Introduction to Cognitive Robotics

Module 4: Robot Manipulators

Lecture 6: kinematics and inverse kinematics of the LynxMotion AL5D arm

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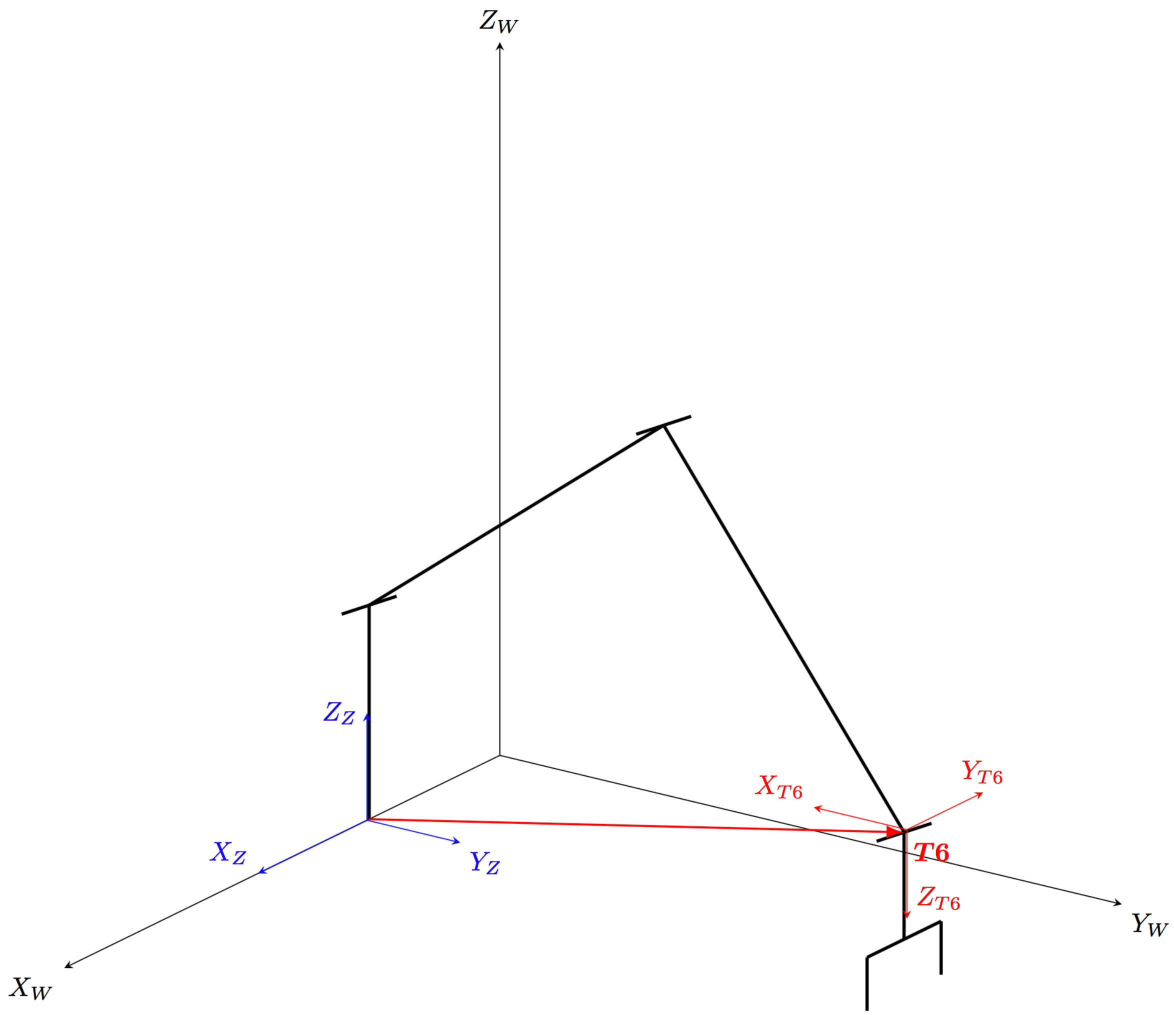


T6 The manipulator wrist

T6 The position of the end of the manipulator with respect to its base at ***Z***

The ***T6*** frame is a computable function of the joint variables

We need to define the relationship between ***Z*** and ***T6*** in terms of the joint angles …
i.e. we need to define the manipulator kinematics

Then we need to find the inverse kinematic solution the allows us to determine the joint values that correspond to a given ***T6***:

The joint variables are computed for a given ***T6*** frame

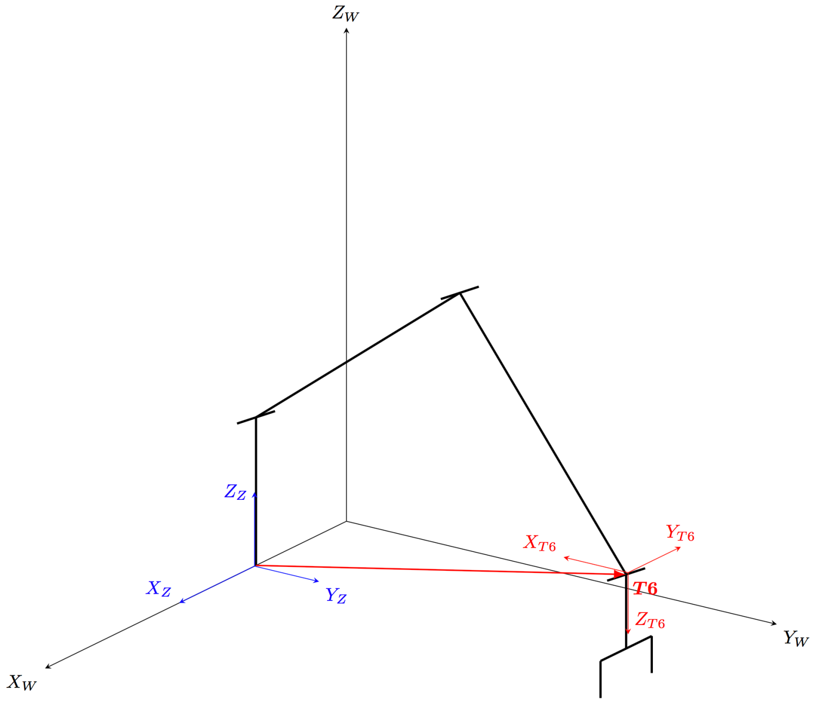
Recall that there is a convention that the ***T6*** frame should be embedded in the manipulator

- with the origin at the wrist

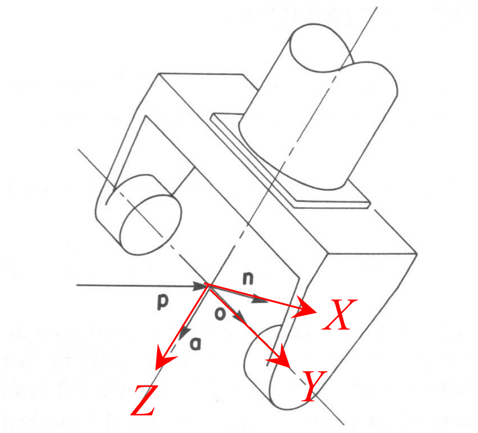
- with the Z axis directed outward from the wrist to the gripper

- with the Y axis directed in the plane of movement of the gripper when it is opening and closing

- with the X axis making up a right-hand system



The same convention applies to the ***E*** frame that is embedded in a two-finger gripper (end-effector … hence ***E***)



(Paul 1981)

Direction of X axis
Direction of Y axis
Direction of Z axis

$$E = \begin{bmatrix} n_x & o_x & a_x & p_x \\ n_y & o_y & a_y & p_y \\ n_z & o_z & a_z & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

n Normal
o Orientation
a Approach

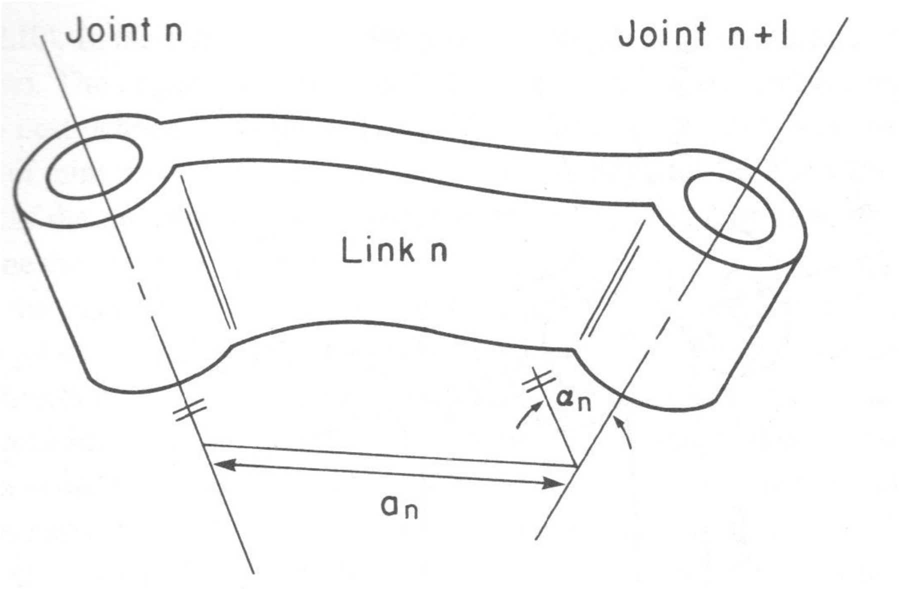
Denavit-Hartenberg Representation

$$T6 = {}^{Z}A_1 \; {}^{A1}A_2 \; {}^{A2}A_3 \; {}^{A3}A_4 \; {}^{A4}A_5 \; {}^{A5}A_6$$

$$T6 = A_1 \; A_2 \; A_3 \; A_4 \; A_5 \; A_6$$

${}^{Z}A_1$ position and orientation of link 1 w.r.t. the base
${}^{A1}A_2$ position and orientation of link 2 w.r.t. A_1
${}^{A2}A_3$
${}^{A3}A_4$
${}^{A4}A_5$
${}^{A5}A_6$ position and orientation of link 6 w.r.t. A_5

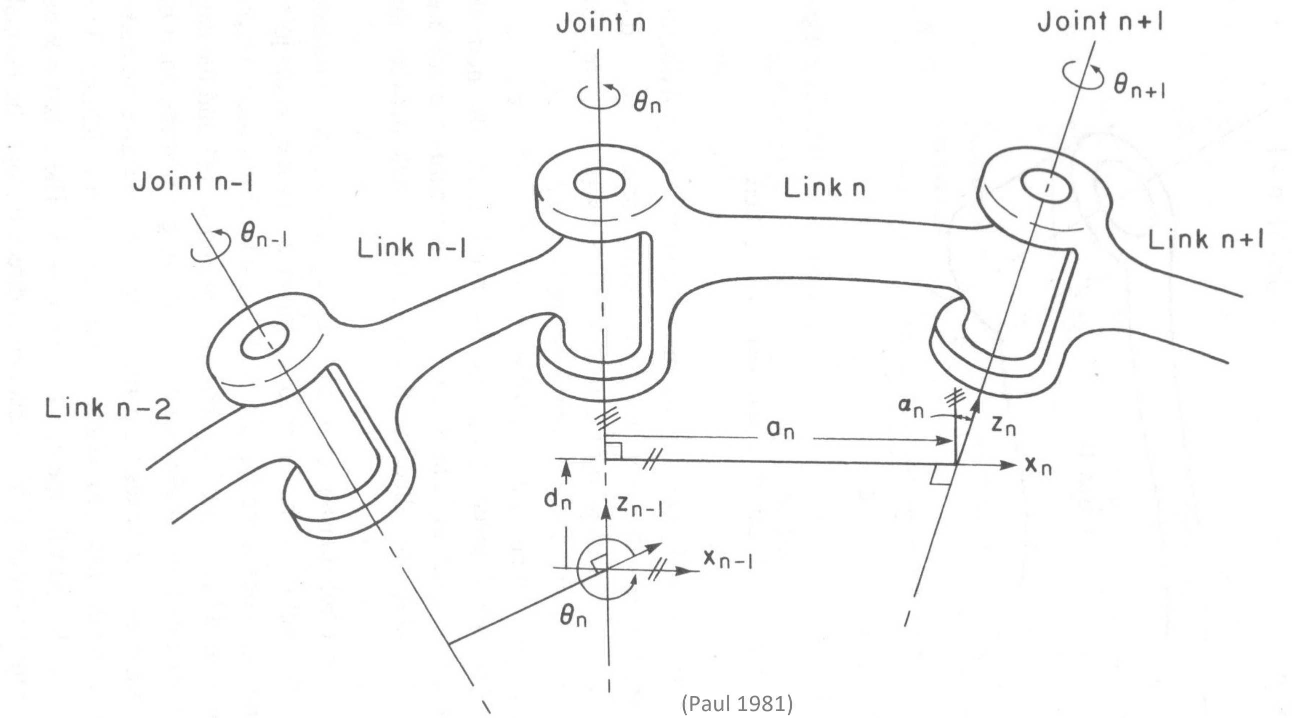
Denavit-Hartenberg Representation



(Paul 1981)

Link **length** (common normal distance) a_n

Link **twist** (angle between the two axes) α_n



(Paul 1981)

A link axis has two normals to it, one for each link

d_n the distance between the normals along the joint n axis
θ_n the angle between the normals measured in a plane normal to the axis

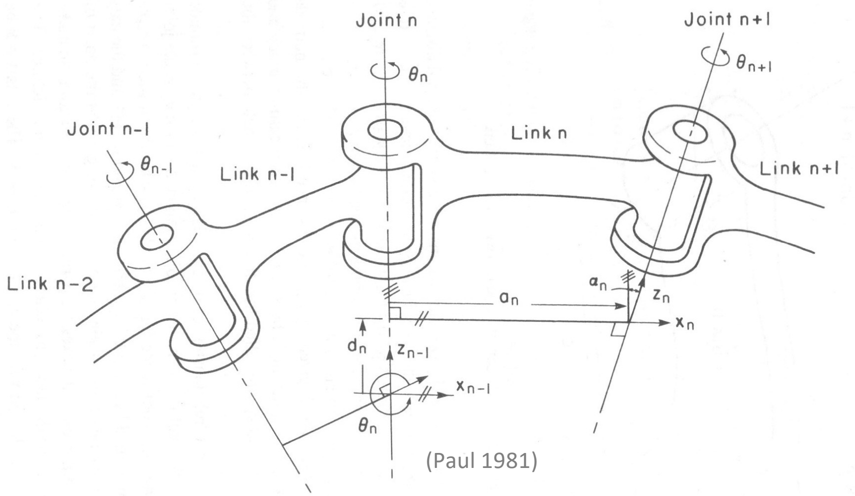
Denavit-Hartenberg Representation

- Assign coordinate frames to each link
- Determine the transformation from link n-1 to link n in terms of the four link parameters a_n α_n d_n θ_n

Denavit-Hartenberg Representation

Assign coordinate frames to each link

- The origin of each coordinate frame of link n is set to be at the intersection of the common normal between the axes of joints n and $n+1$ and the axis of joint $n+1$

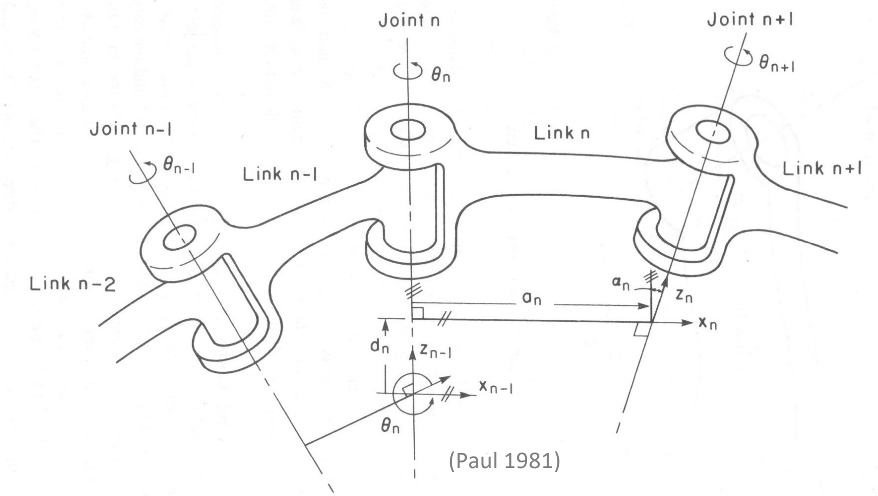


(Paul 1981)

Denavit-Hartenberg Representation

Assign coordinate frames to each link

- In the case of intersecting joint axes, the origin is at the point of intersection of the joint axes

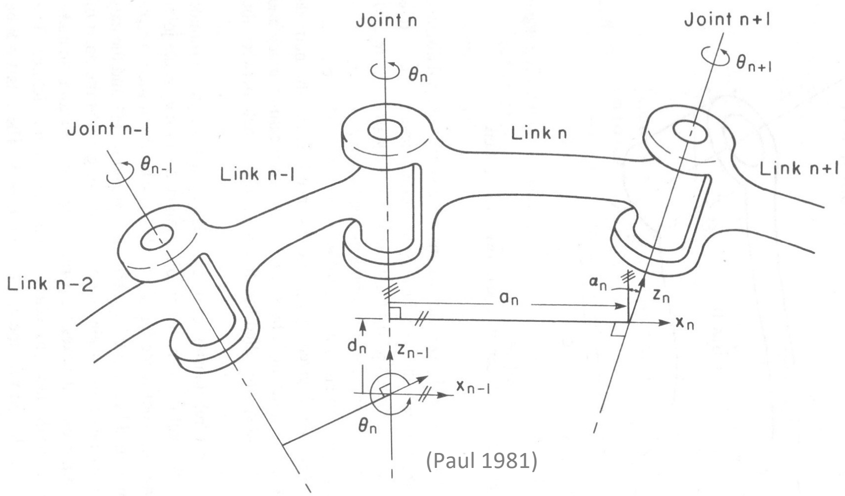


(Paul 1981)

Denavit-Hartenberg Representation

Assign coordinate frames to each link

- In the case of parallel joint axes, the origin is chosen to make the joint distance zero for the next link whose coordinate origin is defined

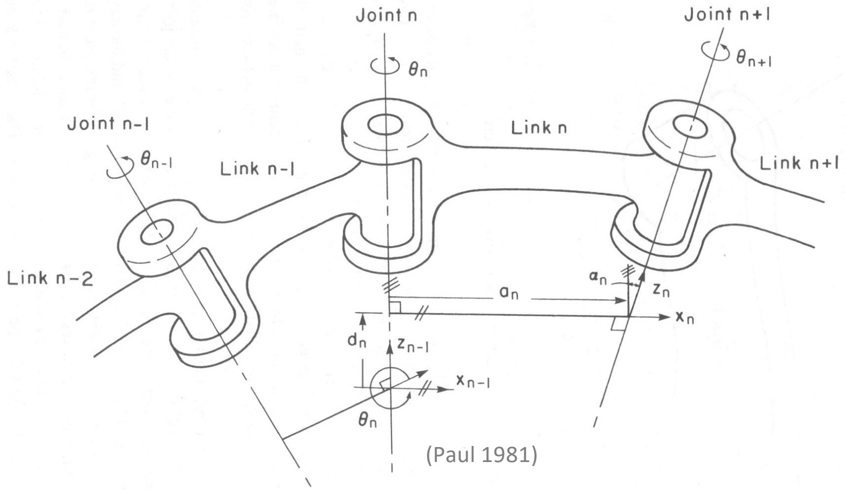


(Paul 1981)

Denavit-Hartenberg Representation

Assign coordinate frames to each link

- The Z axis for link n will be aligned with the axis of joint $n+1$

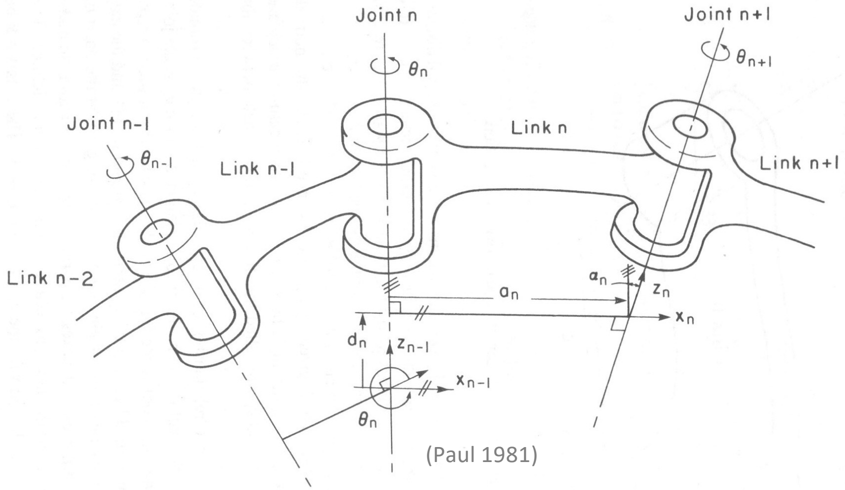


(Paul 1981)

Denavit-Hartenberg Representation

Assign coordinate frames to each link

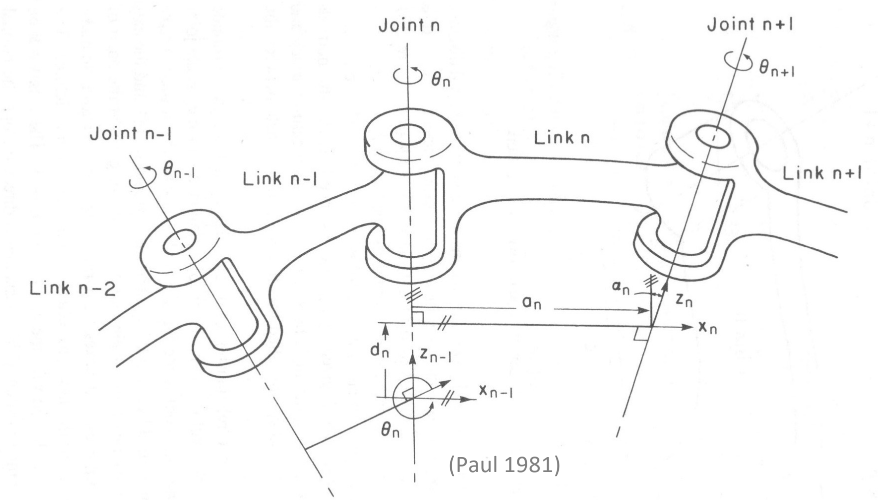
– The X axis will be aligned with any common normal which exists and is directed along the normal from joint to joint $n+1$



(Paul 1981)

Denavit-Hartenberg Representation

- The base is considered to be link 0
- Link 1 is joined to the base (link 0) by joint 1



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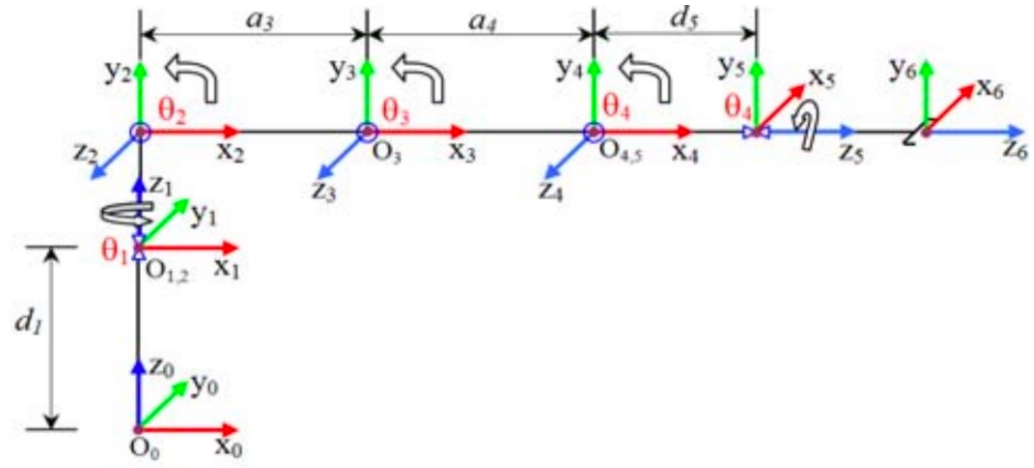

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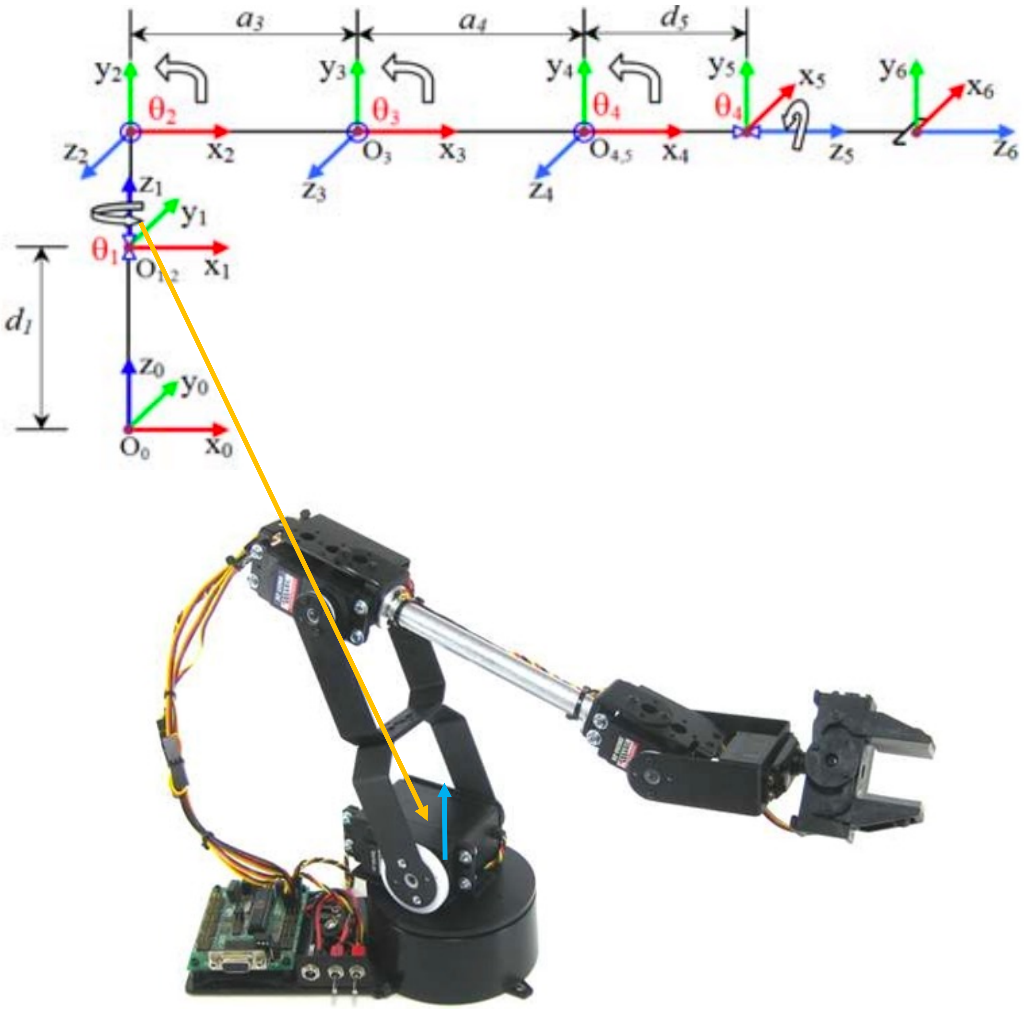

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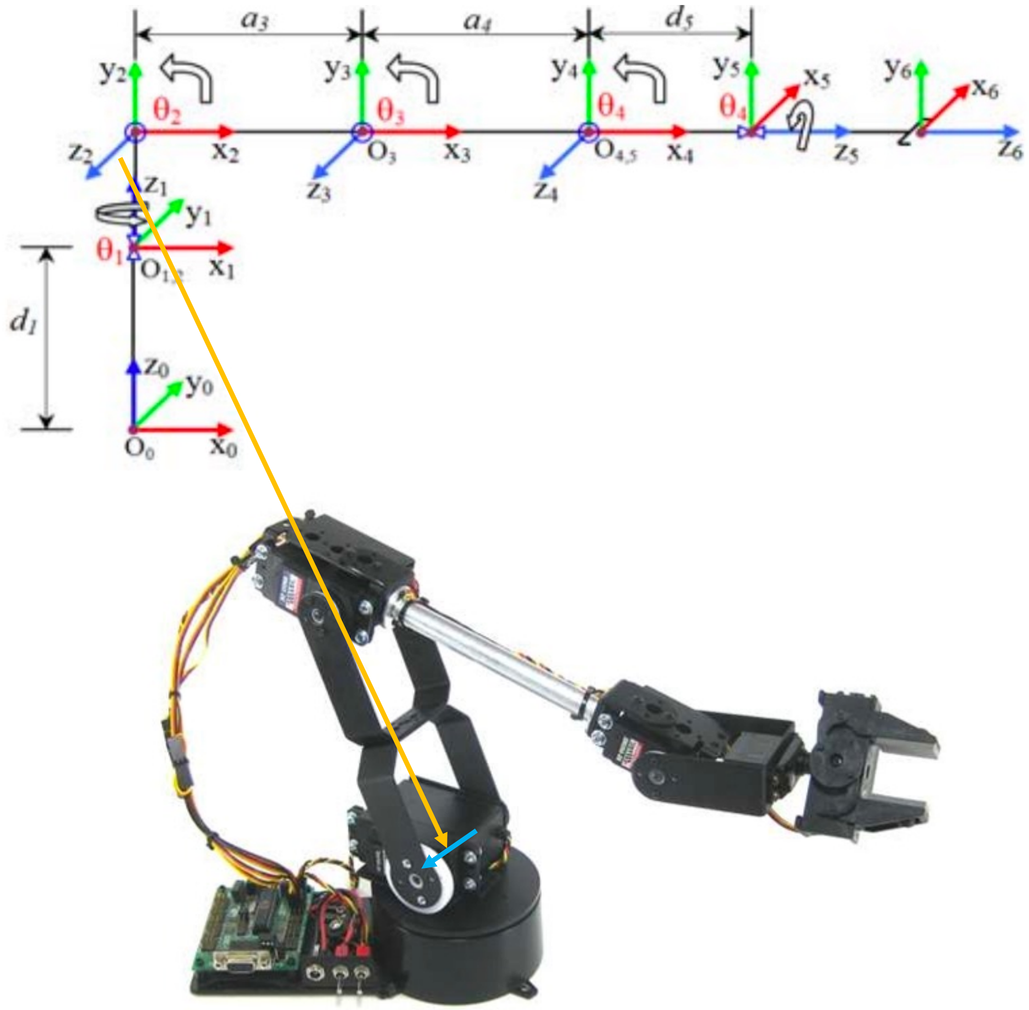

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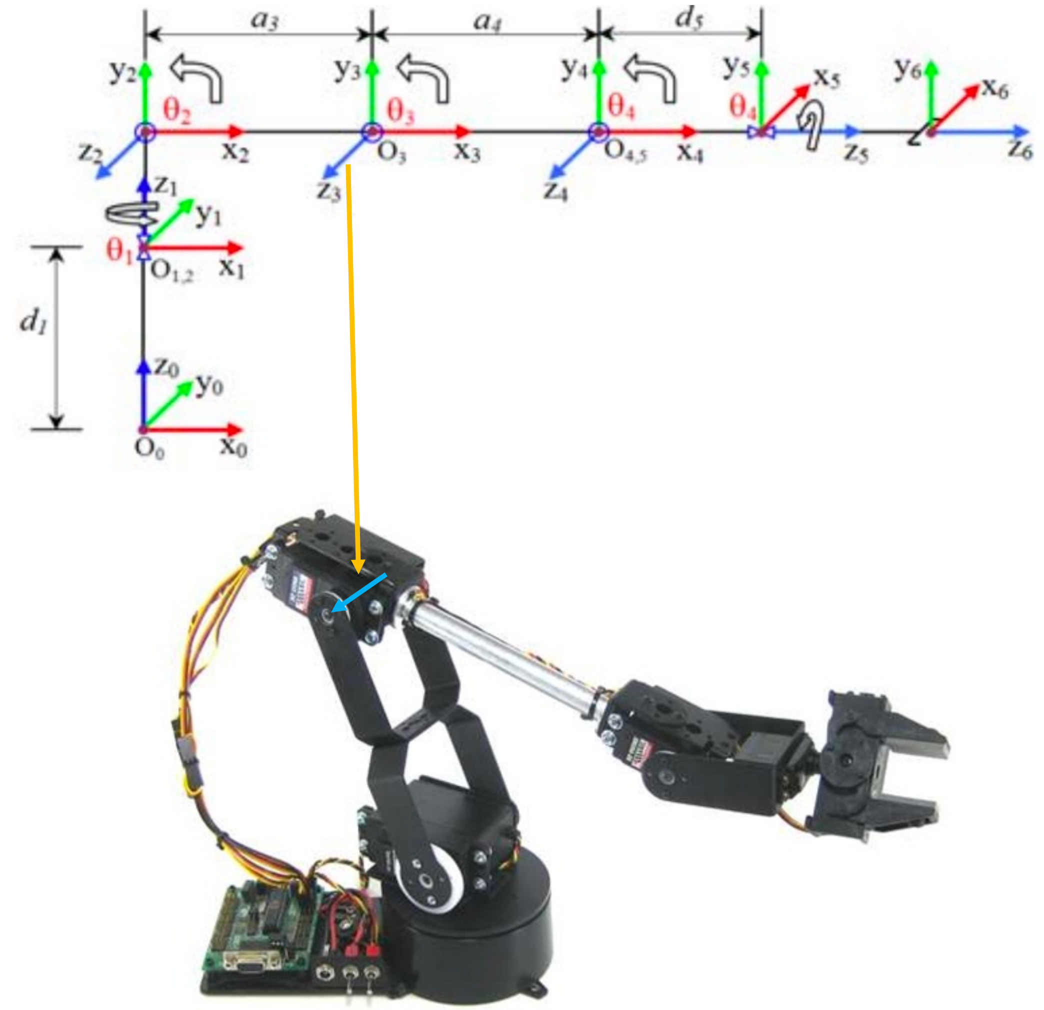

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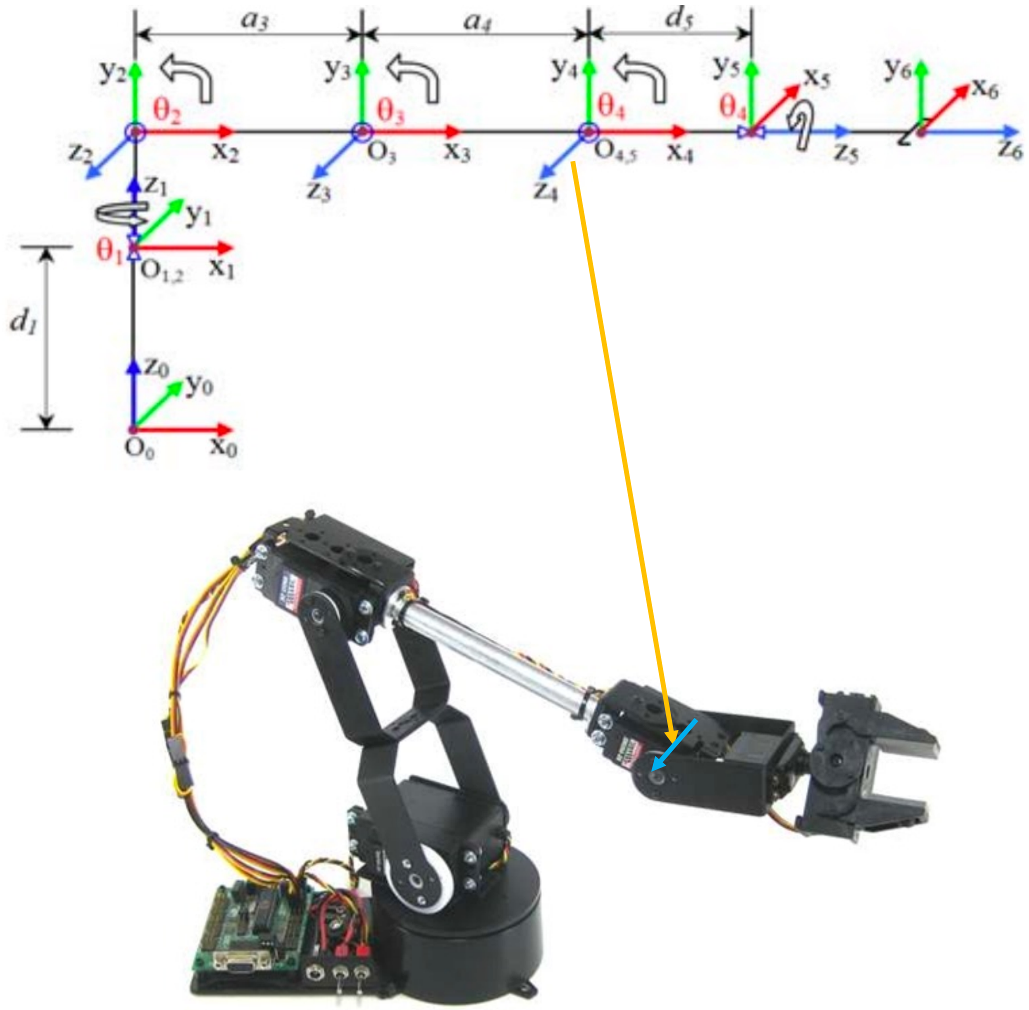

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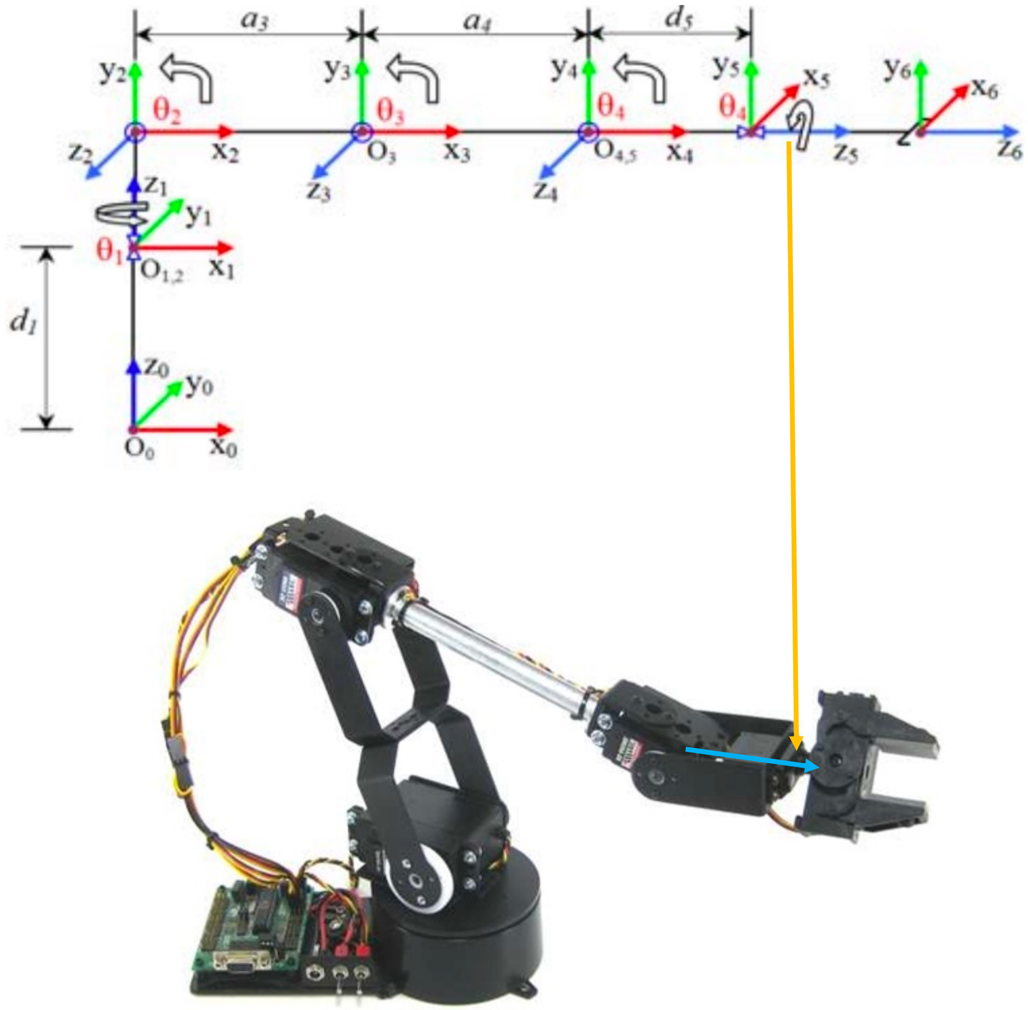

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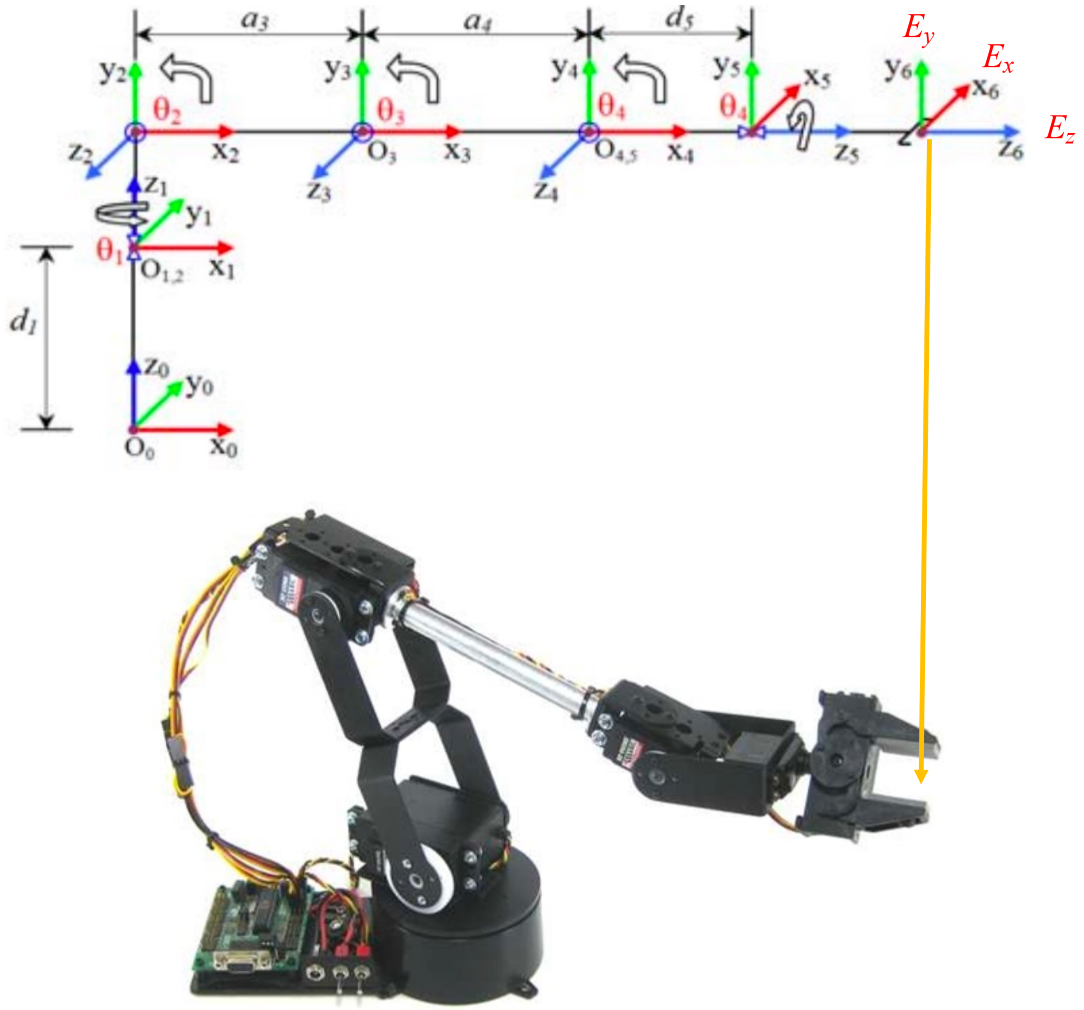

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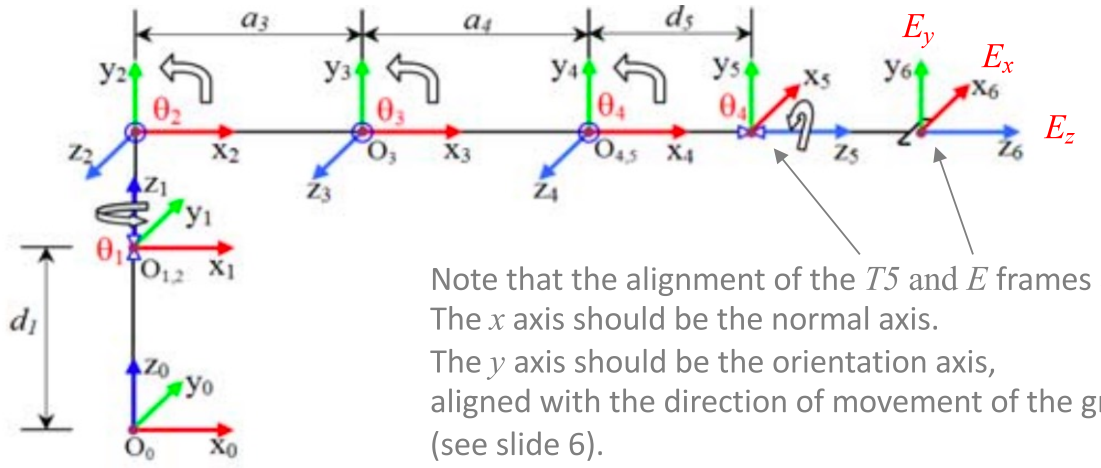

Note that the alignment of the $T5$ and E frames are incorrect.
The x axis should be the normal axis.
The y axis should be the orientation axis,
aligned with the direction of movement of the gripper fingers
(see slide 6).

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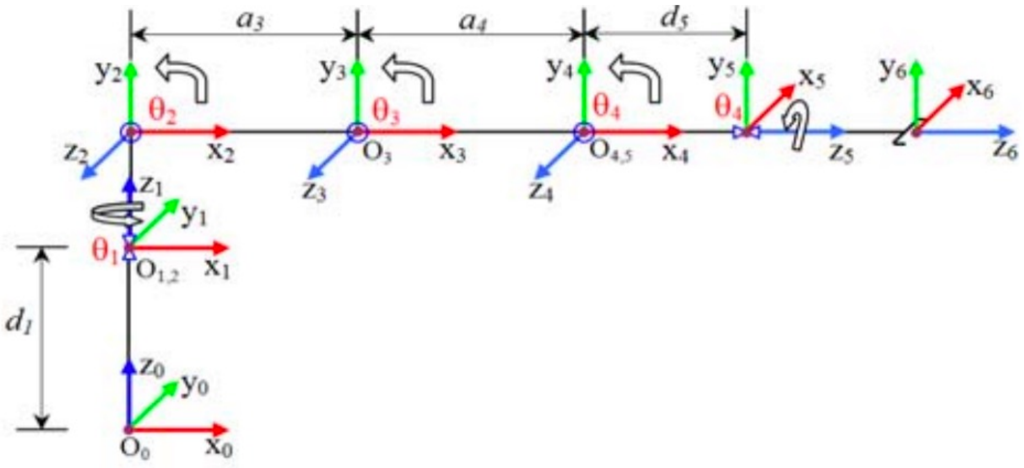

i	α_{i-1}	a_{i-1}	d_i	θ_i
1	0	0	d_1	θ_1^*
2	90	0	0	θ_2^*
3	0	a_3	0	θ_3^*
4	0	a_4	0	$\theta_4 -90^*$
5	-90	0	d_5	θ_5^*
6	0	0	0	Gripper

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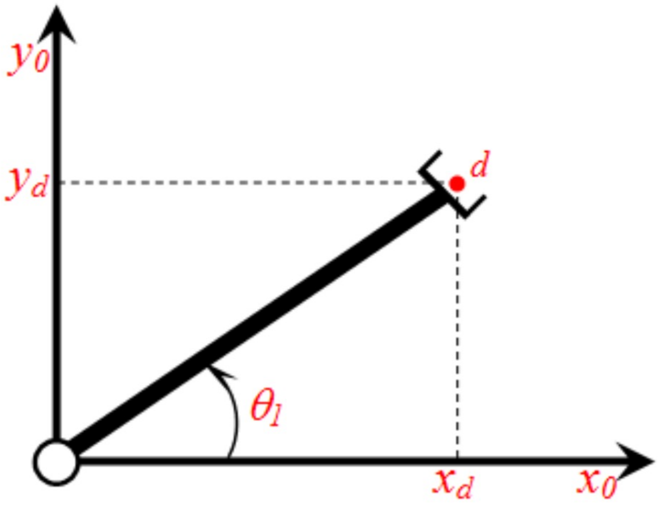

$$\theta_1 = \text{Atan2}(y, x)$$

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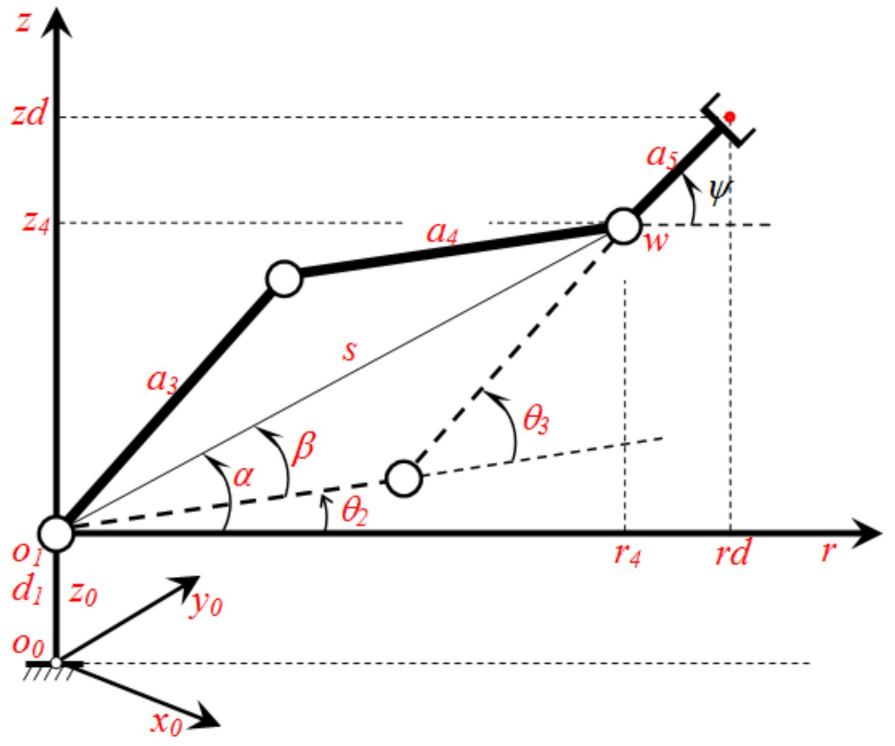

$$r_4 = a_3 \cos(\theta_2) + a_4 \cos(\theta_2 + \theta_3)$$

$$z_4 = a_3 \sin(\theta_2) + a_4 \sin(\theta_2 + \theta_3) + d_1$$

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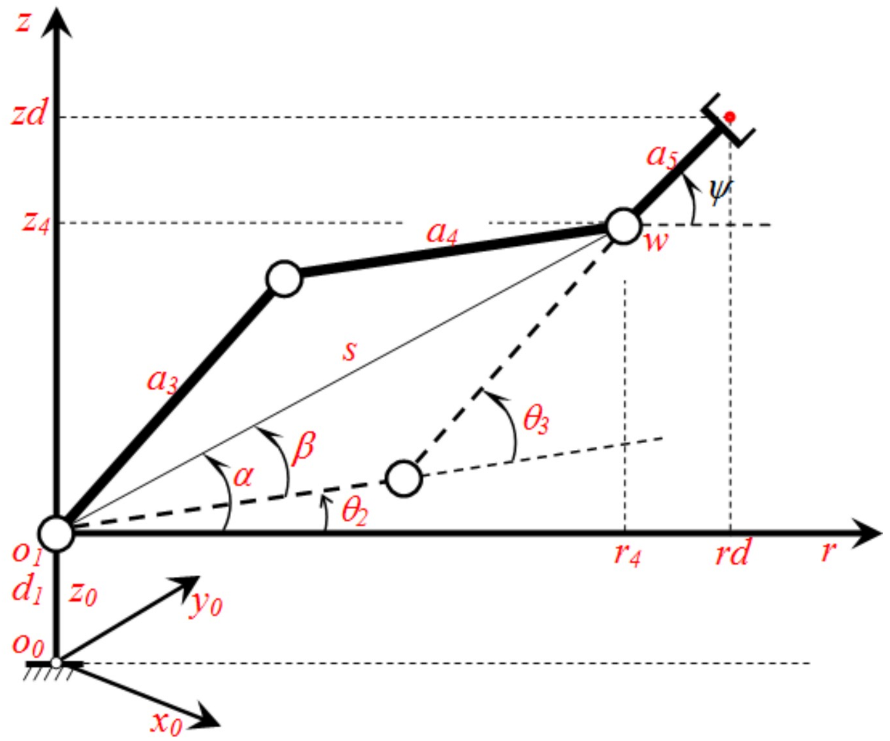

$$\beta = A\tan 2(s^2 + a_3^2 - a_4^2, 2sa_3)$$

$$\alpha = A\tan 2(z_4 - d_1, r_4)$$

$$s = \sqrt{(z_4 - d_1)^2 + r_4^2}$$

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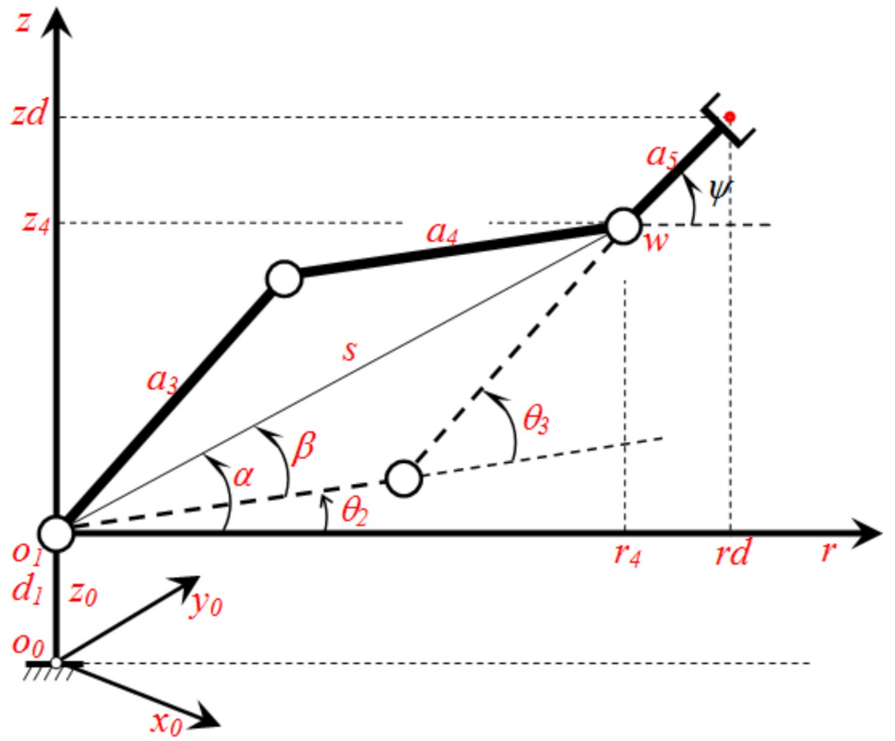

$$\theta_2 = \alpha \pm \beta$$

$$\theta_3 = A\tan 2(s^2 - a_3^2 - a_4^2, 2a_3a_4)$$

$$\theta_4 = \psi - \theta_2 - \theta_3$$

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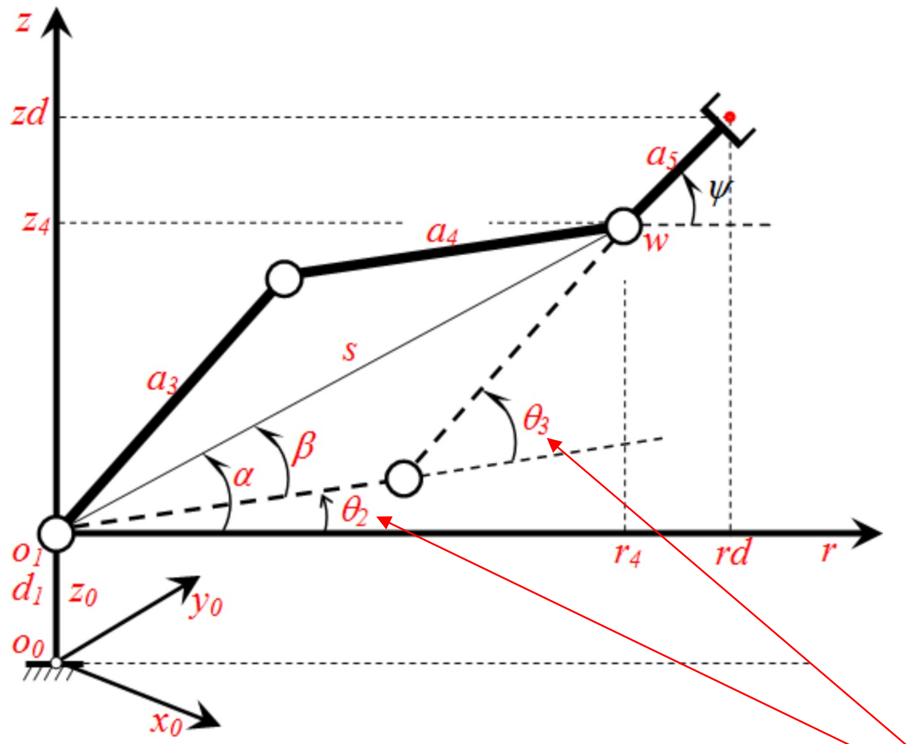

$$\theta_2 = \alpha \pm \beta$$

$$\theta_3 = A\tan 2(s^2 - a_3^2 - a_4^2, 2a_3 a_4)$$

$$\theta_4 = \psi - \theta_2 - \theta_3$$

This equation for θ_3 is valid for the case where $\theta_2 = \alpha - \beta$ i.e. the case shown in the diagram

It should be negated for the case where $\theta_2 = \alpha + \beta$ i.e. the solution used in the inverse kinematics in the code provided in the accompanying software

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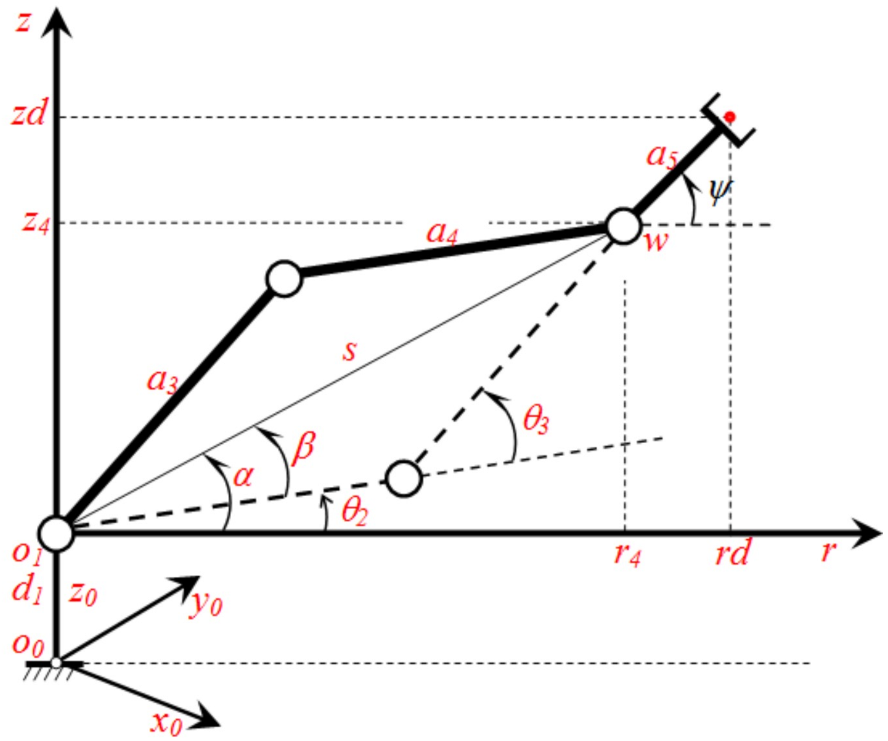

$$\theta_2 = \alpha \pm \beta$$

$$\theta_3 = A\tan 2(s^2 - a_3^2 - a_4^2, 2a_3a_4)$$

$$\theta_4 = \psi - \theta_2 - \theta_3$$

Adjust θ_4 by +90 degrees to if you want to have the T5 z axis aligned with the base frame of reference z axis when T5 is a pure translation

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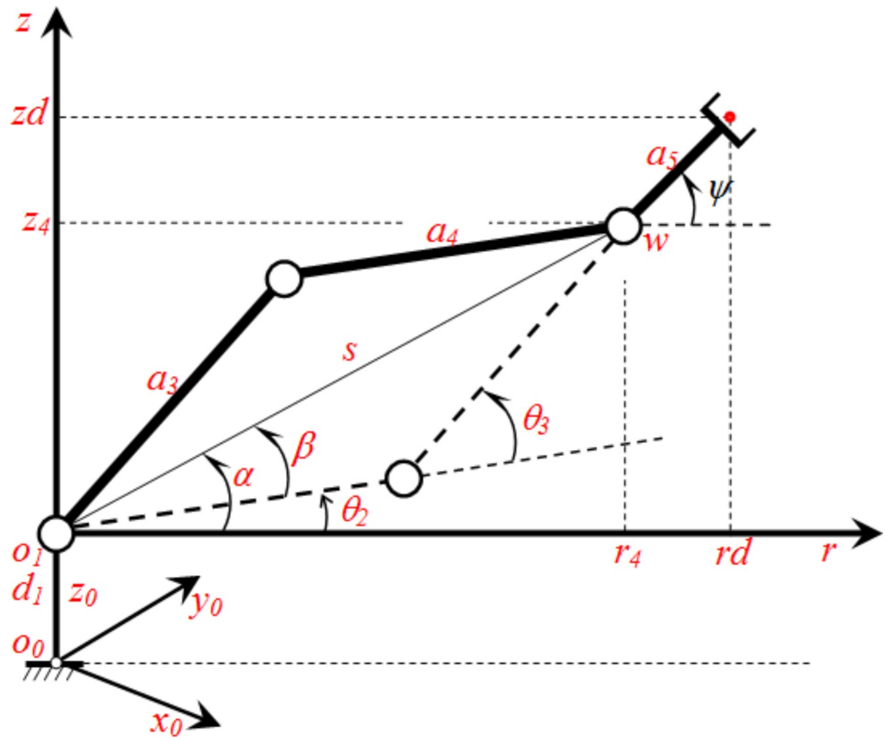

There is no expression for θ
It is assumed that it is given,
and used directly.

Adjust θ by +90 degrees to if you want to have the T5 z axis aligned with the base frame of reference z axis when T5 is a pure translation

$$\theta_2 = \alpha \pm \beta$$

$$\theta_3 = A\tan 2(s^2 - a_3^2 - a_4^2, 2a_3 a_4)$$

$$\theta_4 = \psi - \theta_2 - \theta_3$$

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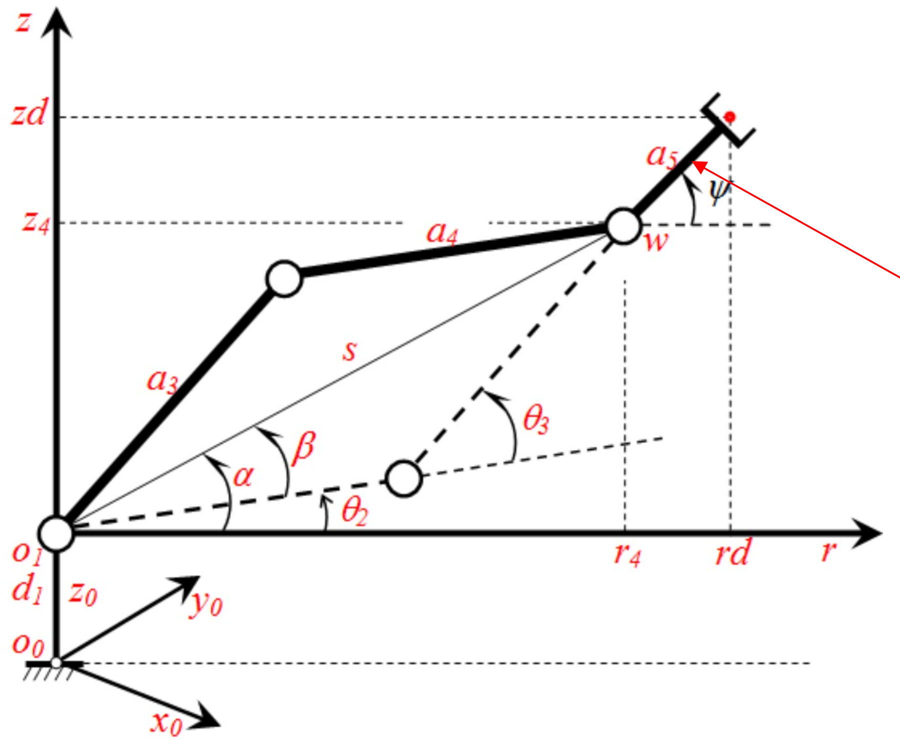

d_5 is not used in this solution (the a_5 is a typo in the figure).
It is effectively integrated into the end-effector frame E so that the origin of $T5$ is in the wrist, where it should be, and not in the gripper as shown.

$$\theta_2 = \alpha \pm \beta$$

$$\theta_3 = A\tan 2(s^2 - a_3^2 - a_4^2, 2a_3a_4)$$

$$\theta_4 = \psi - \theta_2 - \theta_3$$

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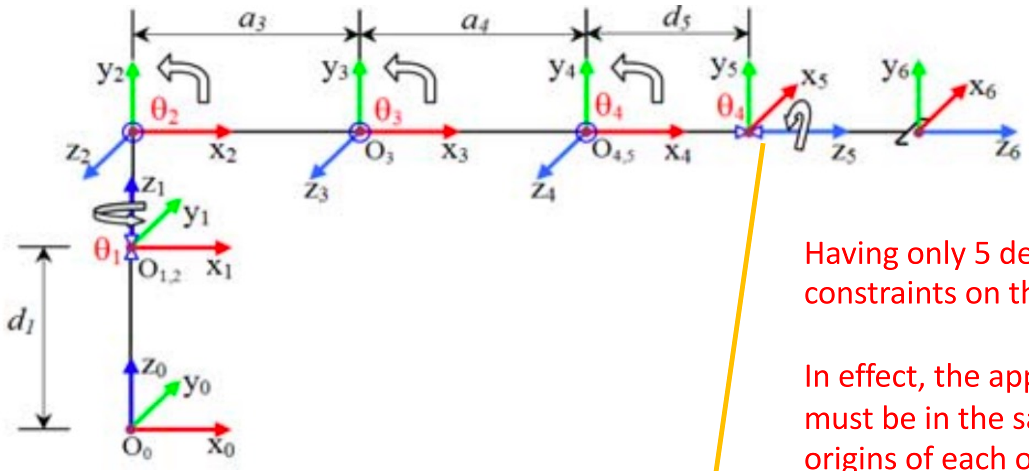

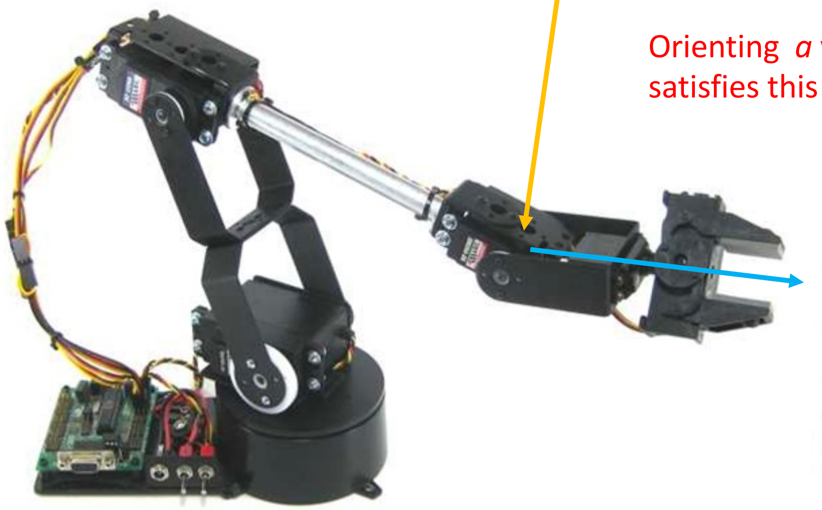

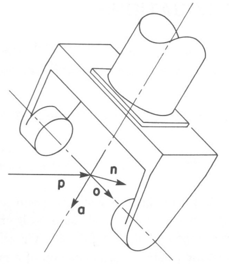

Having only 5 degrees of freedom imposes constraints on the achievable pose.

In effect, the approach vector *a* of the T5 frame must be in the same plane as that formed by the origins of each of the manipulator links.

Orienting *a* vertically upwards or downwards satisfies this requirement.

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