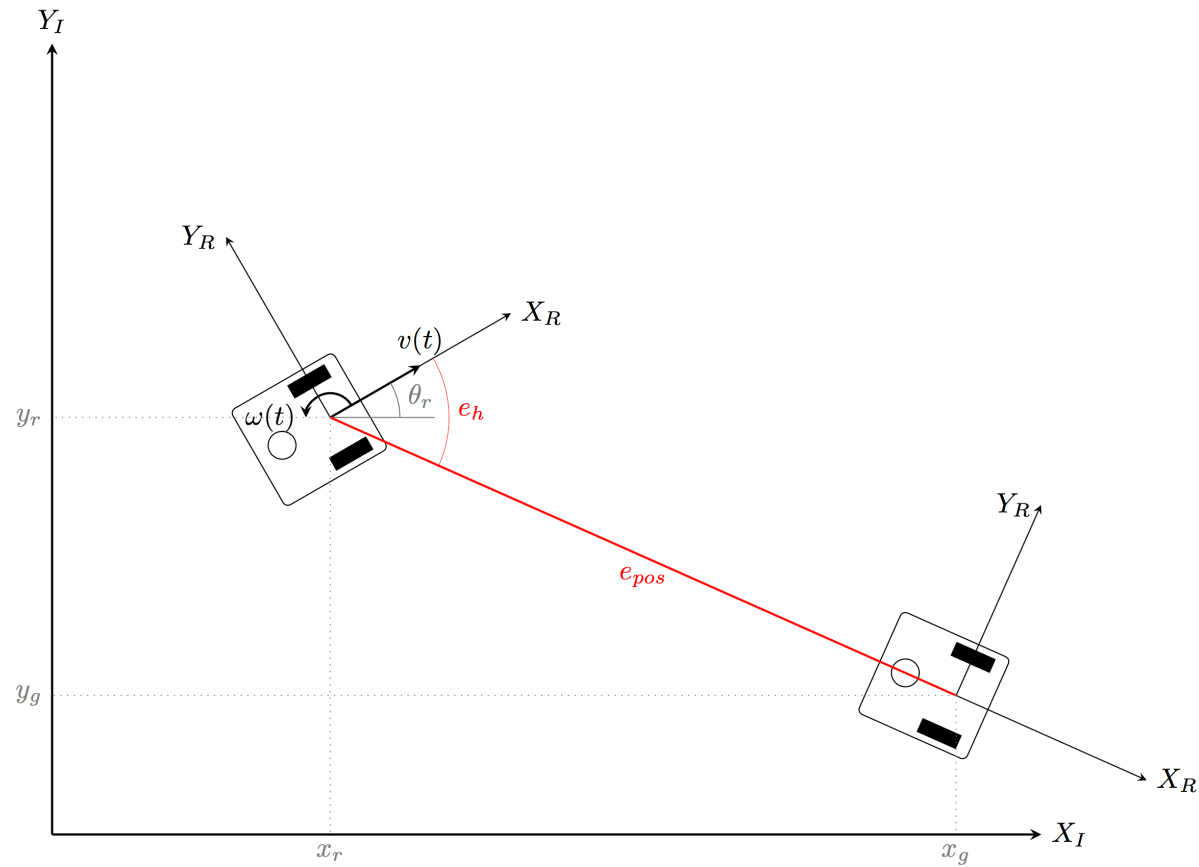
# The Go-to-Position Problem



# The Go-to-Position Problem

- The robot knows its own global current position
  - $(x_r, y_r, \theta_r)$
- It knows the global position of the goal
  - $(x_g, y_g)$
- Compute error in position and heading
  - $(e_{pos}, e_h)$

# The Go-to-Position Problem



# The Go-to-Position Problem

- The robot knows its own global current position
  - $(x_c, y_c, \theta_c)$
- It knows the global position of the goal
  - $(x_g, y_g)$
- Compute error in position and heading
  - $(e_{pos}, e_h)$
- Reduce both errors to zero
  - by generating the appropriate forward and angular velocities  $(v, \omega)$  or, alternatively,
  - by generating the appropriate angular velocities of the wheels,  $(v_R, v_L)$  i.e.  $(\dot{\phi}_1, \dot{\phi}_2)$



# Go-to-Position as a Control Problem

## Solution 1: Divide and Conquer

Decompose 2D problem into two 1D problems

- First, correct the heading:

rotate to reduce the orientation error to zero

- Second, correct the position:

translate straight ahead to reduce the position error to zero

# Go-to-Position as a Control Problem

## Solution 1: Divide and Conquer

Algorithm `goto1(xg, yg)` using proportional control

Global variables: the current robot position and orientation  $(x_r, y_r, \theta_r)$

Arguments:

- the goal position of the robot  $(x_g, y_g)$
- the proportional gains for controlling position and heading

$$K_p^{pos}$$

$$K_p^h$$

- the tolerances on position error  $\Delta_{pos}$  and orientation error  $\Delta_h$

# Go-to-Position as a Control Problem

## Solution 1: Divide and Conquer

Version A: Control forward and angular velocities of the robot

Do

Compute the current position of the robot  $(x_r, y_r)$

Compute the distances from the robot position to the target position  $(d_x, d_y)$

$$d_x = x_g - x_r$$

$$d_y = y_g - y_r$$

Compute the position and heading errors  $(e_{pos}, e_h)$

$$e_{pos} = \text{sqrt}(d_x^2 + d_y^2)$$

$$e_h = \text{atan2}(d_y, d_x) - \theta_r$$

The difference between the desired heading and the current robot orientation

If not oriented correctly, correct heading

Otherwise, correct position

if  $|e_h| > \Delta_h$

set forward velocity:  $v = 0$

set angular velocity:  $\omega = K_p^h e_h$

else

set forward velocity:  $v = K_p^{pos} e_{pos}$

set angular velocity:  $\omega = 0$

or some maximum velocity  $v_{max}$

Send velocities  $(v, \omega)$  to the robot

Pause some time

while  $|e_{pos}| > \Delta_{pos}$

Send velocities  $(0, 0)$  to the robot

# Go-to-Position as a Control Problem

## Solution 1: Divide and Conquer

Version B: Control angular velocities of the wheels

Do

Compute the current position of the robot  $(x_r, y_r)$

Compute the distances from the robot position to the target position  $(d_x, d_y)$

$$d_x = x_g - x_r$$

$$d_y = y_g - y_r$$

Compute the position and heading errors  $(e_{pos}, e_\theta)$

$$e_{pos} = \text{sqrt}(d_x^2 + d_y^2)$$

$$e_h = \text{atan2}(d_y, d_x) - \theta_r$$

If not oriented correctly,  
correct orientation

Otherwise,  
correct position

if  $|e_h| > \Delta_h$

set right wheel angular velocity:  $\dot{\phi}_1 = -K_p^h e_h$

set left wheel angular velocity:  $\dot{\phi}_2 = -\dot{\phi}_1$

Rotate: left and right wheels go in opposite direction

else

set right wheel angular velocity:  $\dot{\phi}_1 = K_p^{pos} e_{pos}$

set left wheel angular velocity:  $\dot{\phi}_2 = \dot{\phi}_1$

or some maximum velocity

Translate: left and right wheels go in same direction

Send velocities  $(\dot{\phi}_1, \dot{\phi}_2)$  to the robot

Pause some time

while  $|e_{pos}| > \Delta_{pos}$

Send velocities  $(0, 0)$  to the robot