

**NON-LINEAR SPATIAL WARPING APPLIED TO THE GENERATION  
OF POST-PERFUSION NUCLEAR MEDICINE AEROSOL  
VENTILATION IMAGES.**

**by**

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The use of  $^{99m}\text{Tc}$ -DTPA aerosols to produce a post-perfusion ventilation image involves the computer subtraction of the perfusion image from a composite image of the perfusion and aerosol ventilation. This requires prior registration of the images in order to avoid artifacts introduced by patient movements between acquisition of the images. The application of a non-linear spatial warping technique to the registration of the lung scans is described. This warping technique models the required geometric mapping between the images using second order polynomial functions of image coordinate. The resultant image grey-level intensity is estimated by bi-linear interpolation. Results of this approach to registration are also described.

### **Introduction.**

Nuclear medicine perfusion and ventilation lung scans are routinely used in the Radiology Department to diagnose the presence of a pulmonary embolism. To confirm the diagnosis a ventilation scan is usually performed. Mismatches between the ventilation and perfusion images confirm a high probability of pulmonary embolism. There are considerable advantages to the use of  $\text{Tc-}^{99\text{m}}$  D.T.P.A. aerosols over the Krypton and Xenon gases as the ventilation agent (Short 1979).

However, a valid image of the isotope distribution by ventilation alone can only be obtained if the perfusion and composite (ventilation + perfusion) Aerosol images occupy the same spatial coordinates. This condition is rarely fulfilled in practice and subtraction artifacts are introduced in the ventilation image as a result of patient movement (Fig. 1).

Orthogonal shifting routines (Dowsett and Perry 1970, Appledorn et al 1980) only partially alleviate the situation. The addition of an image (Fleming 1984, Barber 1980) rotation facility adds greater flexibility to the realignment process but requires the use of marker sources attached to the patient. Otherwise, correct choice of angle and centre of rotation may be difficult. Movements encountered in practice tend to be more complex than just orthogonal, rotational or a combination of both.

In this approach to registration a mathematical model for the spatial relationship between the isotope distribution in the two images is obtained, and its inverse mapping applied to one of these images. Subtraction of the registered perfusion image from the aerosol image yields a better approximation to the ventilation distribution of radionuclide.

#### **Description of the Registration Algorithm.**

Registration of the two images requires that one image; (A) in (Fig. 2) receives a non-linear spatial mapping so that every point in that image superimposes, i.e., maps to its corresponding point in (B). Such a plane-to-plane non-linear mapping is normally referred to as a spatial warping function which, for any given point in the registered image (B), generates the coordinates of the corresponding point in the unregistered image (A).

This relationship is expressed by the following equation:

$$(q_x, q_y) = \{W_x (r_x, r_y), W_y (r_x, r_y)\} \dots\dots\dots(1)$$

where  $(r_x, r_y)$  is a registered pixel in the warped version of image (A) [corresponding positionally to the image (B)] and  $(q_x, q_y)$  is the corresponding unregistered pixel in image (A), (Fig. 2).

Thus, given any registered image point  $(r_x, r_y)$  in image (B), the unregistered coordinates  $q_x$  and  $q_y$  in the image (A) can be generated using the warping functions  $W_x$  and  $W_y$ , respectively.

Since analytic expressions for  $W_x$  and  $W_y$  will rarely be known, a common approach is to model each spatial warping function by an  $n$ th order polynomial in  $r_x$  and  $r_y$  (Pratt 1978) or by polynomials of the form (Hall 1978):

$$\sum_{j=0}^n \sum_{i=0}^n a_{ij} r_x^i r_y^j \dots \dots \dots (2)$$

The system described in this paper uses polynomials of the second order. Now,  $W_x$  and  $W_y$  can be written:

$$W_x(r_x, r_y) = \sum_{i=0}^n \sum_{j=0}^n a_{ij} r_x^i r_y^j \dots \dots \dots (3)$$

$$W_y(r_x, r_y) = \sum_{i=0}^n \sum_{j=0}^n b_{ij} r_x^i r_y^j \dots \dots \dots (4)$$

and, in order to model the (geometric) spatial warping, it is necessary merely to estimate the coefficients,  $a_{ij}$   $b_{ij}$  of each polynomial. If  $n = 2$ , there are nine coefficients for each polynomial; these can be determined by actually specifying the spatial warping for (at least) nine points in the image to be registered. This is accomplished by selecting pairs of corresponding "control" points within each image (Fig. 2).

For each control point pair:

$$\{(q_x, q_y), (r_x, r_y)\}$$

one can form two equations:

$$q_x = \sum_{i=1}^n \sum_{j=1}^n a_{ij} r_x^i r_y^j \dots\dots\dots(5)$$

$$q_y = \sum_{i=1}^n \sum_{j=1}^n b_{ij} r_x^i r_y^j \dots\dots\dots(6)$$

Nine such control point pairs facilitates the construction of nine equations in  $a_{ij}$  and nine equations in  $b_{ij}$  which can then be solved simultaneously to determine the coefficients of the spatial warping polynomial functions.

The solution of an exact system of equations such as this is very sensitive to the choice of control point, however, and inappropriate choices can yield a poor model of the required spatial warping. To overcome this problem, more than nine control points can be used. This produces an overdetermined system of equations which can be solved by obtaining a least square error solution. For equation (5) in matrix notation,

$$Q = AR \dots\dots\dots(7)$$

Q is the observation coordinate vector, A the coefficient vector and R is the independent variable matrix.

In its overdetermined form:

$$Q = AR + E \dots\dots\dots(8)$$

where the vector E represents the error between the observation

coordinates and those obtained by polynomial estimation. This system is solved for  $A$  subject to the condition that  $E^2$  squared be a minimum (Ballard and Brown 1982), i.e.,

$$A = (R^T R)^{-1} R^T Q \dots\dots\dots (9).$$

Similarly for  $B$ , where  $R^T$  represents the transpose of  $R$ .

It has been found empirically that sixteen control points yield an adequate model. These points are normally taken from the periphery of each lung at similar positions anatomically and, presently, these control points are identified manually in each of the two images using a joystick-guided cursor. To help ensure that the spatial transformation is valid for the entire image, these control points should, as far as possible, be uniformly distributed around the edges of the lungs.

The final registered version of the perfusion image is generated by "pixel-filling", i.e., by computing each pixel in the (new) registered image, column by column, row by row. This is accomplished by applying the warping function to the current coordinates in the (new) registered image, determining the (non-integer) coordinates of the corresponding points in the unregistered image and estimating the grey-level intensity by interpolation.

The simplest approach, zero order interpolation, merely chooses the grey-scale value of the nearest neighbouring pixel. This type of interpolation, however, yields images which tend to be "blocky". This method produces reasonable results considering the nature of the images used. First order, bilinear interpolation (Rosenfeld and Kak 1983, Castleman 1978) used in this implementation produces more desirable

results with a slight increase in programming complexity and execution time. In this manner, the new registered image is generated by filling in each pixel in turn.

### **Results and Discussion.**

The application of this warping technique to post-perfusion aerosol ventilation has proved successful and the technique is in current clinical use. Fig. 3 demonstrates the improvement obtained on the phantom lungs after applying spatial warping. Fig. 5 shows the results obtained on clinical data.

Low order polynomials have proved sufficient to cater for the types of misregistration introduced by patient movements between images. The validity of this technique rests, of course, on the selection of representative control points. This dependency is alleviated somewhat by over-determining the specification of the geometric transformation through the use of more control points than is strictly necessary, sixteen per image in this case. This is particularly useful in situations where gross differences can exist between the images. In many situations gross differences between the perfusion and aerosol composite images may be sufficient to confirm the diagnosis of pulmonary embolism and image subtraction may not be required.

The generation of the registered perfusion image, using zero and first order interpolation, takes 3 and 3 1/2 minutes respectively on a Nova IV computer without a floating-point processor.

Current research is directed towards automatic selection of the registration control points, in the first instance by analysis of critical points (i.e., points of zero and maximum curvature) on the lung edges.



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